The 10th International Conference on Technology in Mathematics Teaching
Enhancing Mathematics Education Through Technology

University of Portsmouth
5–8th July 2011

Conference Proceedings
Edited by
Marie Joubert, Alison Clark-Wilson and Michael McCabe
ICTMT conferences

The biennial ICTMT conference is a legacy of Professor Bert Waits’ (Ohio State University) commitment to promote technology in mathematics education. The inaugural conference was held in Birmingham, UK in 1993. The ICTMT conferences aim to bring together lecturers, teachers, educators, curriculum designers, mathematics education researchers, learning technologists and educational software designers, who share an interest in improving the quality of teaching and learning by effective use of technology. It provides a forum for researchers and practitioners in this field to discuss and share better practices, theoretical know-how, innovation and perspectives on educative technologies and their impact on the teaching and learning of mathematics.

ICTMT10: Enhancing Mathematics Education Through Technology

The Tenth International Conference on Technology in Mathematics Teaching took place from Tuesday July 5th to Friday July 8th 2011 at the University of Portsmouth on the South coast of England.

There were 162 registered delegates from 27 different countries and 20 University of Portsmouth student ambassadors took part in the academic proceedings. Over 300 people attended the Bloodhound talk given by Richard Noble.

Thanks go to the local organising committee, and in particular to Alison White for her detailed planning, and to Stephen Webb and Rosemary Shearer for the excellent editing and layout work they did on the abstracts.

Photographs of the conference can be found at
https://picasaweb.google.com/106154666197426987568/ICTMT10#
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Professor Paul Drijvers (Utrecht University)
Professor Colette Laborde (University of Grenoble)
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These proceedings include
Part A: the abstracts of scheduled sessions
Part B: papers submitted after peer review and conference feedback
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Part A Abstracts
Abstracts: papers

Mathematics and technology: exploring teacher educators’ professional development
Maha Abboud-Blanchard, Université de Cergy-Pontoise, France

This paper reports on research which tackles the issue of how designing resources with digital technologies shapes the professional development of secondary mathematics teachers when preparing to become teacher educators. Drawing on previous research works that acknowledge the role of selecting and managing available resources for facilitating the use of technologies in mathematics teaching, I intend to go further in this direction and explore the role of designing such resources in teacher education. I therefore developed an educators’ training course taking into account, on the one hand, their own experiences/habits as secondary teachers in the use of digital technologies, and on the other, the potential difficulties in the integration of technological tools into mathematics learning and teaching identified by the research in this domain. A principal task within this course was first to choose a technological tool, to use it with students and to observe and analyse this technology-based lesson. Secondly, the student educator had to design a resource based on this experience and reflect on its eventual use in teacher education. This paper also discusses how this task was accomplished and proposes an analysis of the resources designed. It also considers the impact of this work on the development of teachers’ representations and alternative teaching strategies and on their understanding of what could be taught in teacher education programmes, and how teachers can be supported in implementing technology resources into their teaching practices.

How prospective teachers’ use of the Cabri II environment can have an effect on the posing of fractal problems
Reda Abu-Elwan, Sultan Qaboos University, Oman

The use of Cabri II in mathematics education has spread considerably in the last few years. Nevertheless, many teachers have not yet completely overcome their fears and suspicions about using it in geometry teaching. Furthermore, there is a generational gap that raises further difficulties: while most of the in-service teachers in Oman have little confidence in the new technologies nor with their application in educational activities, the younger teachers are more acquainted with these tools and appreciate their potential in geometry teaching. Fractal images are very beautiful and stunning. Fractal geometry presents a different form of geometry to that featured in classical geometry. However, it has failed to model and formulise many of the ordinary events and shapes surrounding us.

This study investigates how Cabri II might be effective in developing prospective teachers’ skills in developing new fractal problems for school-level geometry. The research was designed to introduce prospective mathematics teachers into a learning experience with a dynamic geometry environment (Cabri II), making them work in small groups (four students in each group) on developing fractal problems based on ‘Cabri II’ dynamic geometry.

Twenty-five prospective mathematics teachers participated in six activity sessions in topics including circle, triangle, fractal and other problem posing. The experience showed that the participants, after getting used to Cabri II, were able to apply their competence in the construction of interactive educational materials for a classroom situation. Moreover, they could focus on their (technical, mathematical and educational) difficulties and develop some critical reflections and new fractal problems. All of the creative fractal shapes in the proposed problems and images were developed by the participants, for that purpose, using Cabri II.
Cognitive development of pre-service teachers: misconception, cognitive conflict and conceptual understanding

Muhtarrem Aktümen, Ahi Evran University, Kirsehir, Turkey
Zekeriya Karadag, Tufts University, MA, USA
Tolga Kabaca, Pamukkale University, Denizli, Turkey

This paper reports on research investigating a group of pre-service teachers’ misconception of the regular versus equilateral hexagon. Seven pre-service teachers were recruited for the study and asked to construct an equilateral hexagon whose sides are $\sqrt{5}$ units in length. Although they were explicitly asked to create equilateral hexagons, our assumption was that they, or at least some of them, would create regular hexagons instead.

In this qualitative research, participants were asked to construct their artefacts in the GeoGebra environment, which is a dynamic and interactive mathematics learning environment (DIMLE) and to screencast their work by using a software package, Wink. They submitted their work at a public wiki space because the communications between researchers were performed online. The seven pre-service teachers created and submitted 15 constructs in total.

Their work was analysed by using frame analysis method (FAM) (Karadag, 2009; Cengel & Karadag, 2010). Selected students were interviewed to validate our interpretation of the results and to elaborate on the results of the analyses. During the interviews, students were presented with their own construction(s) as well as the constructs of equilateral, but not regular hexagons, developed by researchers.

The preliminary analysis results demonstrate not only the participants’ misconception and the reasons leading them to this misconception but how effectively they use a DIMLE (GeoGebra in this research) while working on a mathematics problem. Moreover, the results also demonstrate how their cognitive conflict emerged, how they re-assessed the situation to solve it, and how they improved their conceptual understanding through this resolution.

Risk-based decision-making by 15-year-old students

Hasan Akyuzlu, Institute of Education, University of London, UK

The judgement of risk is a key factor in our decision-making. Yet, the complex nature of risk means that people find challenging the trade-offs involved in co-ordinating impact, likelihood and the many ethical and value-based judgements entailed (Brandstätter et al., 2006). Pratt et al. (2010) worked with teachers to explore the nature of their risk-based decision-making in a personal dilemma. They found that teachers drew on personal experiences and values but often did not easily trade-off impact and likelihood; in other words, co-ordination was far from trivial. They proposed a new mapping tool to support co-ordination between impact and likelihood. This tool allowed teachers to express views about impact, likelihood and other aspects of the perceived hazards in the decision-making process but required them to locate the relative risk of these hazards. The mapping tool was designed at the end of their study and so to date has not been systematically tested. In the study reported here, I test the conjecture that the use of this mapping tool might facilitate students’ (as opposed to teachers’) risk-based decision-making. Therefore, I used the final previously untested version of the computer-based Deborah’s Dilemma (DD) (Pratt et al., 2010), in order to observe and trace two pairs of 15-year-old students’ thinking about risk during clinical interviews. This paper supports the prior work with teachers that students also drew on personal experiences and values and that, prior to use of the mapping tool, they referred to impact separately from likelihood even though their discussions were thoughtful and rational. Subsequently, the mapping tool provided a resource through which they expressed ideas for the co-ordination of likelihood and impact in terms of the situated notion of a trade-off, articulated as ‘balance’ and ‘cancelling’.
Dynamic representation of mathematics: the case of statistics

Gilles Aldon, Ecole Normale Supérieure de Lyon, France

The topic of module 2 of the EdUmatics project concerns the implementation within suitable software of dynamic representations in mathematics and the didactic engineering necessary to improve them in the classroom. The main aim of this module is to better understand the role of multi-representation for the learning of mathematics and the possibilities given by ICT to represent mathematical objects in a dynamic way. In this module teachers in training will have to explore dynamic representations of mathematics in different ways – cognitive, pedagogical and technological. Although not often taken into account, the case of statistics is very interesting to study because the description of data often requires different kinds of representation, each of them showing and hiding properties; a good understanding of data involves exploring different representations. Furthermore, it involves knowing how to translate one representation into another. Starting from this hypothesis, we elaborate on a class situation in which students were invited to measure their reaction time to a visual stimulus. After a certain number of trials, the first question might be: “What is your reaction time?” In order to answer this question, students have to do a statistical treatment of their data. The questions of the description, comparison and communication of data lead them to define the main characteristics of a statistic, but also to explore inferential statistics. This class activity is one of the three class activities which constitute the basis of the EdUmatics module 2. Starting from the mathematical problem, teachers will have to successively analyse a class situation, understand the role of representations, understand the possibilities ICT offers, build a new scenario taking into account their own teaching conditions, and test it. In this presentation, through the work of students, we will explore the potential of this situation being used in a teacher training course.

Using technological approaches in teaching mathematics: the perspectives of mathematics teachers

Othman Ali Alghtani and Nasser Elsayed Abdelhamied, Tabuk University, Saudi Arabia

The main aim of this paper is to investigate how mathematical teachers use a technological approach in teaching mathematics. To achieve this a questionnaire was prepared and sent to a sample of 120 mathematics teachers. We asked them to describe how they used technological approaches in teaching mathematics. The main results were that (i) most did not have a clear perspective of teaching related to how they use technology in the mathematics classroom and (ii) most did not consider the use of technology when they prepared their action plan.

Starting to work with ICT

Bärbel Barzel and Ralf Erens, University of Education, Freiburg, Germany

With the increasing complexity of ICT it is becoming more challenging for teachers and students to use it. The first steps for colleagues using ICT are getting more difficult, especially if there is more than one programme or one tool that might be considered for use. Thus it is very important when working with ICT to create material in a transparent and clear way, which can be adapted by teachers for their everyday teaching. The material must allow different learning paths so that every individual can find his or her own way of putting it into practice.

The concept of a professional development course as an introduction for ICT use can be split up in two aspects:

a) Getting to know about the technical possibilities and features of the artefact.

b) Getting an idea of how the artefact can be included in some way, which benefits the teaching and the learning of mathematics. This idea does not focus purely on single tasks or a short-term environment but also for a longer period of teaching.

The examples, which have been developed and compiled within the framework of the Comenius project EdUmatics, show on a simple mathematical level that different representations of the tool are interrelated with different approaches to solving mathematical problems (e.g. in a symbolical, graphical and numerical way). As long as different types of learner use different ways for the same
task, it is important that the different parts of the tool with different representations must be introduced in a parallel way without any “pre-dominated” hierarchy or preference given by the teacher.

Reference


Using multiple representations in the classroom – the EdUmatic project

Andreas Bauer and Hans-Georg Weigand, University of Würzburg, Germany

Caroline Bardini, Jacques Salles and Marie-Claire Combes, University of Montpellier, France

The European Development for the Use of Mathematics Technology in Classrooms (EdUmatic) project aims to increase thoughtful integration of ICT in European mathematics classrooms by building and disseminating an online training course for in-service and pre-service secondary teachers, in particular by providing high quality teaching material based on research and experience from the 20 partners involved, who are leading experts in using ICT in the classroom from ten universities and research institutes together with ten secondary level schools across six European countries. The project consists of five different “chapters” (called modules):

1 Starting to work with ICT.
2 From static to dynamic representations.
3 Constructing functions and models.
4 Using ICT in the classroom.
5 Multiple representations.

The Würzburg Group developed module 5 together with the Institute for Research on the Educating of Mathematics (IREM) at the University of Montpellier. This module deals with the use of multiple external representations (MER) in the classroom, the interrelationships between employed software and how to use them wisely in class. The module includes didactic considerations about the use of MER, methodical reflections on how to make thoughtful use of ICT, discussions concerning the theoretical background of MER in the learning of mathematics and ready-to-use classroom activities. Advantages and disadvantages, goals and difficulties of the use of multiple representations are also discussed. In this paper we will give a short overview of the aims and methods of this project and we will present two classroom activities. The first one explores, using appropriate technological environments, the limits of methods used to determine the position of a curve with regard to its tangents. The other compares exponential and linear growth processes using interactive GeoGebra applets or handheld devices. Results of empirical investigations carried out by French and German teachers about these two classroom activities will also be examined.

Design principles for an online algebra course

Christian Bokhove and Paul Drijvers, Freudenthal Institute for Science and Mathematics Education, Utrecht University, Netherlands

Procedural skills and conceptual understanding have been widely debated, especially with regard to algebra education (Schoenfeld, 2004). Meanwhile, the use of ICT in education has increased. In this article we report on a design research study that set out to investigate in what way ICT can be used to acquire, practise and assess algebraic expertise. The general framework of the study combines three elements: algebraic expertise (basic skills and symbol sense), theories on tool use (instrumental genesis) and assessment (formative assessment and feedback). In the first cycle of the study criteria for algebra tools were formulated (Bokhove & Drijvers, 2010a), and a first prototypical version of the intervention was designed (Bokhove & Drijvers, 2010b). Next, the results were used to identify design principles for a digital intervention in the Digital Mathematical Environment called ‘Algebra met
Using cryptology to teach fundamental ideas of mathematics

Thomas Borys, Institute of Mathematics and Computer Science, Karlsruhe University of Education, Germany

Cryptological methods were used in antiquity, but until relatively recently (a few decades ago) it was a science used mainly by governments, the military–industrial complex, secret services and spies. Nowadays, cryptology occurs “almost everywhere” in our lives. Many applications in the fields of computers and communication use cryptological techniques; to mention just a few: secure email communication, internet applications (in particular internet commerce and electronic banking), mobile phones. In this presentation I give a report on my research project “cryptology and fundamental ideas in mathematics education”. In large part it is an epistemological analysis of the question: “Is it possible to teach fundamental ideas of mathematics by using cryptology?” In a first step the role of fundamental ideas as basic guidelines for mathematics education (e.g. algorithm, functional dependence, modelling, number, measuring and ordering) are discussed. In a second step it is studied if and how these fundamental ideas can be illustrated and supported by various techniques of cryptology. Some useful examples for this part of the analysis are the “Fleissner Grille” (named after Austrian inventor Colonel Eduard Fleißner von Wostrowitz) or the Diffie–Hellman method for exchanging keys. For some case studies concerning cryptological techniques, the following interactive internet site is used:

www.ziegenbalg.ph-­‐karlsruhe.de/materialien-­‐homepage-­‐jzbg/cc-­‐interaktiv/	index.htm

Finally, the use of computers or special handheld computers (in particular those supporting computer algebra technology) for educational purposes in connection with modern cryptological techniques is discussed.

A “future” curriculum in a “3rd Industrial Revolution”?

Oliver Bowles, International School of Toulouse, France

In the current mathematics curriculums the memorisation of “rules”, without understanding, can result in exceptional performance1. This session proposes concrete lesson materials, in both computer assisted and non-computer assisted instruction environments, to develop and then assess student’s conceptual understandings and their approach to problems2. It is hoped that these may provide concrete examples from which participants can brainstorm other useful activities considered “essential” in addressing the practical challenges of a “future” curriculum.

I was teaching the binomial theorem recently. Often students can lose from two to four marks out of six, due to “calculation” errors: e.g. 3–x(62x2 – forgetting to include the negative for: 3–x(−)n , forgetting to raise the
‘x’ coefficient to the power: \((2x^2)^3 = 2x^6\), etc. Conceptually they have understood that this expansion is a “choosing” problem that Pascal’s Triangle gives us the number of different ways ‘r’ objects can be selected from ‘n’ etc. Attention to detail (and the grave consequences a lack of rigour/detail can cause) is an “essential” lesson, but are there not more up-to-date activities e.g. the manufacture of precision engineered products and how to estimate such calculations (or a return to proof?) that can be used to instigate such learning (and that do not fall into the domain of CAS or other calculation software)?

The theory of computation (Turing, Von Neumann, Chomsky et al.), the study of what can and can’t be computed, stimulates thought as to which task humans can accomplish more effectively than can a “computing machine”. For educators, this is likely to have a profound impact on what should and should not be taught, particularly in the fields of science and mathematics.

The state of Victoria, Australia, as have certain European states at certain times, currently uses CAS assessment practices – is this the direction of future curricula? What are the counter arguments?

1 It is not the author’s view that the efficient application of a rule is, therefore, evidence of poor “understanding”, only an observation that “rules” can be “memorized” without understanding and that some exercises given in mathematics are less effective than others at, for example, “testing” the proportional understanding, in the case of fractions arithmetic, that underly the rule.


3 www.ted.com/index.php/talks/conrad_wolfram_teaching_kids_real_math_with_computers.html


5 www.mei.org.uk/files/pdf/MEA_CAS_Report_v1a.pdf (p.15): (Half of the 100 universities and colleges contacted by the IB “reacted negatively” to the introduction of CAS in schools/colleges.)

**A long-term educational treatment using dynamic geometry software**

Dirk Brockmann-Behnsen, Institute for the Didactics of Mathematics and Physics (IDMP), Leibniz University, Hanover, Germany

We accompany two classes over a period of two years with an optional extension of another year beginning in year 7. One of the classes undergoes a treatment, the other one serves as a control group. The focus of the treatment is being set on the geometry units within the sequent forms. The treatment class is repeatedly acquainted with various kinds of heuristic aids and principles of operation and has permanent access to dynamic geometry software, as this software can be a “producer of new powerful heuristics” (Hötzl, 1996). We encourage the students to argue in a deductive way about suitable mathematical problems. Our research is trying to establish whether the intensive and appropriate use of the software along with the instructions in heuristics will lead to significant improvement in the students’ performances, and, especially whatever upgrades in the geometric thinking of the students regarding the van Hiele levels can be registered (see Gawlick, 2004).

**Improving middle school teachers’ questioning strategies using video**

Sue Brown, University of Houston-Clear Lake, US

This paper reports the results of a study focused on improving teachers’ questioning strategies. Seventeen middle school teachers enrolled in a graduate mathematics education course were asked to create a teaching portfolio. They videotaped themselves teaching a mathematics lesson to their middle school students, and then chose a 5–10 minute clip from the lesson where they focused on questioning their students. The teachers reviewed the clip, listed each question they asked, and categorised each question according to one of the three Costa’s categories. During class, each teacher presented the clip and the list of questions they asked to their small community of practice (three to four teachers). The community categorised the questions according to Costa’s levels of questioning and brainstormed ways the teacher might ask “better” questions. This process was repeated for a total of three videos. The components of the portfolio are an initial reflection paper on the teachers’ current questioning practices; three of the following: the video clip, a list of the questions with
Costa’s categories identified for each question, and a paper summarising the brainstorming session and based on comments from the community as to what the teacher would do differently in terms of questioning when they teach this lesson next year; and a summary reflection paper on what the teacher learned from the project and any changes that might have occurred in their questioning practices. The portfolio was electronic and in creating it the teachers used Movie Maker, Teacher Tube, and Google Sites.

Online resources for the busy teacher
Douglas Butler, ICT Training Centre, Oundle, UK

Busy teachers generally do not have the time to research and catalogue the fantastic resources that are now just a click away on the web. Online resources can now so easily add variety to a lesson, and Douglas is always on the lookout for material that is pedagogically sound and deserves a place in his “TSM Resources” website: www.tsm-resources.com.

Many of these resources have also been tested to work well on the evolving mobile technology (iPad, iPhone and Android smartphone etc). “Mathematics links”: there are many such collections on the web, but this one uniquely categorises resources by country, helping students to realise what a global subject mathematics is. “Web broadcasting”: most TV and radio stations are now available online, and this page makes it easy to find the main ones in the UK – to be used judiciously of course! “For the busy teacher”: this includes links to useful online clocks and timers, Google Earth, Flash Earth, Jing, and so on – all the useful apps that a busy teacher does not have time to find! “Training opportunities”: a regularly updated calendar of professional development events for mathematics teachers. “Integer lists”: students can be fascinated by large integers. This is a collection of useful lists (to full accuracy) of powers of two, Mersenne primes, Pythagorean triples. “Useful Files”: this includes a collection of Excel files, each with a strong pedagogical focus, and also a great set of data sets (in .xls format) carefully chosen to work well in statistics and data handling classes. There is also a collection of carefully categorised images to import and study in graphing software. Categories include “circular”, “parabolic”, “arches”, and so on.

Using live online tutoring to provide access to higher level mathematics for pre-university students
Tom Button and Richard Lissaman, The Further Mathematics Support Programme, UK

In England, the main pre-university qualifications are General Certificate of Education Advanced Levels (GCE A levels). An A level in Mathematics is taken by approximately 70,000 students each year. Around 15% of these students take an additional qualification in Advanced Level Further Mathematics. The Further Mathematics Support Programme (FMSP), previously the Further Mathematics Network, has been working with schools, colleges and universities across England to ensure universal access for students to this additional A level qualification since 2005. In particular, the FMSP has been providing tuition for students who are currently unable to access the qualification at their chosen school or college. A substantial proportion of this tuition is provided online using a shared whiteboard technology (Elluminate).

Online tuition usually takes the form of weekly, early-evening sessions in which a small group of students meet online with an experienced tutor to learn new mathematics and solve mathematical problems. In addition, students have access to extensive online resources to support their learning. The students are geographically located across England and many of them would not be able to study an additional mathematics qualification without the live online tuition provided by the FMSP. There are over 300 students receiving regular online tuition during this academic year with many more using the technology to access online revision classes.

The FMSP has been providing live online tuition in mathematics for over four years. During this period, through a process of review and reflection, the FMSP has continued to improve and develop approaches to online learning and the provision has been extended. The aim of the session is to share our experiences of teaching mathematics online and consider the potential for further development.
Home technologies: how do they shape beyond-school mathematical problem-solving activity?

Susana Carreira and Nélia Amado, University of Algarve and UIDEF, University of Lisbon, Portugal

In this paper we analyse and discuss the mathematical activity of a 12-year-old participant in a web-based problem-solving competition – Sub14 – promoted by the University of Algarve, Portugal. Ultimately, our purpose is to contribute to the knowledge on the learning processes that occur outside the classroom, constantly and necessarily entangling technologically rich environments.

The theoretical framework supporting our analysis draws on the concept of humans-with-media (Borba & Villarreal, 2005) and the notion of mediational artefact (Wertsch, 1991), which are vital for understanding the unbreakable entity resulting from the interactions between subject, object and action.

The preliminary results are the outcome of an exploratory study, integrated in a broader ongoing investigation. Although the results report to the exploratory study, they are expected to assist in decision making regarding the implementation of the research project.

Hence, we will focus on the work of one participant, Leonor, who has been attending the competition for the past three years. We describe and analyse two problems solved by Leonor in the competition, and try to understand (i) her mathematical problem-solving activity and (ii) how digital tools mediate such activity, particularly regarding the effective expression of her reasoning through the use of iconic signs.

The results show that Leonor uses ‘home technologies’ and, particularly, their graphical flexibility to ‘engineer’ her reasoning and develop her very own strategy in the problems given by Sub14. The computer is not only a means to present a neat solution to the problems. Instead, the tool becomes part of the solution’s creation process and it is being used as a ‘native language’ to think with, act with and communicate with. We suggest, therefore, that Leonor is a person-acting-with-mediational-artefacts (Wertsch, 1991), and argue that the computer is a significant mediational artefact in her problem-solving activity, which indicates that Leonor’s problem solution might be considered a ‘computer-mediated-solution’.

Researching teachers’ experiences of introducing multi-representational handheld technology – what and how do they learn?

Alison Clark-Wilson, University of Chichester, UK

This research presentation presents selected outcomes from a longitudinal doctoral study that sought to gain a deep insight into how a group of English secondary school mathematics teachers learnt to integrate a complex multi-representational technological tool, the Texas Instruments Ti-Nspire handheld device, into their classroom practice. Using Verillon and Rabardel’s (1996) theory of instrumented activity as the theoretical base the teachers’ instrument utilisation schemes are described, which suggest a clear trajectory for their evolving use of the tool. The presentation will describe a number of classroom tasks devised by the teachers to illustrate how both the range and integration of the technological applications (dynamic geometry, dynamic graphing, spreadsheets etc.) expanded over the timescale of the project. The content of this session will be of interest to teachers, researchers and developers who have an interest in the teacher development implications for new technologies and the design of related teacher development courses and materials.

Mobile apps in mathematics education

Timothy Collinson, University of Portsmouth, UK

The iPad (and more recently the iPad 2) has often been accused of being for the consumption of information only. Great for reading, web browsing, movies and the like, it in fact also enables creativity in a multitude of ways as many are finding out. In addition, schools, colleges and universities are discovering educational uses, or developing their own apps to support such usage. Some are trialling free devices for every student in a class or year, some are simply preparing for increasing numbers of students to be carrying and expecting to use their own devices.
This session will look at some of the apps that are available for mathematicians and those learning mathematics. We explore using apps ranging from free calculators to deluxe digital texts that exploit the potential of the device to its full. We will also consider more widely mobile apps and learning and whether this is a future that will grow or just a passing fad whipped up by Apple’s clever marketing. Has the ‘slate’ returned to the classroom? Or is this just a disruptive technology?

Attractive mathematical induction

Aija Cunska, University of Latvia

The method of mathematical induction can be compared with progress. We start with the lower degree and as a result of logical judgments we come to the general conclusion (result). As man always tries to advance, tries to develop his ideas in a logical way, consequently, nature itself makes man think in an inductive way.

The inductive method plays a significant role in understanding the principles of mathematics. Although the range of the problems concerning the usage of the mathematical induction method has grown, in school syllabi very little attention is paid to the issue.

The majority of students are visual learners. Therefore, if mathematical induction teaching methods are improved, more and more students would become interested in it. This is a powerful and sophisticated enough method to be acceptable for the majority.

This ‘century of information’ offers our society completely new opportunities in nearly all fields. However, the field of education is the one where the new technologies provide the greatest advantages.

Multimedia provides the opportunity to create teaching aids that combine text, pictures, sound and video, as well as quickly helping one to find necessary information. Professional usage of multimedia appliances turns the learning process into an exciting process of cognition.

For students the learning process sometimes may seem boring, but we can attract their attention with the help of information technologies. This can be done by creating multimedia learning objects. The multimedia learning objects can make the learning process more exciting, visually more perceptible and more specific. In that way teachers can work easier and faster, paying more attention to practical assignments. The created multimedia learning object “Mathematical induction” serves as successful evidence of that statement.

Prospective teachers’ curricular interactions and beliefs with regard to computer algebra systems

Jon D. Davis, Western Michigan University, US

It is widely recognised that teachers’ beliefs shape their use of technology in the classroom (Kendal & Stacey, 2002; Noss & Hoyles, 1996; Philipp, 2007; Schmidt, 1999; Walen, Williams, & Garner, 2003). This study reports on the beliefs (Richardson, 1996) and curricular interactions of six prospective secondary mathematics teachers (PSTs) with regard to computer algebra systems (CAS) as evidenced by intended curricula (Stein, Remillard, & Smith, 2007) they developed from three different US written reform-oriented mathematics textbook lessons. Four categories were used to describe PSTs’ interactions with written lesson elements (keep, adapt, supplement, and omit). The PSTs showed remarkable variability in their intended curricula despite working from the same written curriculum. For instance, the percentage of the written curriculum lesson elements that the PSTs retained within their intended curricula from one textbook lesson varied from 19% to 95%. A Friedman rank test (Corder & Foreman, 2009) showed that PSTs’ intended curricula were statistically different from one another within each of the three curriculum lessons. The PSTs possessed a variety of beliefs concerning CAS that emerged from their intended curricula. For instance, one PST viewed the CAS through a symbolic manipulation lens. That is, she did not believe that technology could be used to assist students in learning other goals such as identifying patterns. Consequently, she routinely adapted textbook activities so that students executed procedures by-hand first and then used the CAS to check their work. Another PST, however, believed that the CAS could fulfill a variety of different roles that depended on textbook lesson goals. At times, he delayed students’ CAS use so that he could
focus on students’ symbolic manipulation skills, while at other times the technology provided students with opportunities to learn about mathematical concepts.

References


Using live online technology to engage mathematics teachers in professional development
Sue de Pomeraï and Sharon Tripconey, The Further Mathematics Support Programme, UK

The Further Mathematics Support Programme, previously the Further Mathematics Network, has been working with schools, colleges and universities across England to ensure universal access for students to Further Mathematics A level since 2005. The FMSP has provided tuition for students in schools and colleges that cannot offer this additional A level themselves. However, as student numbers taking Further Mathematics grow and universities increase the demand for potential STEM students to study Further Mathematics, schools and colleges have been encouraged to offer and teach it themselves. Until now, a shortage of experienced teachers in this area has been a significant barrier to this and the FMSP has recognised that it is crucial that mathematics teachers have the opportunity to develop their mathematical knowledge and teaching skills to meet the growing demand for teaching Further Mathematics.

Since 2007 our range of online courses for teachers have increased opportunities to access professional development (PD). Using an online shared interactive whiteboard which enables communication through audio, instant messaging and via handwritten mathematics, small groups of teachers meet online weekly with a tutor for five to ten weeks, allowing time for reflection and consolidation. The online sessions cover mathematical content, pedagogy and facilitate the exchange of ideas between the teachers, creating a supportive and collaborative working environment. In addition, delegates are given access to extensive online resources for teaching and learning and have extended access to tutor support and peer support through email and online forums.

By gathering feedback from teachers and through review and reflection, the FMSP has continued to improve and develop approaches to online PD, including a current trial using the technology to facilitate discussion sessions for teachers undertaking study at masters level.

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The aim of the presentation is to share our experiences of online PD and consider the potential for further development.

**Student problem solving achievement in a CAS environment**

David Driver, Brisbane State High School, Australia

A calculator with Computer Algebra System (CAS) capability can be used as a teaching and learning tool and/or as an assessment instrument. In the classroom, it can be used by both the teacher and the student as a functional tool and/or as a pedagogical tool. In the examination room, it can be used by the student as a functional tool.

When it is used as a pedagogical tool in the classroom to aid student learning and understanding of the mathematics involved and as a functional tool to undertake repetitive tasks, it can subsequently be used in the exam room as a functional tool to aid in the manipulation involved in problem-solving tasks. When the CAS is used in all of these ways, the effect on student achievement (as measured by the exam) can be quite profound.

When the student is first introduced to the CAS this also has an impact on the future gains in student achievement.

In this action research project, the CAS calculator was introduced to students and used to varying degrees in years 10, 11 and 12. The effects of the timing of this introduction and the level of use of the CAS by students in the classroom on their final year results were examined and reported here.

There is some evidence of an increase in the benefit to students (as measured by achievement in both mathematical procedures and problem solving) by introducing the calculator in Year 10 rather than Year 11. This gain is evident across the range of student ability.

Although a Casio Classpad was used in this study, it is anticipated that similar effects would be observed with alternative technologies.

**Inspired connections in mathematics lessons – new pedagogy for new technology**

Allan Duncan, University of Aberdeen, UK

Does the use of handheld technology that allows dynamically linked multiple representations of mathematics concepts impact upon the dynamics of classrooms? Do teachers change the way they teach particular lessons? Do they alter the way they teach in general? What is the impact of changes in pedagogy or didactics on students’ motivation, interest and ways of working? This presentation will report briefly on the findings of research, published in 2010, which asked teachers to report on lessons in which they used new technology. It will then focus on particularly successful lessons that clearly demonstrate the benefits to be gained by using both new technology and new teaching approaches. Links between different representations of mathematics concepts are highlighted. Connections between different areas of mathematics are emphasised. The teachers described how the use of the technology, together with a more investigative teaching approach, with more opportunities of questioning and discussion, both between teacher and students and among students themselves, led to a deepening in understanding, an increased pace of learning and a surprising increase in motivation and engagement across all ability levels.

**Interactive white boards (IWBs) to support mathematical learning: current practices and open problems**

Eleonora Faggiano and Rosa Laura Ancona, Department of Mathematics, University of Bari, Italy

With this work we intend to focus on the use of IWBs as methodological resources to support mathematics teaching and learning activities.

As research has already shown – see for instance Mousley et al. (2003) and Moss et al. (2007) – in spite of perceptions, it cannot be taken for granted that technological advances, alone, change essential aspects of teaching and learning, simply because they can bring about opportunities for change in pedagogical practice.
We contend that an IWB could be used as a ‘semiotic mediator’ of the process of mathematical knowledge construction. Our hypothesis, in particular, is that this can be done if the teacher, starting from adequate learning experiences, creates suitable ‘a-didactic situations’ aiming to gradually promote the construction of meanings of mathematical objects.

The basic assumption is that the main aim of the educational design in mathematics is to foster the progressive construction of a personal heritage of mathematical knowledge, skills and attitudes which have to be meaningful, stable and fitted for the use in problematic situations both internal or external to the mathematics.

As a theoretical framework we refer to: the theory of didactic situations (Brousseau, 1997); the notion of ‘mathematics laboratory’ as a Renaissance workshop (UMI-CIIM MIUR, 2004); the concept of ‘semiotic mediation’ (Bartolini, Bussi, & Mariotti, 2008); and the ‘instrumental approach’ – starting from Vérillon and Rabardel (1995) – and in particular, within it, the concept of ‘instrumental orchestration’ proposed by Trouche (2003). Some case studies (in Italian schools) will be analysed, according to the theoretical framework and the research hypothesis, focusing on the teacher and his/her needs in using the IWB in classroom activities.

Reviewing the impact of technology on the development of a mathematics curriculum from two cases in China and Singapore

Lianghuo Fan, School of Education, University of Southampton, UK

The last two decades or so have witnessed the increasing impact of technology on mathematics teaching, learning and assessment. The rapid increase of this impact is particularly evident in the development of mathematics curricula. In this article, the author will draw on his recent experience as chief editor of two series of secondary mathematics textbooks developed and published in China and Singapore respectively, examine the impact of technology on the development of a mathematics curriculum, and discuss the future direction of using technology in curriculum development. Based on the two cases of curriculum development in China and Singapore, this article argues that modern information and communication technology has not only impacted what to teach and how to teach, but also why to teach. In relation to this, technology must be reflected and, more importantly, embedded into the development of mathematics curricula including textbooks, which is a most important pedagogical resource for teaching and learning. How this can be achieved is also discussed in the paper, mainly with the examples from the two cases in China and Singapore.

Modelling and spatial reasoning

Gregory D. Foley, Ohio University, USA

Modelling and Spatial Reasoning (Modspar) is a year-long professional development (PD) course that addresses technological pedagogical content knowledge (TPACK) in discrete, continuous, and geometric modelling and in spatial reasoning – areas of current weakness in the U.S. teaching workforce. The course balances tasks, tools, and talk. Using instructional and assessment tasks with a high level of cognitive engagement is the pedagogical aim. The tools for Modspar are computer algebra, graphing, and geometry applications, including spherical and three-dimensional interactive geometry software. The Modspar course balances these tools and tasks with talk, that is, with mathematics language development through high-level classroom discourse. Modspar is part of a two-year PD program called Advanced Teacher Capacity (ATC), a research and development project that investigates the question: “How can teacher professional development improve instruction in innovative post-Algebra II courses for high school seniors and juniors?” Using Boston and Smith’s (2009) instructional quality assessment, the research during the 2010–2011 school year has focused on the instructional tasks and classroom discourse of Modspar participants as a function of whether they have access to classroom sets of TI-nspire CAS handheld computers. The session will provide course details and preliminary research results.
Learning the ropes: how the technology of sailing and seamanship can enhance the teaching and learning of mathematics

Dot French, Community College of Philadelphia, USA

In the sport of sailing, traditional practices of navigation and seamanship have been greatly impacted by the introduction of new electronic technologies that are radically altering the seascape. For example, GPS and digital chart-plotters are being widely adopted, although dead reckoning and celestial navigation are still essential skills for mariners embarking on serious coastal or trans-ocean voyages. Correspondingly, in mathematics education, electronic learning resources are creating new ways of teaching and learning mathematics. How does the inclusion of nautical content and technology, along with hands-on, activity-based methods, affect the ways in which students, especially those in pre-calculus and geometry courses, learn mathematics? Can activities that refer to navigation and sailing help create productive environments for studying mathematics in context? This paper discusses outcomes and attitudes of community college students who study mathematics using nautical references and technology, as well as hands-on activities, model-making and traditional methods. Students’ opinions on the use of technology for learning mathematics are explored, as are their opinions on how mathematics relates to the world outside the classroom.

Exploring impulse and momentum using handheld technology

Ian Galloway, Science Learning Centre South East, University of Southampton, UK

Students at secondary school rarely encounter motion graphs beyond displacement or velocity against time. Yet in order to understand Newton’s third law it is useful to be able to graph force against time. This is relatively straightforward using any datalogging system. Most students, and teachers, have a limited understanding of the third law which, if not rectified, causes problems for the further study of dynamics.

In this presentation, using TI-Nspire, Vernier force plates and force probes we will explore the concepts of impulse and momentum. Previous work has shown that presenting ordinary events such as jumping up and down on the spot as a force-time graph is met initially with misunderstanding. Cognitive conflict results in students reappraising the situation and reaching a much greater understanding of what is taking place. Force, and consequently impulse, is an abstract quantity which deserves more attention within the curriculum. As J.W. Warren writes, “it [force] has long been regarded as a simple concept...and insufficient consideration has been given to ensuring that it is taught correctly”. Using modern digital technologies it is now possible to address the problem in a new way.

Computer-aided assessment of mathematics and statistics for first year economics students

Martin Greenhow, Department of Mathematical Sciences, Brunel University, UK

This paper will focus on exploiting computer-aided assessment (CAA) developed under the Metal project in a formative/summative mode within a first-year Mathematics for Economics module at Brunel University comprising over 300 students. Three year’s worth of results will be presented to demonstrate the positive impact on students’ perception of their learning and on the actuality of their learning as measured (in various methodologies) by their examination scripts. Such positive effects are underpinned by attributing marks for the CAA, thereby rewarding student engagement, coupled with the need to pass the exam component as well as the whole module, which forces the students to focus on their learning rather than simply on marks accrual. Indeed, the students (correctly) view the CAA as a learning resource in its own right and spend most of their time studying the very complete feedback screens. Repeating tests and group work is allowed, even encouraged, since each question realisation uses random parameters that are carried through to all aspects of the question (stem, key, mal-rule based distractors, MathML equations and SVG diagrams). This technology is exportable to other subject disciplines, as are most of the decontextualised questions that are mainly of A level standard.

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The general nature of the approach will be demonstrated by presenting a new project, DeStress, a statistics equivalent of Metal, but with a wider intended target of statistics within social sciences. Here the focus will be less on manipulative skills (data is loaded into Excel) but more about the meaning of statistics. One challenging area is interpretation of charts and graphs where data still needs to exhibit certain characteristics after randomization has taken place. Again, most of the underlying statistics will be of A-level standard but dressed up in contexts that will be meaningful to students of Politics, Economics, Sociology and Geography.

**Improving understanding in ordinary differential equations through writing in a dynamic environment**

Samer Habre, Lebanese American University, Beirut, Lebanon

Research on writing in mathematics has shown that students learn more effectively in an environment that promotes this skill and that writing is most beneficial when it is directed at the learning aspect. Writing however necessitates proficiency on the part of the students that may not have been developed at earlier learning stages. Research has indicated though that the burden placed on teachers and learners to master this skill is compensated by the mathematical learning in such an environment. Techniques to successfully integrate writing in the mathematics classroom can be varied. This presentation is on a study conducted at the Lebanese American University on students in an introductory differential equations class in which a reformed approach is adopted, be it in the topics discussed, the textbook used, the technology employed, or the assignments/exams given. More precisely, the presentation explores the effect of writing on improving student understanding of particular topics in differential equations and investigates the development of the students’ writing skills.

**Teachers’ beliefs in the use of the calculator in Brunei primary school mathematics**

Ahmad Fadzillah Hanafiah, Ministry of Education, Brunei, Darussalam

The Ministry of education of Negara Brunei Darussalam is introducing calculators for teaching and learning purposes at primary level. Elsewhere, this has been the subject of heavy debate for many years, if not decades — from arguing whether the calculators should be permitted in the classroom to how calculators can be used effectively in the classroom (Hembree & Dessart, 1986). This paper explores the beliefs, perceived knowledge and practices of primary teachers concerning the use of calculators in the teaching and learning of mathematics, which were identified by means of a questionnaire consisting of 20 items. The items were derived from the survey instrument used by Brown et al. (2007), and divided into four categories namely catalyst beliefs, teachers’ knowledge, crutch beliefs and teacher practices.

**Good calculus problems for the TI-89 calculator and an online homework system: a decade of use in a university mathematics department**

Mako E. Haruta, University of Hartford, USA

Ten years ago, the University of Hartford Mathematics Department introduced the Texas Instruments TI-89 graphing calculator into the calculus sequence, prompting a rethinking of the mathematics curriculum in light of the CAS feature, and discussions on how to best integrate the technology without sacrificing student conceptual learning. The CAS feature of the TI-89 calculator can turn many traditional calculus problems into button pushing exercises that require little mathematical understanding. In response, department faculty members have continued to modify existing problems and develop new and challenging problems that creatively assess student understanding of concepts while allowing full use of the technology during testing, on homework and in the classroom. In tandem, the Mathematics Department also piloted and eventually adopted the free, interactive, online homework system WeBWorK. Results have showed a dramatic increase in the quantity of homework assignments completed as well as a rise in faculty–student interactions via email and in office hours. Student response to the online system has been positive. Administrative features such as the ability to ‘rebuild’ an individual set allow for creative pedagogical enhancements to student learning. Both technologies are required in all calculus courses and have successfully expanded into
higher courses such as Differential Equations, Linear Algebra, and Advanced Engineering Mathematics.

**Exploring swinging and aerial movements**

André Heck, Universiteit van Amsterdam, Netherlands

Daan Knobbe, Nic Nijdam and Onne Slooten, OSG West Friesland, Netherlands

Peter Uylings, Universiteit van Amsterdam, Netherlands

Daan Knobbe was a secondary school student at pre-university level with gymnastics as a serious hobby. He and his fellow student Nic Nijdam investigated the mathematics and physics of gymnastics swing and flight elements. They not only did this to meet the Dutch curriculum requirement of carrying out a large (80 hours) research project, but also to satisfy their curiosity regarding the sports science subject. Under the supervision of their physics teacher they collected and analysed experimental data of swinging and aerial movements. In particular, they recorded and analysed the motion of a backward somersault from a floor exercise and the motion of swinging around a high bar.

We present the results of their research work, which resembled the practice of sport scientists. Video analysis of the rotation during the backward somersault shows the increased speed when the body is tucked. Video measurement offers the opportunity to investigate the angle changes of hip and shoulder joints during a backward giant circle on the high bar. A gymnast flexes after reaching the lowest point and extends before the highest point. A mathematical model of the gymnast and the high bar can be used to explore the gymnast’s body motion to understand the timing of flexion and extension.

Mathematics and physics come together in the presented research project at a rather high, but manageable, level provided that the students have adequate tools. The work is a nice illustration of authentic experiences of secondary school students in doing sports science that they will never forget.

**A jump forwards with mathematics and physics**

André Heck and Peter Uylings, Universiteit van Amsterdam, Netherlands

In this presentation we focus on human body motions such as bouncing on a jumping stick, rope skipping, hopping and making kangaroo jumps, skipping, and running. Students can record the movements on video and use their video clips to investigate the motions with suitable video analysis and modelling software. We discuss some mathematical models of these motions using basic physics and we compare modelling results with experimental data obtained from video measurements. The highlight is the application of the model of an inverted, planar spring–mass system: this rather simple model works well qualitatively and quantitatively for the complex motions of hopping, skipping and running at moderate speeds. The examples of video analysis and modelling activities give a good impression of the potential of the subject of human gait for practical student work and as a context for applied mathematics and physics at secondary and undergraduate level.

**Adapting the game-based learning software Racing Academy for use in engineering education**

Ya Huang, Department of Mechanical and Design Engineering, University of Portsmouth, UK

Jos Darling, Department of Mechanical Engineering, University of Bath, UK

Richard Joiner, Department of Psychology, University of Bath, UK

Following the development and implementation of the game-based learning software Racing Academy in Mechanical Engineering courses at the University of Bath since 2008, the software was adapted for 160 Year 1 Mechanical Engineering students at the University of Portsmouth in 2010. Racing Academy employs principles of engineering dynamics to simulate and display on a PC, in real time, a car drag race in which students modify their car by selecting components, including choosing an engine, tyre and gearbox from a set menu. A work sheet was designed based on configuration provided by the software to try and lead students through the process of playing Racing Academy.

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The aim was to complete a drag race in the minimum time and display the time histories of velocity and acceleration. A lab report on how they had improved their lap times was assessed.

The display and ensuing analysis of velocity and acceleration in real time is expected to make more intuitive connections between physical observations of a race and the mathematical operation of integration and differentiation. These operations are fundamental ingredients in engineering dynamics education.

The students and staff at the University of Portsmouth involved in this project provided feedback based on questionnaires collected before and after the task. The questionnaires were designed in the same protocol as those used at Bath. Feedback at both institutions found improvement in students’ engineering understanding, but there was no increase in students’ motivation towards engineering after playing Racing Academy. This paper compares the results collected at the two institutions.

Pre-service teachers’ understandings of learning to use information technologies in secondary mathematics teaching

Rosalyn Hyde and Julie-Ann Edwards, University of Southampton, UK

One of the biggest challenges facing pre-service teachers is that of learning how to make effective use of digital technologies in the classroom to enhance the learning of their students. For initial teacher educators, the challenge is to enable the development of teachers who have the capability to respond flexibly to new technologies and who are able to evaluate and reflect on the impact of such technologies on learning.

This paper reports on the first part of an on-going research project examining ways in which pre-service mathematics teachers can more effectively develop skills in using digital technologies in enhancing teaching and learning in the classroom. It examines the evidence collected from a cohort of pre-service teachers at the end of a one-year postgraduate initial teacher education course. The pre-service teachers were asked about their experiences, both on school placements and in university. We examined their learning about using technology, the difficulties they faced and the type of experiences from which they think successive cohorts of pre-service teachers would benefit.

The pedagogical understandings of the pre-service teachers regarding their use of technologies are examined and considered in the context of their learning experiences on the course. The paper also begins to chart the learning journeys undertaken during their course experiences towards these understandings and points to ways forward in enabling more effective learning in the use of technologies for mathematics teaching.

Automated assessment and feedback on MATLAB assignments in computational mathematics

Alan Irving, Department of Mathematical Sciences, University of Liverpool, UK
Adam Crawford, engCELT, Loughborough University, UK

We report progress on an automated system for the assessment and provision of feedback on computational mathematics assignments using MATLAB. Each assignment product is designed as a self-contained MATLAB function file, which accepts externally supplied test data and whose output contains the requested assignment results. The latter can include intermediate as well as final results so that partial credit and more useful feedback can be given. The assessment code checks whether the submitted function runs and, if it does, tests the requested output against a correct reference code, using a variety of inputs. The results, anonymised total marks and individual feedback files, are made available via the VLE and the web. Useful marking details and summaries are provided for the tutor to assist with assembling generic feedback for the whole class. An additional facility allows the tutor to convert the marking script into a modified binary version that the students can use to get preliminary feedback on how well their code performs against the final marking criteria. This helps prevent the student performing blind submission of code that is doomed to failure because of some simple programming error. We have also developed code that compares the active portions of all submitted code to check for collusion. It assigns a numerical correlation coefficient to similar pairs of files and then, where necessary, collects these into groups which exhibit common coding and so
The indirect impact of using modern technology, especially the calculator and the Internet, in reducing mathematics anxiety

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The relatively recent introduction of new technology into the mainstream of education has been accompanied with hot debates. The terms “technological tools” or “ICT” encompass the range of hardware and software with not necessarily the same effects on the educational process and this makes the debates so burning and complicated. Hence, we cannot approve one rule for all the cases and educators are supposed to investigate them separately. It seems, and my teaching experience also supports, that there is a complex relationship between technology-aided mathematics education and mathematics anxiety. If this term refers to a normal state of uneasiness and distress about future events, then this seems quite natural. Unfortunately, for many students, this anxiety associated with mathematics and/or tests may be an “intense fear or dread”, which may seem abnormal. We, as educators, are supposed to foster an environment where students and faculty become confident and competent problem solvers, not without anxiety, but with a natural and rational extent of anxiety and technology may make a dramatic impact on this task. The aim of this study is to investigate the effects of using the calculator and the Internet in teaching and learning secondary level mathematics on reducing mathematics anxiety. To achieve the aforementioned goal, to measure mathematics anxiety researchers developed scales dealing exclusively with mathematics anxiety. Examples of such scales are the Mathematics Anxiety Rating Scale (Richardson & Suinn, 1972), the Mathematics Anxiety Rating Scale – Revised (Plake & Parker, 1982) and the Mathematics Anxiety Questionnaire (Wigfield & Meece, 1988).

Theatrical or efficient use of ICT in mathematics teaching?

Antonín Janěaøík and Jarmila Novotná, Faculty of Education, Charles University, Prague, Czech Republic

The use of computers in mathematics education at primary and secondary school levels and in pre-service teacher training is topical. Computers have become a tool of motivation and foster comprehensible interdisciplinary links between mathematics and other subjects. However, the use of computers in teaching asks for new approaches to exposition and to mathematical content. This might be one of the reasons why recent studies in mathematics education show that, despite many national and international actions aiming at integration of ICT into mathematics classrooms, such integration in schools remains underdeveloped. The rate of this integration increases markedly slowly when compared to the speed of evolution of the technology.

One of the causes for this state is the huge diversity of ICT resources, which often leaves teachers unsure of which to use, and when and how to use them. Another important retarder of successful use of ICT in teaching is a lack of information on the potential advantages and dangers of inclusion of activities using ICT into teaching. Despite the fact that ICT has a huge potential in teaching, examples from practice show that in many cases theatrical examples contribute very little to development of mathematical knowledge and may even be counter-productive.

Integration of computers into teaching should always be governed by the principle of its efficiency. It is appropriate to use computers only in those situations in which it really brings benefit, opens new perspectives or significantly decreases the amount of time needed for technical calculations. The aim of this paper is to demonstrate the potential of both of these ways of use of ICT on concrete problems and herewith to show examples of improper use of ICT.
Implementing a dynamic geometry approach in classrooms

Zhonghong Jiang, Texas State University, US

This paper describes the theoretical foundations and key elements of an approach to high school geometry that utilises dynamic geometry (DG) software and supporting instructional materials to help students construct mathematical ideas. The presentation will focus on how that approach will be operationalised in the classrooms and a research study that examines the efficacy of the DG approach.

Dynamic Geometry is active, exploratory geometry carried out with interactive computer software such as the Geometers’ Sketchpad and Cabri-Geometry. The theoretical foundations of the DG approach are the constructivist perspective and van Hiele’s learning model. As the key elements of the DG approach, the teacher should lead the students to construct geometric objects, perform actions (drag, measure, transform, and/or animate) on the constructed objects, observe what characteristics change and what remain the same, investigate mathematical relationships, form and test conjectures, receive immediate feedback, think mathematically and prove (or disprove) conjectures.

A four-year study is funded by the US National Science Foundation to conduct repeated randomized control trials of the DG approach. The study compares effects of that intervention with standard instruction that does not make use of computer tools. The basic hypothesis of the study is that the DG approach results in better geometry learning for most students. The study tests that hypothesis by assessing student learning in 76 classrooms randomly assigned to treatment and control groups. Student learning is assessed by a standardized geometry test, a conjecture-proving test, and a measure of student beliefs about the nature of geometry and mathematics in general. Teachers in both treatment and control groups also receive relevant professional development. Fidelity of implementation for the experimental treatment is monitored carefully. Data for answering the several research questions of the study are analyzed by appropriate HLM methods. This presentation will report on the first two years of the study.

Working in the 21st century – moving teacher professional development online

Angela Jones, Ministry of Education, New Zealand

2010 has been a time of change for secondary education in New Zealand as schools and teachers implement the New Zealand Curriculum. As part of a three-year implementation the Ministry of Education (MoE), in association with the New Zealand Qualifications Authority, has been reviewing all curriculum-related standards so that they are aligned to the New Zealand Curriculum, to resolve issues of duplication between standards and ensure credit parity.

As most teachers are only now becoming familiar with our new achievement standards, sample assessment resources (tasks) are vital. In recognition of this the MoE is providing two resources for each new Level 1 standard. The New Zealand Association of Mathematics teachers (NZAMT) has a history of providing secure assessment resources (to support the old standards) to teachers.

I recently instigated a series of professional development (PD) for teachers in New Zealand involving working with teachers and advisors writing additional assessment resources for use by the wider teaching community via NZAMT. In the absence of direct funding a face-to-face writing workshop was impractical. Writing was undertaken voluntarily by advisors and teachers in the virtual space – made possible via the use of an Elluminate virtual meeting room and a free educator wiki. These writing workshops have involved a high level of professional development for those involved particularly as understanding the standard has to be the first priority before valid assessment tasks can be written. We have finished two cycles of writing and completed the development of three resources to the final editing stage. These results, obtained with minimum budget, sector buy in and high PD are extremely positive.

The aim of the presentation will be to share the experiences from these writing workshops, discuss feedback from teachers and consider future opportunities for online professional development.
Using Grand Challenges within technology enhanced learning (TEL) to frame research and practice in mathematics teaching with technology

Marie Joubert, University of Bristol, UK

There is now a wide research literature concerning the use of technology in the teaching and learning of mathematics. General text books, guidance for teachers, national curricula and other teacher materials almost always include advice and guidance related to the use of technology (ICT) and practitioner journals frequently include accounts of how technology has been used in a particular setting. Both research and practice have commonly framed their understanding of the landscape of 'what is going on out there' in terms of the software or hardware used (e.g. dynamic geometry or handheld technologies) or in terms of the mathematical area addressed (e.g. graphs and functions, geometrical transformations.)

This presentation/paper takes a different approach, borrowing the three Grand Challenge themes from the European Network of Excellence, STELLAR, to frame our understanding of the research and practice presented within this ICTMT10 conference. The three Grand Challenge themes are:

1) Connecting learners, which is concerned with the issues and questions that arise from the increased connectedness of learners through the use of, for example, the Internet.

2) Orchestrating learning, which aims to understand the opportunities and challenges for teachers when technology is introduced into their classrooms.

3) Contextualizing learning, which focuses on how the use of technology provides new and different learning contexts for teaching and learning.

I argue that the use of this Grand Challenge framing will provide a new and different understanding of the current landscape and future challenges for research and practice in the use of technology in mathematics teaching, and the presentation will conclude by considering what a mid-term research agenda might look like for our community.

Teachers’ scenarios with the use of digital tools in mathematics as a means of redefining the teacher–curriculum relationship

Elissavet Kalogeria and Chronis Kynigos, Educational Technology Lab., School of Philosophy, University of Athens, Greece

Giorgos Psycharis, Department of Mathematics, University of Athens, Greece

The present study analyses 19 scenarios developed by mathematics teacher educators-in-training, during their training course at the University of Athens. The subject of the course was the pedagogical use of digital tools in the teaching of mathematics. Scenarios were used as one of the methods for increasing reflection.

A scenario with the use of technology in the teaching of mathematics should include a series of crucial aspects concerning the teaching and learning process, such as the mathematical concept on which it focuses; the students’ difficulties in relation to it; the mathematical concepts that are going to be embodied in the software microworlds; the added value given by technology; the kind of meanings the students are expected to develop; the social orchestration of the classroom; and time–space parameters and the teaching management.

The inclusion of all the above aspects goes beyond the classical teaching plans and gives a strategic character to the process of developing a scenario. It brings the teacher to the centre of attention and demands him to undertake an active role in the development of new, innovative curricula, enriched with the use of technology.

This study describes the analysis of scenarios with the use of the exploratory software “Turtleworld” and tries both to record the features of scenario design and investigate the role of this process in the teacher–curriculum relationship.

According to the findings of our analysis, teachers deconstructed and reconstructed the formal curriculum, depending on the needs of their scenarios and the mathematical concepts that were
chosen to be embodied. By using the software tools they created many representations for those concepts, connected them accordingly and organized new, widened conceptual fields for them.

Teaching mathematics online: creating a rich learning environment

Ilona Kletskin, University of Toronto, Canada

In today's rapidly changing educational landscape, online courses are becoming an increasingly prevalent component of undergraduate instruction. Online courses are extremely attractive in the flexibility that they offer to learners, but providing an engaging learning environment online can prove to be a challenge. In this talk, I will discuss my experiences developing and teaching an online first-year Calculus course.

Looking at issues of course design and management, assessment, and student engagement, I will discuss the challenges and opportunities that come with the online environment, and how tools such as WebCT blogs and Adobe Connect can be used to enrich the online classroom. Whether you might be teaching an online course in the future or already have years of expertise, this presentation will allow an occasion to discuss best practices. I will conclude with some feedback on the online experience as well as an overview of the tools and changes I hope to implement in future online courses.

The role of interactive assistance in discovering geometrical theorems at secondary school

Magdalena Kucio, Gimnazjum nr 9, Kraków, Poland

There are many interesting geometric theorems essential for didactics. Many of them refer to the analysis of the properties of geometrical figures. Some of them can be analysed and expanded during extra-curricular activities for pupils who show mathematical skills or interests.

Teaching practice shows that solving geometrical problems causes many difficulties for pupils. The question is whether a well-chosen interactive tool can help to remove these barriers. The possibility of providing pupils with materials prepared and sent by the teacher over the Internet is the additional advantage of such a solution.

In my lecture I will focus on research of the interactive teaching materials created with the help of the GeoGebra program. I will also present examples of such tools and ways of using them. The research shows that there are not only positive aspects of using such teaching aids and I would also like to share my observations of some possible dangers connected with this type of teaching.

Combining theoretical frameworks to investigate the potential of computer environments offering integrated geometrical and algebraic representations

Jean-Baptiste Lagrange, Université Paris Diderot and University of Reims, France

Giorgos Psycharis, University of Athens, Greece

Many authors now stress the advantage of considering sensual experience of dependencies as a basis for students' understanding of the idea of function and report on teaching experiments aiming to connect this experience to the algebraic notion of function. Some computer environments offer integrated geometrical and algebraic representations and functionalities to support this approach to functions. As for us, our aim is to study how dedicated software and classroom-based modelling activities can help students to construct the chains of functional meanings connecting representations of functions at different levels. We used two software environments that have been developed under different theoretical frameworks in different research and national contexts. One of these (called Turtleworld) is a piece of geometrical construction software that combines symbolic notation, through a programming language (Logo), with dynamic manipulation of geometrical objects by dragging on sliders representing variable values. The second software (called Casyopée) offers a dynamic geometry window connected to a symbolic environment specifically designed to help students to work on functions.
In order to make sense of the respective potential of these environments for functional meaning making we exploit a grid designed to classify various activities about functions, and show potential connections between these. We also consider two teaching experiments (designed and implemented): one with Turtleworld in the Greek context inspired by constructionism, and one with Casyopée in the French context inspired by the theory of didactical situations. The paper aims to broaden views about the conceptualisation of functions by students by establishing links between these teaching experiments and by coordinating the underlying frameworks, constructionism, theory of didactical situations and also instrumental approach. We adopt “cross analysis” as a methodology built within the project ReMath as a means to progress in connecting and integrating theoretical frameworks in technology enhanced mathematics.

**Assessing mathematical problem solving behavior in web-based environments using log file analysis**

Moshe Leiba and Rafi Nachmias, School of Education, Tel Aviv University, Israel

Problem solving can be described as being composed of three dimensions: the problem, the process, and the outcome. Over the years, mathematical problem solving research has focused on describing the process, as well as understanding attributes affecting it, and assessing its outcomes. Most of the research in this field is qualitative, and this is understandable due to the fact that the cognitive and meta-cognitive investigation involved in problem solving are complicated to trace. Nowadays, when many problem solving environments are implemented using the web, innovative research methodologies may be applied for assessing problem solving behavior in large populations. These innovative research methodologies rely on log file records, which are automatically and continuously collected by Internet servers, that document (almost) every action taken using three basic parameters: what was the action taken, who took it and when. The main purpose of this research is to explore cognitive and meta-cognitive processes during problem solving activities in online environments being used in elementary schools in various areas of mathematics (geometry, fractions, series, etc.).

The core of this research entails the development of a correspondence scheme between the logged traces of the students and the observed problem solving behavior. A mixed method involving both qualitative and quantitative analysis has been chosen for this research. Problem solving behavior is assessed by means of qualitative research, using think-aloud protocols as well as actual learning behavior in the online learning environment using log files (N = 5). Data from both sources are then triangulated, aiming to reflect on students’ problem solving behavior documented in log files; patterns in problem solving processes and factors affecting them will be investigated using quantitative (data mining) methods (N > 1000).

**E-assessment in mathematics and statistics**

Jeremy Levesley, University of Leicester, UK

Sally Barton, University of Nottingham, UK

Chris Sangwin, Birmingham University, UK

Bill Foster, Newcastle University, UK

This session introduces an exciting new project funded by HE STEM involving 20+ HE institutions and focusing on the use of e-assessment. Many HE institutions are using e-assessment in mathematics and statistics and have reported benefits in pedagogy, feedback and resource saving especially at the teaching-intensive early stages of degree courses. However, there is little sector-wide structured dissemination, which results in a duplication of effort not only in implementation strategies, but also in the writing of questions and the creation of local e-assessment systems. There is now a need for this experience to be consolidated, shared and jointly reflected upon by the mathematics and statistics community, so that the potential contribution to mathematics and statistics teaching and learning as a whole can be evaluated. A sector-wide approach has been adopted and this project brings together a large group of university practitioners (20+), with professional providers of e-assessment technology as well as the eAssessment Association, to coordinate the sharing of UK experiences. The key aims are to:

Edited by Marie Joubert, Alison Clark-Wilson and Michael McCabe
• Demonstrate benefits to institutions, staff and students.
• Address barriers to introducing sustainable and effective e-assessment.
• Give supportive and evidence-based guidance.

Outcomes
• Information for all STEM disciplines on present benefits and developments in e-assessment for mathematics and statistics in HE. This includes open-source e-assessment systems.
• Sharing of best practice through wide-scale dissemination to all STEM disciplines.
• Understanding barriers to the sustainable use of e-assessment in STEM disciplines and to potential future developments in HE.
• Criteria and implementation strategies in terms of pedagogy and resource implications, tested by our case studies at Nottingham and Leicester, helping departments and schools introduce sustainable e-assessment into their curriculum.
• A web-based e-assessment advisory resource for mathematics and statistics.
• Models for staff development in using e-assessment.

Video tutorials in teaching mathematics
Matija Lokar, Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

At the Faculty of Mathematics and Physics, University of Ljubljana, students use many computer-based mathematical tools. In a special course entitled ‘Computer tools in mathematics’ various tools and the possibilities of their application in practical problem solving are studied. A part of the resources available to students are e-resources in the form of interactive tutorials, which explain the basic features of the tools used and demonstrate how they are used in the solving of different mathematical problems. The talk presents an approach towards creating such tutorials where GeoGebra is used. The software used in developing the tutorials is a freeware program, DebugMode Wink. These tutorials are produced by capturing screen snapshots while using GeoGebra. On each screenshot explanation boxes, audio, titles, navigation buttons and more can be added. The usage of navigation buttons allows the user of the tutorial to follow the construction more easily and to adjust the speed of the presentation to match his/her level of understanding. The e-resources created for this course are also used in the NAUK project (www.nauk.si), where free e-resources from mathematics, physics, logic and computer science can be found, and also in the EU Comenius project European Development for the Use of Mathematics Technology in Classrooms (EduMatics).

Online support for a distance-learning mathematics course
Tim Lowe, Department of Mathematics and Statistics, The Open University, UK

The Open University has recently replaced its introductory mathematics module. The new module was launched in February 2010 and is offered twice each year, with at least 1500 students expected per presentation. Whilst based around printed course materials, the module makes extensive use of the Open University’s Moodle-based VLE to support and encourage learning.

In addition to online student forums and short videos of tutors explaining worked examples, several topics are supported by online applications to allow students to investigate the mathematical concepts involved.

Each of the 14 units has an associated “Practice Quiz” to enable students to check and consolidate their understanding of the content of the unit. Students can take these quizzes as many times as they wish and will (probably) get a different set of questions each time. Up to three attempts at each question are permitted, with feedback and graduated hints being given after each incorrect attempt to facilitate and encourage independent learning. Complete worked solutions to each problem are also provided.
Lateral (steering) motion of the car. Maple calculus and animations use accurate parameters from the fluid dynamics needed to optimise the car design. To illustrate some of the intermediate level to find the acceleration, speed and distance travelled right through to the advanced computational generation. If the project succeeds its impact is expected to be even greater, just as the Apollo 11 Moon landings inspired a younger generation and encouraging them to follow careers in these areas. For the project to succeed, the overarchingle objective of the project is to stimulate UK growth and investment in STEM (Science, Technology, Engineering, and Mathematics) subjects. Bloodhound SSC is already attracting the interest of a younger generation and encouraging them to follow careers in these areas. If the project succeeds its impact is expected to be even greater, just as the Apollo 11 Moon landings inspired a generation.

Mathematical activities relating to Bloodhound range from the use of arithmetic, algebra and calculus to find the acceleration, speed and distance travelled right through to the advanced computational fluid dynamics needed to optimise the car design. To illustrate some of the intermediate level mathematics, a final year undergraduate project has used the Maple computer algebra system to explore the horizontal (acceleration and deceleration), vertical (suspension), circular (wheels) and lateral (steering) motion of the car. Maple calculus and animations use accurate parameters from the design specification of the car to simulate its motion. Many other extensions to this work are possible.
During the past five years, there has been a major UK commitment at government, university and school level to investment in STEM subjects. Although the Bloodhound SSC team is only funded by modest donations from corporate sponsors and members of the public, its educational legacy may just as important for inspiring young minds.

ICTMT delegates will be able to view a full-scale Bloodhound show-car outside the Portland Atrium from Monday to Wednesday of the conference. The Bloodhound driver, Andy Green, will be giving an invited talk about the project on Wednesday evening at 18.00.

Bloodhound SSC www.bloodhoundssc.com

Mathematics ... and the land speed record! Final year mathematics project by Chris Blow http://userweb.port.ac.uk/~mccabeem/projects2011/chrisblow.pdf

Research informed teaching projects in pro-am astronomy with Maple, MATLAB and GeoGebra

Michael McCabe, University of Portsmouth, UK

Graham Bryant, David Harris, Dave Briggs, Steve Knight, Carol Bryan and Robin Gorman, Hampshire Astronomical Group

Since 2006, the UK research informed teaching initiative has provided funds (£10.1 million in 2010/11) for linking teaching and research, including the engagement of undergraduate students in research activities. While research into mathematics education is well established, taking mathematics education into research at university undergraduate level is extremely difficult.

There is a longstanding tradition of collaboration between professional and amateur astronomers, which began more than a century ago with the first measurement of a galaxy redshift (Henden, 2006). This paper reports on a project funded by the University of Portsmouth since 2009 to enable final year mathematics undergraduates to engage in research informed activities within pro-am astronomy. Progress in these projects has been achieved through the use of mathematical software (Maple, MATLAB, GeoGebra and Excel) to model recent astronomical observations.

Student motivation has been increased by providing the additional personal support of a mentor at the local Clanfield observatory. Students have been helped to collect their own observational data for analysis and mathematical modelling.

One specific project, to which students have actively contributed, is in trying to unravel the mystery of Epsilon Aurigae, a star which has been observed since the mid-19th century. Epsilon Aurigae is eclipsed by a proto-planetary nebula every 27 years for a period of two years. The latest eclipse ran conveniently from August 2009 to May 2011 and has attracted international attention (Hopkins & Stencel, 2009). Mathematical modelling of the transit uses several different numeric, algebraic and geometric methods to produce a light curve for comparison with both local and international observations. For example, Monte Carlo simulations using Matlab have generated a realistic model of the Epsilon Aurigae system.

A similar project has been the observation and modelling of exoplanet transits (Haswell, 2010), the passage of planets beyond our Solar System across the line of sight towards their parent stars. Around 25% of the 500+ exoplanets detected during the past 15 years have been discovered by this method. The circular geometry of the eclipsing planet simplifies model calculations, although the observations are harder to make. Projects to observe asteroid rotation, solar activity and lunar topography are closer to home, but equally challenging.

Traditionally university undergraduates work on “clean” virtual simulations, allowing results to be guaranteed, but these projects give experience of the difficulties and frustrations associated with unpredictable scientific activities. Students are able to visit the observatory regularly both to gain experience of the practical issues involved and to discuss their progress. By analysing data and testing out different possible mathematical models, students can make a genuine contribution to pro-am astronomy in partnership with others.

References
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www.hposoft.com/Campaign09.html

Introducing basic mathematical modelling concepts using sporting applications in MATLAB
Chris Mills, University of Portsmouth, UK

Customised MATLAB scripts were used as an interface to demonstrate basic mechanical and modelling principles using sporting applications. Students began by applying constant acceleration equations to the simple sporting activity of kicking a football and were asked to determine the range of a kick based on initial angle and velocity conditions. The simple MATLAB projectile model was used to verify their calculations. The subsequent task involved optimisation of parameters within the constraints/simulation bounds to determine optimal performance criteria. What was the maximum distance the football could be kicked? What combinations of angles and velocities could be used to make a 20 metre pass?

Following this a more complex model of the cushioning properties of landing mats was introduced, which was based upon ongoing research in the area. After introducing the fundamental mathematics of modelling springs and dampers, students were asked to optimise the landing mat parameters to minimise the risk of injury within given constraints. A combination of basic mathematics, knowledge of the sporting activity and hands-on experience with the computer models seemed to engage students and help them visualise key biomechanical and modelling concepts within the 2010–2011 cohort.

Secondary school mathematics learners constructing geometric flow-chart proofs with a web-based learning support system
Mikio Miyazaki, Faculty of Education, Shinshu University, Japan
Taro Fujita, Faculty of Education, University of Plymouth, UK
Youichi Murakami, Sun First, Japan
Naoki Baba, Toyono Junior High School, Nagano, Japan
Keith Jones, School of Education, University of Southampton, UK

As international research confirms, many secondary school students can find it difficult to understand and construct mathematical proofs. In this research project, we are developing a web-based learning support platform (available in both Japanese and English) for learners who have just started learning formal proof in geometry: www.schoollmath.jp/flowchart_en/home.html

In designing this learning platform we are adopting flow-chart proofs that include both open and closed problems in geometry that involve the properties of parallel lines and congruent triangles. By using technology based on Adobe Flash, learners complete proofs by dragging sides, angles and triangles to cells and our system automatically transfers figural to symbolic elements so that learners can concentrate on logical and structural aspects of proofs. The system identifies errors by referring to a database of acceptable answers classified into four categories. Learners then receive relevant feedback in accordance with the four types of error.

In this paper, we introduce our system through the theoretical and pedagogical underpinnings we are using and illustrate how the system impacts on the teaching and learning of proof. We will also show how flow-chart type proofs with open and closed problems are suitable for learners who have just started learning formal proof in geometry in lower secondary school. Then, we introduce fundamental technological ideas and the principles of feedback given to students when they make errors during their proof studies with the system. We show some examples of learners’ activities with
the system selected from our pilot studies, illustrating how it helped the students’ proof construction processes and promoted their mathematical thinking.

Solving contextual problems with the spreadsheet as an environment for the development of algebraic thinking

Sandra Nobre, Escola Básica 2, 3 Prof. Paula Nogueira, Olhão, Portugal
Nélia Amado* and Susana Carreira* (* presenters), University of Algarve, Portugal and University of Lisbon

Several authors (e.g., Ainley et al., 2004; Dettori et al., 2001; Rojano, 2002) view the spreadsheet as a powerful tool in mathematical problem solving, particularly in the development of algebraic thinking embedded in problem solving activities. One of the gains of connecting algebraic thinking and the use of spreadsheets is the creation of a significant environment to induce students into algebraic language. The spreadsheet has proved to be a relevant pedagogical resource in the construction of algebraic concepts, especially in what concerns working with functional relations, sequences and recursive procedures that are extensively used in solving mathematical problems. Using the spreadsheet in the context of problem solving emphasises the need to identify the relevant variables involved and fosters the search for variables that depend on other variables, resulting in composed relations. The definition of intermediate relations, by means of spreadsheet formulas in intermediate columns, meaning the decomposition of more complex relations in chained simpler ones, is a special feature inherent to the use of the spreadsheet that amounts to important results in solving algebraic contextual problems (Carreira, 1992; Haspekian, 2005). Moreover, the spreadsheet offers an algebraic organization of the structure of a problem through an apparently arithmetic approach (Haspekian, 2005).

In this paper we report and discuss a contextual problem solving task relating the number of seats to the number of people in a restaurant, which was proposed to a class of 8th grade (13–14 year-old) students. These students had been developing a reasonable experience in the use of the spreadsheet to model relations within contextual problems and chose to use this tool to solve the aforementioned problem, engaging in the process of translating relations between variables and combining them in chained models, while working with fractions, multiples, and expressions. We intend to highlight the role of the tool in students’ processes of variable identification and translation of the problem conditions, their numerical approaches to algebraic models and their experimental forms of finding solutions to equations.

References


Old proofs, new technologies
Margarida Oliveira, Escola E.B. 2,3 Piscinas Lisboa and Universidade do Minho, Portugal
Suzana Nápoles, Universidade de Lisboa, Portugal
This paper focuses on a study that is being developed, in a school in Lisbon, on the impact of technologies on the teaching and learning of mathematics.

The objective of the work presented is to understand how designing simple computer programs, or translating a mathematical algorithm into a programming language, can improve student learning of mathematics, when compared with conventional teaching. It provides multiple and complementary representations to teach a mathematical concept.

It also aims to illustrate how computer programming can be considered a credible tool to deepen understanding of mathematical concepts and their specific applications, and to help develop students’ capacity to solve problems in mathematics.

It is crucial to understand the importance of mathematical representations in the teaching and learning processes as tools for the comprehension of the history of mathematics both in its cultural and scientific facets.

The methodology used involves three groups of students: one of the third cycle of basic education (7–9th grades), a high school class (10–12th grades) and a third group of college students. It will take place during three consecutive years. An important factor for its success is to develop a systematic and continuous work from the very beginning of the school year, so that the basic procedures involved and the programming needed to develop applications can be rapidly internalized by the students.

The teachers involved in this project are regular teachers; the one for the third cycle is an early career teacher, the secondary school teacher has considerable experience, and we have a college professor.

The work takes place in teams who carry out regular meetings to make necessary adjustments and to evaluate progress.

Alongside the work with students, the two school teachers (third cycle and secondary education) will also receive training. These teachers will develop materials for their students, thus allowing assessment of how this kind of work can be extended to all teachers.

The paper presents two typical examples present in the national curriculum for mathematics education. One concerns the demonstration of the well-known Pythagorean theorem and the other the resolution of a certain type of second degree equations given by the Arab mathematician al-Khwarizmi.

Designing windows for researching students’ experiences of dimension
Nicole Panorkou and Dave Pratt, Institute of Education, University of London, UK
This study explored the experiences of dimension among young school children. A phenomenographic research study was designed (Marton, 1981) gathering meanings of dimension from 24 students during four situations. Data were collected using clinical interviews (Hunting, 1997), accompanied with the design of tasks using Elica software, physical objects, the film Flatland, and Google SketchUp in each of the four situations respectively. Within this overall methodology, there was a need to structure that experience. Whereas in some phenomenographic studies the aim has been to consider the depth of knowledge by forming a hierarchy between surface and deep approaches of a phenomenon, here the connection between setting and meaning emerged as the key-structuring factor. It therefore became appropriate to analyse that setting from the perspective of it being a ‘window’ on children’s experience of dimension and to see that experience as articulated in terms of ‘situated abstractions’ (Noss & Hoyles, 1996).

This paper presents an analysis of each situation by taking into account the representation of dimension in the tasks, the types of vocabulary resources available to students, and the level of
students’ involvement and illustrates how each setting differentiated the generation of meanings among the children. An exploration of the potential and the constraints of the tools showed that SketchUp, together with the use of its dimensional tools, offered particular affordances for the mathematical expression of dimension, even though the children were very young (10 years old). Building on the idea of ‘designing for abstraction’ (Pratt & Noss, 2010) and the ways that designing and modelling fosters the utility of mathematical concepts (Ainley, Pratt, & Hansen, 2006; Simpson, Hoyles, & Noss, 2005), the examination of the four situations gave an insight into what makes a window expressive both to the student phenomenon and the researcher–student relationship.

**Teachers’ attitudes and beliefs about using ICT in teaching mathematics**

Sirje Pihlap, University of Tartu, Estonia

Several research studies confirm that using ICT in teaching mathematics motivates students in their learning process and facilitates their understanding of the subject (Baki & Güveli, 2008; Luik, 2004; Pihlap, 2009, 2010). Even though it has been possible to use computers in teaching mathematics for a long time, in many states there is a problem as the use of computers in classrooms is still low (Watson, 2002; Prei, 2010). In Estonia computers have been used in teaching mathematics for about 15 years. During the last 10 years, however, they have been used more widely. According to the present Estonian curriculum, it is not obligatory for teachers to use computers in teaching mathematics. It depends on the teacher if, and how, the ICT possibilities are used in mathematics lessons. A qualitative study was conducted to find out mathematics teachers’ attitudes and beliefs towards using ICT. Based on the interviewing of a focus group it is explained how teachers assess the necessity of using ICT and its impact on learning results and learning motivation. Also, the factors that determine a teacher’s decision on whether to use computers in teaching mathematics is clarified. Finally, it is pointed out as to what kind of help is needed by teachers to integrate computers into mathematics instruction.

**Using interactive GeoGebra-based educational assistance for introducing concepts connected with averages – preliminary research results**

Marzena Plachciok, Gimnazjum nr 32, Kraków, Poland

The mathematics curriculum contains many interesting concepts connected with different notions of averages. Some of them do not seem to be too difficult for pupils, but are nevertheless rarely used. The main difficulty seems to be the transfer from numeric accounts and symbolic exemplars to geometrical conceptions. The geometrical interpretation of averages, Cauchy’s theorem and similar statements present a barrier to understanding that can sometimes be difficult to overcome even for stronger pupils.

The fact that teachers do their best during classes is not enough to overcome the problem. Here the idea of didactic assistance, which is available “outside the classroom”, becomes relevant. Interactive material, accessible to pupils via the Internet, may be helpful because not only can it explain but it can also widen topics using additional materials. Students can then work with it in their own individual way.

In my lecture I shall demonstrate such a didactic conception by showing how the interactive GeoGebra software package, published on an e-learning platform, can be used to introduce concepts connected with averages. This assistance includes not only traditional material and classic tasks, but goes beyond the range of the current curriculum. In my research I concentrate on 16–17 year-old pupils. The results of my research show not only the positive aspects of using such help but also the risks of teaching in this way.

**Analysing T-algebra solution files to improve student support**

Rein Prank, University of Tartu, Estonia

T-algebra (http://math.ut.ee/T-algebra) is an interactive learning environment covering four areas of school mathematics: calculation of the values of integer expressions; operations with fractions; solving linear equations, inequalities and linear equation systems; and operations with polynomials.
The student solves tasks step-by-step. Each solution step in T-algebra consists of two substeps: 1) Selection of the operation from the menu and marking the operand(s) in expression. 2) Entering the result of the operation. The program verifies each substep and displays error messages. The student should correct any mistakes before going on to the next substep. Explicit input of the intentions of the student in substep 1 enables them to avoid entering the results of meaningless conversions. The program also contains an automatic solver and is able to give hints in any situation. T-algebra saves students’ solutions (both finished and unfinished) in solution files together with all error and help request situations. The main program and additional teacher tools for reviewing the work of the group on an assignment enable: the viewing of solutions and error/help situations as they looked on the student’s screen; the creation of student/task tables of solved/unsolved tasks, step counts, solution times, error counts, hint counts; the creation of message/task tables to indicate how many times different error messages have appeared (ordered by frequency); and the performing of data mining for appearance of concrete expressions, messages, error categories, and sticking points in the solution process. This paper describes how we used the reviewing facilities to identify the difficulties caused by T-algebra itself. As a result, we made some changes in the operation of the student program, reformulated several instructions for steps and error messages and developed additional recommendations for teachers on how to obviate certain difficulties. For associated documentation email: rein.prank@ut.ee

Didactic computer games as a tool for discovering reductive reasoning

Tadeusz Ratusiński, Pedagogical University of Kraków, Poland

Reduction is a very effective type of reasoning, especially for solving mathematical problems. One of the primary aims of school teaching is to demonstrate such a way of thinking and argumentation to pupils. However, it is hard to teach reduction in a natural way in a school setting. Could some particular type of mathematics problem help with this? Well-chosen educational computer games could perhaps provide a solution. They can discreetly provoke situations in which pupils discover the reductive method in order to win the game. In my lecture I’ll show the results of research in the use of educational computer games for developing reductive reasoning in 10–13-year-old pupils. I have tested a few specially prepared PC games based on Flash technology “outside the classroom”. The results highlight the positive aspects of using such a tool, especially when students must change their way of thinking from visual perception to numerical inference.

Using sport to engage and motivate students to learn mathematics

Carol L. Robinson, Loughborough University, UK

In recent years concerns have been expressed about the level of student engagement in the learning of mathematics at university. There can be a wide variation in the level of prior knowledge of students and many do not appreciate the importance of mathematics for their course. Often they are taught in large classes and poor attendance at lectures and tutorials is not uncommon. Universities are finding that they have to look at ways of addressing the issues of how to motivate and assist such students in their learning of mathematics. This paper describes how technology has been used to motivate students of Sports Technology in the learning of mathematics at Loughborough University. Applications from the world of sport are introduced whenever it is appropriate and MATLAB is taught to enable the students to solve realistic problems. The mathematical background of the students involved is varied and the required pre-requisite is a GCSE grade A in mathematics. Group projects include modelling the velocity of a downhill skier, the height a pole vaulter can clear, the effects of lift and drag on the length of drive of a golf ball, and the size of parachute required to ensure a smooth landing. All of these require the use of the MATLAB. In-class engagement is enhanced by the introduction of electronic voting systems. Questions involving sporting applications can be posed in-class and immediate feedback received. The effect of introducing such material, on attendance and progression rates, and student engagement is also reported. For associated documentation email: c.l.robinson@lboro.ac.uk
Mathematical modelling with technology: the dynamic role of representations

Ornella Robutti, Ferdinando Arzarello and Francesca Ferrara, Dipartimento di Matematica, Università di Torino, Italy

Our research deals with the use of different technologies in problem-solving activities at secondary school level.

The activities are of two kinds: one deals with sequences of natural numbers, the other with geometry and motion. The overall aim of the activities is to find a model (respectively for a sequence and for a geometric configuration) to describe the situation, and to represent this model in various ways (using a table, a recursive function or a close formula for the sequence, and using a construction or a Cartesian graph for the geometric situation).

The tasks have been part of a teaching experiment where students used not only paper and pencil, but also technological tools: a spreadsheet in the case of sequences, and GeoGebra and TI-Nspire for geometry. Results of the teaching experiment were analysed, in order to prepare materials for teacher training in a Moodle platform for e-learning (EdUmatics – Comenius Project).

Our analysis focused on the passage from static to dynamic representations and back, to observe how technologies may foster dynamic cognitive processes of students in the solving of mathematical problems. Analysed data tell us that the dynamic features of technology support the genesis of conjectures, and their validation, along with the choice of independent and dependent variables.

Furthermore, the use of dynamic representations in modelling situations enhances the dialectic between the empirical side and the theoretical side of mathematical objects. For example, in TI-Nspire, the Data Capture function (used to model a situation) allows data collections in a way similar to physical samples. On the other hand, DGS let students choose various quantities as independent variables, allowing for different dynamic representations of one situation, and for the exploration of corresponding mathematical objects.

References


Mathematics lecturers’ views of the advantages and disadvantages of electronic and traditional assessment

Peter Rowlett, University of Birmingham, UK

This presentation will report on a study into mathematics lecturers’ views of the advantages and disadvantages of assessment and feedback with and without the use of computers. The study has captured the views of users and non-users of e-assessment systems at universities with large cohorts of academically strong students and those with relatively small cohorts of relatively weak academic backgrounds. This research is particularly focused on identifying when lecturers feel traditional or electronic assessment is more effective and appropriate and why they think this is so. Questions were around the themes: the lecturer’s use of technology in assessment and appropriateness of traditional and electronic assessment; student experience and benefit; practicalities of setting and marking different assessment types; and, efficiency and effectiveness of traditional and electronic assessment methods.

This session will present the findings of this research and the issues that are raised relating to appropriate and effective use of e-assessment in HE mathematics programmes. Particularly, this
research aims to identify when lecturers believe e-assessment and automated feedback can have a positive effect on learning and student experience, when traditional offline assessment is preferred, and why.

**Teachers engage in peer tutoring and course design inspired by a professional training model for incorporating technologies for mathematics teaching in Mexican schools**

Ana Isabel Sacristan, Center for Research and Advanced Studies (Cinvestav), Mexico

Ivonne Sandoval, National Pedagogical University, Mexico

Nadia Gil, Federal Administration of Educational Services – DF (SEPDF), Mexico

Two years ago, at ICTMT 9, we presented results from a long-term professional development Master’s programme for in-service teachers that focussed on the incorporation of technologies for mathematics teaching and learning. In the past five years, we have had the opportunity to document the impact of the programme on the participants’ academic lives.

In the previous paper we discussed the participants’ experiences and transformations during the development programme. Here, with the benefit of another two further years of data collection and analysis, we go deeper into how the participants engaged, by their own initiative, in peer training (inspired on our own model) and course design for the use of DT. We also present some of their own findings, some of which illustrate the challenges of incorporating technologies for teaching mathematics in Mexico at different levels.

In particular, the distance between reality and the will of official policies is highlighted by the experiences of two of the participating teachers who belong to Mexico’s “Tele-secondary” (Telesecundaria) School program – a 40-year old distance-education model of the Ministry of Education, where learning is structured through learning and content guides and multimedia resources, with one teacher–promoter for all subjects, which was renewed in 2006 to include computer media labs and materials. Despite such reforms, it is difficult for teachers, mainly due to lack of training and resources, to incorporate technologies, particularly for mathematics (where technology use has tended to remain on a technical level).

The teachers who participated in the above-mentioned Master’s programme developed new perspectives on how to incorporate technologies in a meaningful way in mathematics education; they then spontaneously engaged in tutoring fellow teachers, designing training courses for them, and using many ideas from the pedagogical model that we had implemented with them (which centred on collaborative reflection and discussion of daily experiences when attempting to incorporate technologies in mathematics teaching practice).

**The problem of the digital divide for mathematics teachers in developing countries**

Ana Isabel Sacristan, Sandra Evely Parada and Lourdes Miranda, Center for Research and Advanced Studies (Cinvestav), Mexico

In this digital era, there is a need to incorporate technologies into educational practices. In our country (Mexico), there have been many policy and educational reforms with this aim. However, many factors create a gap between this political will and the reality of school teachers, particularly in non-developed countries, where resources, access to digital technologies and training are scarce. Our aim was to look at how mathematics teachers use technologies in their practice. In doing this, we became aware of some deep limitations that teachers have.

In 2010, we worked with a community of 71 middle-school mathematics teachers and asked them to sign up to online forums and web resources that we had set up for them. We were surprised that this was a big obstacle. Most teachers admitted not knowing how to use a web browser; over a third of them did not even have an email account; and we also observed that most teachers had difficulties using the hardware (the mouse, the keyboard, and so on). Although these teachers admitted their limitations, they also expressed their fear of using technologies, of exposing their lack of digital
competencies and of asking for help; a vicious circle, as these fears prevent teachers from developing the competencies they need. Despite remedial actions on our part, many difficulties have continued.

In another part of the research, we surveyed 140 high-school mathematics teachers who do have basic ICT skills, and observed many of them. Less than two-thirds of them use technologies at all in their lessons, generally only once per school year, and then usually just for facilitating the construction of graphics.

These results point to the gravity of the digital divide in Mexico: digital competencies and use are scarce, and create an obstacle towards harnessing technologies for enriching the mathematical teaching and learning.

Some approaches to advanced mathematical education in a multiprofile lyceum

Nikolay M. Salnikov

State Educational Institution of Moscow Lyceum 1586, Russia

In this talk I’m going to give an overview and some details of the educational techniques in mathematics adopted in the lyceum 1586. The lyceum is working with students of the five final grades specialising in science. Within the framework of educational experiment special programs and courses were developed and introduced for these grades. I’m going to focus on three of them.

1) Mathematics. I’ll briefly explain the main philosophy of teaching mathematics in our lyceum and about special programs for advanced studying.

2) Computer modeling. The course basically consists of explaining some ideas of numerical methods and then applying them to concrete models coming from the area of interest of the student (physics, chemistry, biology, and so on). This idea was developed in collaboration with the Department of Biology of Moscow State Lomonosov University. It permits one to explain advanced mathematical results in a manner understandable even for lyceum students.

3) Probability and statistics. A recent course, introduced in all the grades of our lyceum. In wise combination with programming it offers the possibility to deeply understand random events.

I will also comment on the possibilities this approach is giving for establishing the interdisciplinary relations, and on some experience in teaching of mathematics and application of it at Moscow State Lomonosov University. I shall focus on concrete examples of the application of our methods.

VITALmaths – a bank of video clips for autonomous learning of mathematics

Duncan Samson, Education Department, Rhodes University, South Africa

Helmut Linneweber-Lammerskitten, School of Teacher Education, University of Applied Sciences Northwestern Switzerland

Marc Schäfer, Education Department, Rhodes University, South Africa

VITALmaths is a collaborative research and development project between the University of Applied Sciences Northwestern Switzerland (PH FHNW) and Rhodes University. The project involves the development, dissemination and evaluation of short mathematical video clips designed specifically to encourage the autonomous learning of mathematics. The project has three main objectives: (1) to produce short video clips designed specifically for the autonomous learning of mathematics; (2) to establish and maintain a website to house and freely distribute these video clips; and (3) to establish a research agenda around their use and efficacy. The video clips are silent, purposefully very short (typically less than three minutes) and specifically incorporate natural materials using a stop–go animation technique to develop and explore a variety of mathematical ideas and themes.

These themes are developed in a progressive manner in a way that purposefully avoids specific pedagogical imperatives or predetermined outcomes. These video clips can be used in the preparation of lessons, for personal conceptualisation of mathematical concepts, and as motivational and explanatory tools, with the emphasis lying on teachers and learners using them as autonomously and independently as they wish. A dedicated website has been established to house this growing databank of video clips (www.ru.ac.za/ VITALmaths) from which the video files can either be freely
downloaded or streamed. This paper engages with a number of theoretical and pedagogical issues relating to the design, production and use of these video clips. In addition, synergies between the autonomous learning imperative of the project and the potential autonomous affordances offered by mobile technology are explored. A number of video clips will be shown in the presentation.

**An assessment package for Maxima**

Chris Sangwin, University of Birmingham, UK

Many automatic assessment systems for mathematics use a computer algebra system (CAS) to automatically generate structured problems, establish the mathematical properties of students’ answers and generate feedback. Since Fenishal’s work in the late 1960s there have been many different implementations. Examples include MathWise, Aim, MapleTA, and STACK. CAS is usually designed to “do a calculation” rather than “establish a property of an object”. These are subtly different problems, and existing desktop CAS are more or less suitable for establishing properties. In this paper we report research into which properties are most useful for assessment of mathematics problems, and present an independent “Assessment Package” for the CAS Maxima which establishes them. This package has been factored out from the STACK CAA system to enable it to be used independently. We also report some theoretical limits on the extent to which such algorithms can be guaranteed to terminate, and provide examples of their use.

**A technologically enhanced mathematics curriculum for teacher education: an exploratory study**

Fernando Luís Santos School of Education Jean Piaget, Almada, Portugal

António Domingos, Faculty of Science and Technology, New University of Lisbon, Portugal

Along with the challenges that the Bologna Process has brought to Higher Education in Portugal policy makers have expressed growing concern about science, technology and mathematics education and, with less satisfactory results in some international reports, there are big changes happening. The introduction of a new mathematics curriculum for basic education is reflected in teacher training, either by the particular definition of a new kind of student or the need for new methods of teaching and learning of mathematics.

It is argued that the curriculum should provide a sufficiently strong mathematical background and flexibility so that pre-service teachers can handle and create conditions for students to learn mathematics based on the three problems facing mathematical training: to identify content relevant to mathematical education; to understand how knowledge should be learned; and what we need in order to teach mathematical concepts to children.

Theories such as advanced mathematical thinking, self-regulated learning, as well as an increased use of technology in the classroom based on methodologies studied in the context of STEM (Science, Technology, Engineering and Mathematics) education, support this paper as a starting point for a broader investigation.

It is necessary to assess the changes to the mathematics curriculum in pre-service teacher training, in terms of content, methodologies and pedagogy, and this paper presents the rationale and structural changes to the curriculum in a basic education course at a College of Education in Portugal by a technologically enhanced learning environment. It focuses on a preliminary study of the use of Moodle as a tutoring environment using forums, demonstration videos and problem solving.

Preliminary results indicate statistically significant differences for students who use the new curriculum with technology and those without this support in geometry.
The redesign of a quantitative literacy class: student responses to a lab-based format

Nicole Scherger, Elgin Community College, IL, US

The purpose of this study was to observe students’ retention, success, and attitudes towards mathematics in a quantitative literacy course taught in a lab-based format, utilizing Microsoft Excel. Quantitative literacy develops skills in problem solving, logical analysis, use of mathematical models and functions, statistical and graphical representation of data, and decision making. Work on this course was driven by renewed attention given to the importance of quantitative literacy. The seminal work in this area, Mathematics and Democracy: The Case for Quantitative Literacy (National Council on Education and the Disciplines, 2001), calls on colleges to re-examine how they prepare their students to be citizens capable of dealing with the numerical and quantitative needs of the future.

Because of the growing need for today’s students to not only be competent consumers of quantitative information, but to also be technologically literate, the redesigned course implemented the daily use of Excel in classroom demonstrations, group activities, and individual assignments, and utilized data from many fields of study. The teaching techniques used in the course, including collaborative groups, practical content, open communication and dialogue, and the use of writing, are reflective of feminist pedagogy, which was the philosophical framework of the study.

Retention data revealed that the retention rate in the redesigned course was 90.3%, compared to 84.7% in the traditional section; grades data revealed the success rate in the redesigned course was 72.1%, compared to 66.1% in the traditional sections. Results of mathematics attitude surveys showed marginally significant growth (p = .05) in students’ attitudes towards the relevance and utility of mathematics in the redesigned course compared to the traditional course. Students taking the redesigned sections additionally showed significant growth compared to their traditional counterparts in their attitudes towards real world application problems (p < .01), the use of computers (p < .01), and their consideration of taking additional mathematics courses (p = .04).

Change of and transfer between representations – especially between digital and paper-and-pencil representations

Barbara Schmidt-Thieme, University of Hildesheim, Germany
Hans-Georg Weigand and Andreas Bauer, University of Wuerzburg, Germany

The flexible use of representations is a major goal in mathematics and mathematics education. Mathematical knowledge is very much related to the use and change of, as well as the transfer between, representations. There is a wide range of research to this topic, e.g. constructing adequate representations, interpreting representations, working with representations.

Most of this research concentrates either on representations that are generated by computers or representations or notations on paper-and-pencil work. A main obstacle concerning the understanding of representations, however, is the discrepancy between these two types of representation. How do they fit together? Which ones can be seen as equivalent? How is this related to the mathematical content? How do they differ in restricting or opening mathematical actions?

This is particularly a problem if students work with handheld technology in the classroom. Usually they have to give notes of the solution on a piece of paper, especially when they take a test. In a long-term empirical investigation we experienced many problems concerning the relations and the transfer between notations on the computer screen and on paper. There are also some mathematical obstacles produced by digital representations.

To investigate these obstacles one needs a theoretical framework for representations of mathematical ideas and concepts that will be able to integrate both ‘digital’ and ‘paper-and-pencil’ representations. In this talk we shall present a framework based on cognitive psychology and linguistics, which will enable us to categorize different representations.

Subsequently, we shall discuss and analyze some examples of how students switch representations.
This will lead us to some reflections on how to describe the competence of using representations and changing representational systems and how to support students in achieving this competence.

The effect of using Google SketchUp when teaching fifth graders about the surface area of composite solids

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The main purpose of this study was to investigate the learning effect of Google SketchUp software when integrated into the teaching instructions for fifth graders finding the surface area of composite solids. This research was conducted under a quasi-experimental method, with a non-equivalent pre-test–post-test control group design. There were 111 fifth graders (the subjects) collected from four classes in Keelung city, and divided into the control group (n = 55) and the treatment group (n = 56). All the subjects accepted a mathematics proficiency test on the surface area of composite solids before the teaching experiment was conducted. The treatment group accepted the ‘SketchUp auxiliary instructional model’ and the control group accepted the ‘tradition instructional model’ for finding surface area of composite solids. Two-way ANCOVA was applied to investigate the learning achievement performed by the two groups on finding the surface area of composite solids. The results were as follows:

1) Students who accepted the ‘SketchUp auxiliary model’ had better learning results than students who accepted the ‘tradition instructional model’.

2) In each group, there was no significant difference in learning effect caused by gender, but a significant difference was shown in the boys’ learning results between the groups.

3) There was a significant difference between teaching methods and learning attitudes. The students who had accepted the ‘SketchUp auxiliary instructional model’ exhibited an improvement in their attitudes towards learning mathematics.

The use of technology in teaching secondary school mathematics

Duduzile Sibaya, University of Zululand, South Africa

David Clarke, International Centre for Classroom Research, Australia

Patrick Sibaya, University of Zululand, South Africa

Technology is recognized worldwide as being valuable in the teaching of science and mathematics. Research conducted in this field has focused on the pedagogical use of technology in schools, access to technology and the impact of technology on the academic achievements of learners. The purpose of this study is to investigate the different types of technology that mathematics teachers use in teaching secondary school mathematics, and how this technology is integrated in the teaching of mathematics. To this end, a random sample was chosen consisting of 350 secondary school mathematics teachers in the Empangeni district, KwaZulu-Natal, South Africa.
Computerised assessment and student attitudes

Colin Steele, University of Manchester, UK

Computerised assessment in mathematics has been used at the University of Manchester since about 1998 (and possibly before) using a variety of programs and platforms, gradually moving towards more sophisticated ones.

With each new development, new possibilities emerge in terms of the kinds of question that can be asked and the kind of processing that can be done to a student response. However, this is accompanied by a cohort of students needing to operate the system in a slightly different way to previous cohorts. There is also a process among the staff in finding the best mode in which the students can carry out assignments successfully.

When taking part in a computerised assessment, students have to enter answers in some way, e.g. select an option from multiple-choice, enter a number, put in a mathematical formula etc. Each situation entails some amount of discipline, and some students may show more than others.

A similar situation exists with the mode in which the assessment is done. Clearly, there are some situations where supervision is desired but if students are to try assessments several times and build up understanding, there must be a system where such repeated practice is rewarded.

This presentation looks at the various forms of computerised assessment in mathematics used at UMIST and the University of Manchester, concentrating on the reaction of students to the assessment.

3D modelling in teaching and learning geometry

Petra Surynková, Charles University, Prague, Czech Republic

This article addresses an application of computer modelling in the teaching and learning of geometry. Our aim is to increase the interest of students in studying synthetic geometry at secondary schools and colleges. The main field of our interest is descriptive geometry – geometric constructions, projections, the geometry of curves and surfaces. One possible approach to improving the study of geometry is the integration of computer software in the teaching process. This seems to be interesting, attractive and motivational for students. The use of computers in education is modern. We use the Windows program Rhinoceros, a NURBS-based package for visualization, for proving geometric problems in the plane and in space or for demonstrating the application of geometry in practice. We deal especially with the geometry of curves and mainly surfaces. We now have a large collection of pictures that illustrate examples of surfaces in building practice and architecture. We also show the use of software on some concrete examples. Of course, we refer to the advantages and also the disadvantages of computer modelling. We mention how to use geometric modelling systems for image creation or illustrations of geometric problems. The outputs can be used in some publications and also for home schooling and e-learning. We have web pages with a database of 3D models and geometric tasks in the plane and in space, and we provide access to these resources to our undergraduate students.

An overview of approaches for producing mathematics question banks and the automatic creation of numerical calculation questions in Questionmark Perception using macros in Excel

Giles Tewkesbury, Simon Chester and David Sanders, University of Portsmouth, UK

Computer-based testing (CBT) has been used at various levels at most universities in the UK. Computer-based assessments can be used for end-of-unit summative testing and also for formative testing during a unit to help students prepare for end-of-unit examinations.

Computer-based tests can consist of a number of questions selected from banks of questions. It is often desirable for examiners to randomise elements of some questions so that students are unlikely to be presented with a question that they have attempted previously. In this way, students are less
likely to ‘learn the test’ – reciting answers to questions without understanding the underlying subject matter. Furthermore, this approach discourages plagiarism.

Questionmark Perception (QMP) provides a friendly interface for creating questions. However QMP has very limited functions for producing numerical questions and cannot create questions containing random values.

This paper presents a ‘QML Generator’ spreadsheet that has been created to generate randomised questions based upon a question template. Questions generated from this spreadsheet can then be imported into QMP as a bank of questions. The use of this spreadsheet is described in this report.

The paper then evaluates the functionality of this new system along with five other CBT solutions with respect to numeric questions. A summary comparison of these evaluations is presented.

**Order and chaos: interactive computational activities for the classroom**

Maria Joana Torres and Ricardo Severino, Universidade do Minho, Portugal

It has long been believed that typical students learn better through contemporary approaches to questions originating in physics problems that allow experiments. This belief motivated us to develop interactive computational didactic materials about contemporary mathematics that can be used both in the classroom and in mathematics clubs in school. Most topics in mathematics curricula are presented in an abstract setting, with no physical meaning, which makes students feel no motivation and enthusiasm for what they see as a ‘dead’ discipline. However, since Galileo Galilei and Isaac Newton it has been possible to search for mathematical models that help us to understand natural phenomena. Indeed, it was this ability to simulate the reality, and to predict its temporal evolution, that proved to be the key to the success of science itself. Dynamics (or dynamical systems), the study of how physical systems evolve with time, is a contemporary research field that has the profit of being comprehensible by young students. Furthermore, it allows the introduction and exploration of many of the topics in the students’ mathematics curricula. Dynamical systems are present in many different branches of science, which allows interdisciplinarity, such as biology, economics, ecology, medicine, meteorology, astronomy and computer science. Moreover, most problems have a physical insight. These features of the field of dynamics lead us to elaborate a series of interactive computational activities based on dynamical systems topics such as evolutionary models, ordered pattern formation, generation of fractals and many interesting features of “chaos theory”. Another important goal to achieve with these activities was to give mathematics an experimental/laboratory component, which is rarely present. In fact, all the interactive computational didactic materials developed include simulations with applets designed for this purpose. Last, but not least, students can enjoy the beauty of mathematics since they can, for instance, generate wonderful pictures by iteration of a simple function with the help of a computer.

**Mobile computer laboratory for teaching mathematics**

Jana Vavřinová, University of South Bohemia, Czech Republic

In mathematics teaching, it is appropriate to use new technologies. However, teachers have to often solve technical problems such as inadequate school computer facilities or their unavailability. This usually happens when a teacher wants to use them. In our school, we have solved this issue by providing the school with the mobile computer classroom, which consists of 15 MacBooks. This paper summarises the advantages and disadvantages of its use in mathematics lessons. Moreover, it states which key competencies can be developed by working with the mobile computer classroom. In the last part of the paper, I present specific examples of mathematics lessons in which our mobile computer laboratory was used.
New technologies in the next decade

Hans-Georg Weigand, University of Würzburg, Germany

The advantages and disadvantages of the use in mathematics lessons of digital technologies (DT), especially of computer algebra systems, have been widely discussed. What will be the meaning of DT in the next few years? What is the basis for an answer to this question and how might it be possible to get a vision of possible developments?

A first aspect might be an evaluation of developments in the past. How was the situation in the middle of the last century when we introduced computers into mathematics lessons? Can we learn from past developments?

A second aspect is the evaluation of the present situation. The 17th ICMI Study Mathematics Education and Technology: Rethinking the Terrain was published recently by C. Hoyles & J.-B. Lagrange. It gives an evaluation of the present situation concerning the use of DT and attempts to present a basis or a vision for the development of DT in the forthcoming years. The word ‘vision’ is used quite often: a vision for the development of the software, the hardware, the pedagogical landscape, the mathematics in the classroom, the learning and the teaching. The last paragraph of this book is called ‘Future directions’.

I think the book is not a vision, but it may serve as a basis for visions (Weigand, 2010). The talk is a critical reflection of past and current developments, and it gives some hypotheses of possible gainful developments:

1. DT will support the process of thinking in relations, e.g. relations between different subjects, between 2D and 3D geometry, between the past the present, and this will be a central condition for improvements.

2. The gainful use of DT in the classroom requires a master plan for the integration of all kinds of DT into the learning and teaching process.

Benchmarking and mastery: integrating teaching, learning and assessment

Roy Williams, University of Portsmouth, UK

E-Assessment in mathematics had been established in several universities, over the past decade or more. Much work has been done in developing materials for assessment, on a range of different platforms. This work has now matured, by and large, to the stage where e-assessment can include algorithmic question creation, good graphics display, and the ability to write answers in mathematical notation. More recently, some academic publishers have started to develop and provide more diverse, if not always fully integrated, packages of resources, including e-books, e-assessment, and a range of associated materials and resources, often in ‘cloud’ based platforms. We are now at a stage where we can explore the possibilities for a fully integrated system, which is based in teaching and learning, rather than in assessment. This paper addresses two related aspects of the issue: the technical and the conceptual shifts that are required to integrate teaching, learning and assessment, particularly in the initial years of university mathematics courses. It should be possible to do this if we move away from a paradigm of testing to one of benchmarking and mastery instead. We will describe current practice at the University of Portsmouth, and outline a framework for a flexible and fully integrated, multi-modal system, which includes teaching, tutoring, portfolios, EVS’s, e-books, interactive links between resources, interactive reports to service students, lecturers and tutors, and a flexible system to add to and customise previously authored materials. The paper will also explore the options of proprietary, open source, and commercial modes of provision, and some ideas on possible business models, across these sectors.
Linking IT-based semi-automatic marking of student mathematics responses to pedagogical objectives

Khoon Yoong Wong, Kwang-Shin Oh, Qiu Ting Yvonne Ng, Beng Chong Teo and Kalimuthu Kanchiyappan, National Institute of Education, Singapore

The purposes of a system to auto-mark students' responses to mathematics test items are to expedite the marking process; to enhance consistency in marking; and to alleviate teacher assessment workload. We propose that a semi-automatic marking system better serves pedagogical objectives than a fully automatic one. The two pedagogical objectives to be addressed are that teachers should know about the range of students’ solutions and that they should provide meaningful feedback to students through utilization of customizable feedback. Both objectives align with using assessment data for learning. Our proposed IT-based system consists of a marking component and a feedback component, and it will provide closer linkage between IT-based marking and these pedagogical objectives.

The mathematics items are classified into three types: closed, semi-open and open. The closed items are select-response types such as MCQ, or they require only a specific number or word as the answer. These questions will be automatically marked as right or wrong. For the semi-open or open items (construct-response), the system provides several possible answers based on a marking scheme, and the teachers select from these answers through a drop-down menu or add their own answers to the menu. This cuts down on marking time, but more importantly it alerts teachers to the alternative answers given by their students in a systematic way. To insert feedback for the students, the teachers can select from an initial feedback pool or add their own comments.

At the end of the marking, the system can output detailed results by student or by question. Further item analysis can be conducted based on the output. On the basis of these results, the research team and the teachers will plan follow-up activities to help students master the contents, for example, by using student errors as a springboard for further learning.

Interactive self-paced learning using Mathematica

Yakov Zinder and Tim Langtry, University of Technology, Sydney, Australia

Mathematica is a computer algebra system with powerful graphical and computational capabilities and an impressive ability for performing symbolic calculations. This computer algebra system has proven to be a useful instrument in teaching practically all fields of mathematics. An extensive body of literature reflects the breadth of experience in teaching with Mathematica that has been accumulated in universities all over the globe. In saying this, it is important to stress that, in the majority of cases, Mathematica is used only as a tool for visualisation or as important software for any career in mathematics, science or engineering. In the latter case, Mathematica becomes not a medium for, but rather the object of, teaching. Although the richness of Mathematica’s computational environment is widely acknowledged, very little has been done to utilise its potential as a platform for a new teaching modality. This new generation of Mathematica-based learning support software will open new horizons for teaching mathematics and related subjects, but also imposes significant challenges for the instructor, both technical and pedagogical in nature. This talk will address challenges of both types. The pedagogical challenge for the instructor is to develop new procedures for teaching, wherein the strategies appropriate to face-to-face teaching are replaced or augmented with novel strategies for designing an interactive computer-based learning experience for the student. The technical challenges are closely linked with the pedagogical ones. In particular, the instructor’s efforts must be shifted from the development of software to the development of the learning environment. This requires software that combines both an end-user focused approach with flexibility that allows the implementation of a variety of teaching strategies. We demonstrate how these challenges were addressed in our university and discuss directions for future research.
Abstracts: workshops

Developing mathematical understanding with ICT in the classroom
Nicola Bilsby, Eltham College, UK

In this workshop I will share my research findings and encourage colleagues to reflect on the use of TI-Nspire and Navigator, the wireless connectivity system in the classroom, and consider strategies for assessing how effectively ICT has been used to support the development of mathematical understanding.

How can ICT motivate pupils to want to learn a difficult subject?
Douglas Butler, ICT Training Centre, Oundle, UK

There are many software titles around now, and the purpose of this workshop will be to let delegates find out what makes Autograph different. Firstly, it was created in the classroom and its essential quality, which teachers like so much, is that it helps students to learn and consequently to enjoy a difficult subject. At the same time it can help the teachers to enjoy teaching the subject!

Images and data can be seamlessly incorporated to allow topics of immediate interest to be blended into a classroom lesson with a distinct STEM focus.

The workshop will look at how the judicious use of “slow plot” and Autograph’s “scribble tool” can foster real understanding. These tools are particularly well suited for use with interactive boards, and also suite the ‘walk- about’ slates. The key is pedagogy, and the aim must always be to make sure that students are made to feel involved at every stage, so that a lesson never becomes a “show and tell” environment.

The approach works just as well for the younger students as for the more advanced mathematicians. Pupils who are settling into concepts such as gradient (slope), functions, inequalities, transformations, and right up to differential equations and 3D linear algebra can all benefit from visualisation, but only if it is handled in an interactive and collaborative way.

The workshop will also show how easy it is to create web resources from Autograph in statistics, 2D and 3D, so that teachers and students can share ideas on the Internet.

On your bike: practical application of mathematics
Sarah Chapman, Advanced Skills Teacher, Hayesbrook School, Kent, UK

Explore how to increase students’ awareness of the mathematical world and facilitate a deeper understanding inside the classroom through a practical application of riding a bike.

This workshop will provide an opportunity for delegates to analyse the mechanics of a bike so that they can confidently deliver a practical, functional lesson involving a number of grade C topics. In this session a bicycle will provide the starting point for work on ratios, compound measures and circles. The workshop is particularly relevant for STEM-based subject teachers at secondary level and will incorporate ICT and multimedia.

Lights, camera, mathematics: how the use of digital cameras can support secondary school mathematics learners
Dave Eacott, Park Community School, Hampshire, UK
Lucia Threadgill, The Petersfield School, Hampshire, UK

The Hampshire Leading Mathematics Teachers Information Technology Development Group has been established for a number of years. The group is made up of teachers from secondary schools in Hampshire, UK working with the Hampshire Mathematics Advisory Team. The group has specifically set out to establish new sets of resources which support the use of technology to improve the
teaching of mathematics in schools; the focus is on learners using the technology but all resources could be adapted for use as a teacher demonstration tool.

There is an increasing awareness that accessibility of software has become a barrier to the use of technology in mathematics. This workshop provides the opportunity to try out some of the resources we have developed to make use of digital cameras, a handheld solution to technology that negates the need to have access to a dedicated technology room.

This is an interactive workshop where you will get directly involved with the activities. The workshop will start with a short demonstration of how a low-cost video camera can be used to enhance pupils’ mathematical learning by giving them opportunities to assess and feedback on their work as well as providing a very useful tool for future discussion with any group of learners. You will then learn the best way to plan your footage, experience how easily you can use low cost cameras to film your footage, show your footage on a whiteboard and edit your footage for later use. You will then get the opportunity to try out the cameras to develop some footage of mathematics learning of your own.

**Around the world in 60 minutes using Google Earth: helping secondary school mathematics learners to develop their understanding of bearings**

Helen Humble, Amery Hill School, Hampshire, UK

Dave Eacott, Park Community School, Hampshire, UK

The Hampshire Leading Mathematics Teachers Information Technology Development Group has been established for a number of years. The group is made up of teachers from secondary schools in Hampshire, UK working with the Hampshire Mathematics Advisory Team. The group has specifically set out to establish new sets of resources which support the use of Technology to improve the teaching of mathematics in schools; the focus is on learners using the technology but all resources could be adapted for use as a teacher demonstration tool.

There is an increasing awareness that both cost and accessibility of software have become a barrier to the use of technology in mathematics. This workshop provides the opportunity to try out some of the resources we have developed to make use of the Google Earth software. The software has the advantage of being free to users and therefore accessible, without licensing restrictions, to learners in their homes.

During the workshop you will find out how to access the Google Earth software and learn how the basic functions work. You will have an opportunity to trial some of our resources which support the teaching of bearings and measurement, using local maps. You will also be shown ways in which the software can help demonstrate the use of bearings in ‘the real world’, particularly looking at airports, and finally you will be able to follow and create a treasure hunt that goes all around the world, supporting learners to appreciate the importance of accuracy when measuring bearings over long distances.

**Wirelessly connecting TI-Nspire CX handhelds in the classroom to share mathematical ideas**

Cindy Hunt, Davison CE High School, UK

The arrival of TI-Nspire Navigator CX will require teachers to evolve their practices with the previous system, reworking previously developed lesson activities to take advantage of new functionality. In this session, you’ll find out about some early lessons, and hopefully get a peek inside a secondary mathematics classroom!
A classroom activity – just how fast does ‘Bloodhound’ go?

Pip Huyton, Independent Mathematics Consultant, UK

This workshop presents an opportunity for delegates to explore the use of ICT with digital images, video and mathematical modelling, to discover the speeds at which the ‘Bloodhound Supersonic Car’ travels.

The workshop is particularly aimed at teachers of secondary/FE/UG mathematics/science subjects.

The software that will be used during the session includes Vernier LoggerPro 3 for data logging and video analysis and Promethean’s ActivInspire specialist education software.

A 30-day free trial version of the software Vernier LoggerPro3 Demo can be downloaded from www.indis.co.uk/education/software.htm, in addition ActivInspire Personal Edition can be accessed for free through www.PrometheanPlanet.com/ActivInspire.

Cars – Maths in Motion

Angela Jones, Ministry of Education, New Zealand

Val Brooks, Deputy Director, Stockton CLC (retired)

As a result of meeting Val Brooks in the UK, Angela instigated a project in New Zealand involving the use of an interactive mathematics program (Cars – Maths in Motion) with a class of 13–14-year-old boys to promote engagement in mathematics. This is a piece of software designed to help encourage students to grasp basic mathematical skills and to use them on a practical level, by involving them in a competitive simulation of a series of Grand Prix races at school, national and international level.

Students identify a variety of mathematical concepts implicit in the learning experience, and describe ways in which the software presents opportunities to encounter, apply or develop these ideas. Working in small groups, they are given details of a race track and other information that includes variables such as the number of laps and the weather forecast. They then have to measure and calculate the length of each straight and the angle of each bend. Add to this driver temperament, mathematics modelling, estimation, decimals, percentages, scale, ratio, long multiplication and division, strategy and teamwork and you have a mathematics project that keeps students engaged and highly motivated, and appeals to all ability levels across KS2, 3 and 4 (UK) and curriculum levels 3, 4, 5 and 6 (NZ).

The aim of the workshop will give a hands-on experience for teachers to begin to understand how the project is motivating and engaging to anyone involved and will also give opportunities to discuss how the use of the software can be integrated into the curriculum with benefits both in the UK and internationally. The teacher’s heightened awareness capitalises on the novel context to promote increased opportunities for learning and participation in mathematics.

Information on the Jaguar Cars – Maths in Motion challenge for schools can be found at www.mathschallenge.co.uk and at http://home.btconnect.com/cambs-software/MIM.html

Technology for mathematics teaching: taking the concerns and interests of practitioners to inform a research agenda for technology enhanced learning

Marie Joubert, University of Bristol, UK

The STELLAR Network of Excellence in Technology Enhanced Learning (TEL) research aims to inform the mid-term research agenda for TEL. This workshop draws together the concerns and interests of the participants of the workshop in order to inform this research agenda. The workshop will invite participants to consider three ‘Grand Challenge’ themes within TEL: 1) connecting learners, which is concerned with the issues and questions that arise from the increased connectedness of learners through the use of, for example, the Internet; 2) orchestrating learning, which aims to understand the opportunities and challenges for teachers when technology is introduced into their classrooms; and 3) contextualizing learning, which focuses on how the use of technology provides new and different learning contexts for teaching and learning.
By focusing discussion within these three areas we will together develop a set of overarching research topics, within which we will put together a set of focused research questions, also outlining the background and rationale for these questions. STELLAR will use the output of the workshop to develop the ‘Grand Challenge Problems’ which will guide the future directions of research in TEL. By participating in the workshop participants will ensure that the voice of mathematics educators is represented.

**Three-dimensional geometry in a virtual 3D world: using Google SketchUp to support secondary school mathematics learners to appreciate three-dimensional shapes, plans and elevations**

Randall Jull, Brune Park Community College, Hampshire, UK

Chris Martin, Hampshire Mathematics Advisory Team, UK

The Hampshire Leading Mathematics Teachers Information Technology Development Group has been established for a number of years. The group is made up of teachers from secondary schools in Hampshire, UK working with the Hampshire Mathematics Advisory Team. The group has specifically set out to establish new sets of resources which support the use of Technology to improve the teaching of mathematics in schools; the focus is on learners using the technology but all resources could be adapted for use as a teacher demonstration tool.

There is an increasing awareness that both cost and accessibility of software have become a barrier to the use of technology in mathematics. This workshop provides the opportunity to try out some of the resources that we have developed to make use of the Google SketchUp software. The software has the advantage of being free to users and therefore accessible, without licensing restrictions, to learners in their homes.

During the workshop you will find out how to access the Google SketchUp software and learn about how the basic functions work. You will have an opportunity to trial some of our resources which support the teaching of geometry and space without the limitations of young people’s spatial awareness. We will look at calculating surface area where you can look around every surface, consider opportunities for teaching three-dimensional Pythagoras, create nets of complex shapes without needing to manually construct them first and make scale models to construct an icosahedron through using the golden section.

**A geometry tool for everyone: using GeoGebra software to support secondary school learners with understanding geometry and algebra**

Chris Martin, Hampshire Mathematics Advisory Team, UK

Greg Wilson, Cowplain Community School, Hampshire, UK

The Hampshire Leading Mathematics Teachers Information Technology Development Group has been established for a number of years. The group is made up of teachers from secondary schools in Hampshire, UK working with the Hampshire Mathematics Advisory Team. The group has specifically set out to establish new sets of resources which support the use of Technology to improve the teaching of mathematics in schools; the focus is on learners using the technology but all resources could be adapted for use as a teacher demonstration tool.

There is an increasing awareness that both cost and accessibility of software have become a barrier to the use of technology in mathematics. This workshop provides the opportunity to try out the GeoGebra software. The software has the advantage of being free to users and therefore accessible, without licensing restrictions, to learners in their homes.

The GeoGebra software has been developed by others and in this workshop we act as advocates of how this software can be used to enhance mathematics teaching. During the workshop you will find out how to access the GeoGebra software and learn about how the basic functions work. You will have an opportunity to see demonstrations on how it can be used and trial some resources to get you thinking. You will be able to work through a series of activities exploring three key areas of the software: understanding geometric construction, developing algebraic understanding, and
transformations. These activities are aimed at providing you with a basic understanding of how the software can be used and encourage you to explore further yourself. Links and direction to other resources available will also be provided on the day.

Seizing the opportunity of using online learning for UK mathematics support

Sue Milne, ELandWeb Limited, UK

Leslie Fletcher, School of Computing and Mathematical Sciences, Liverpool John Moores University, UK

The recently published (January 2011) report to HEFCE by the Online Learning Task Force sets out:

- Ways to encourage:
- Flexibility in UK provision
- Online pedagogy
- How to:
- Support institutions to take full advantage of rapidly developing technology and rich sources of content
- Ensure quality provision to meet rapidly changing student demands
- Seeks:
- A stronger understanding of the potential of web-enabled learning and the use of social media
- Greater prioritisation of teaching partnerships between technologists, learning support specialists and academics
- An end to the ‘not invented here’ syndrome
- And concludes that:
- Good practice must also be shared
- There is no point duplicating effort to create content that is already available and proven to work
- Institutions can build on the existing open educational resources initiative to achieve economies of scale and efficiencies

This workshop will consider how these aims can be realised within the mathematics support community and ways in which technology can help. Specific examples of good practice will be sought, from prospective participants and elsewhere, and ways in which they can be represented, disseminated, discussed and tested will be explored. The usefulness of technological support such as LAMS for capturing and coding good practice and the OU’s Learning Space for curation and dissemination will be discussed and analysed using real examples.

The proposers of this workshop were heavily involved in the FETLAR project so are aware of the scope of existing open source resources in mathematics support. They have been involved in online learning and support in mathematics for many years, so are well placed to take forward the agenda set out in the Task Force report.

A web-based learning support system to help secondary school mathematics learners construct geometric flow-chart proofs

Mikio Miyazaki, Faculty of Education, Shinshu University, Japan

Taro Fujita, Faculty of Education, University of Plymouth, UK

Youichi Murakami, Sun First, Japan

Naoki Baba, Toyono Junior High School, Nagano, Japan

Keith Jones, School of Education, University of Southampton, UK

Edited by Marie Joubert, Alison Clark-Wilson and Michael McCabe
This workshop provides the opportunity to try some of the geometrical proof problems that we have designed to support secondary school mathematics students as they build their knowledge of mathematical proof. Our web-based learning platform uses flow-chart proofs and includes both open and closed problems involving the properties of parallel lines and congruent triangles. By using Adobe Flash-based technology, learners complete proofs by dragging sides, angles and triangles to cells and our system automatically transfers figural to symbolic elements so that learners can concentrate on logical and structural aspects of proofs. The system identifies errors by referring to a database of acceptable answers classified into four categories. Learners receive relevant feedback in accordance with the four types of error.

In the workshop, participants have the opportunity to experience our system by working on various geometric proof problems. For example, an introductory problem by which the user can construct four different proofs, another open problem but this time with two steps to the solution, and a proof problem involving the base angles of an isosceles triangle.

The workshop provides the chance to discuss interface design, the use of open and closed problems in the teaching and learning of proof, the effectiveness of feedback given by web-based systems, how to internationalise such a system, and so on. Video clips of learners using our system are also available.

The impact of technology on the way mathematics and statistics have been taught in the last decade

Ghada Nakhla, The Sixth Form College, Solihull

Workshop participants will explore:

• The uses of graphics calculators in teaching A-level Mathematics and Statistics (using the idiot’s guide).
• Does the calculator/technology enhance students’ conceptual understanding of mathematics and statistics?
• Does the calculator/technology make higher grades accessible to lower ability students doing A-level mathematics?
• Are exam boards integrating this technology into their mark schemes?
• Will the way we use technology in the mathematics classroom affect the way we will assess students in future?

The workshop will involve sharing good practice of the use of technology in the mathematics classroom. This will involve hands-on experience and discussion on what lecturers/teachers think of the impact of technology in enhancing students’ conceptual understanding.

Animated questions

Jim Noble, International School of Toulouse, France

Dynamic geometry and similar dynamic software have been a major influence on my own understanding of mathematics and consequently my own teaching. Start with the notion that all squares actually meet the minimum requirements to qualify as all other quadrilaterals. (National variations in the definition of a trapezium or trapezoid do provide an interesting challenge to this notion.) How often does this notion upset students who somehow want rectangles and squares to be discretely different from each other? I suggest that a mathematical object or phenomenon is best described by its properties and that these properties can best be explored in a dynamic environment. The ability to bend, stretch, and explore a dynamic situation demands that we consider generalities and their limits where static representations provide us only a particular case. Why, then, are so many questions in mathematics classrooms asked through a static, fixed and often printed medium? Developments in technology have prompted me to explore the setting of ‘animated questions’ where students are shown short animations of particular mathematical phenomena and asked to explore and define them by attempting to recreate them. In this workshop I propose to engage the audience with a number of examples of these animated questions exploring geometry, functions, sequences...
and more. In exploring the problems I hope the group is prompted to recognise the benefits of asking questions in this way in terms of exploring generality, engagement and problem solving.

I work in a school in which students carry their own laptops with them at all times, which aids such experimentation, but this is not a prerequisite for being able to ask these questions. What it does do is make me increasingly curious about how long it will be before external assessment tools for mathematics will be set using technology as a medium and thus allowing a broader, more versatile style of questioning of which these animated questions are just an example.

**Integrating STEM and inspiring STEM activities with TI-Nspire technology**

Adrian Oldknow, University of Chichester, UK

Linda Tetlow, Independent Mathematics Consultant, UK

STEM is a matter of current concern worldwide. TI has produced a series of STEM booklets for TI-Nspire technology in the UK. This session will look at some of the examples around which strategies will be discussed for using TI technology as a catalyst for STEM subjects working together. The session will also explore using data from a variety of sources about topics such as the weather, nutrition and census information to engage students’ interest in analysing real world problems.

**Brunel’s bridges, boats, books and box**

Peter Ransom, Independent Mathematics Consultant, UK

This workshop will focus on STEM activities based on the work of Isambard Kingdom Brunel, exploring various practical activities with TI-Nspire CX handhlds.

**Matching, speed dating, human sculptures and curve stitching: developing secondary school learners’ understanding of line graphs using graphical calculators and digital cameras**

Lucia Threadgill, The Petersfield School, Hampshire, UK

Helen Humble, Amery Hill School, Hampshire, UK

The Hampshire Leading Mathematics Teachers Information Technology Development Group has been established for a number of years. The group is made up of teachers from secondary schools in Hampshire, UK working with the Hampshire Mathematics Advisory Team. The group has specifically set out to establish new sets of resources which support the use of Technology to improve the teaching of mathematics in schools; the focus is on learners using the technology but all resources could be adapted for use as a teacher demonstration tool.

There is an increasing awareness that accessibility of software has become a barrier to the use of technology in mathematics. This workshop provides the opportunity to try out some of the resources we have developed to make use of graphical calculators and digital cameras, a handheld solution to technology that negates the need to have access to a dedicated technology room.

This is an interactive workshop in which you can take part in a number of activities that are aimed at enhancing learners’ understanding of line graphs using digital cameras and graphics calculators. You will need to be willing to get up and be involved throughout the workshop. The activities will include the use of digital cameras as a tool to record and discuss outcomes from modelling activities and the use of graphical calculators to help learners to understand the structure of linear equations through trial and improvement.
Mathematics in the palm of their hands: developing the use of iPods and other mobile technology to support secondary school mathematics learners

Greg Wilson, Cowplain Community School, Hampshire, UK
Randall Jull, Brune Park Community College, Hampshire, UK

The Hampshire Leading Mathematics Teachers Information Technology Development Group has been established for a number of years. The group is made up of teachers from secondary schools in Hampshire, UK working with the Hampshire Mathematics Advisory Team. The group has specifically set out to establish new sets of resources which support the use of Technology to improve the teaching of mathematics in schools; the focus is on learners using the technology but all resources could be adapted for use as a teacher demonstration tool.

There is an increasing awareness that accessibility of software has become a barrier to the use of technology in mathematics. This workshop provides the opportunity to try out some of the resources we have developed to make use of iPods and other mobile technology, a handheld solution to technology that negates the need to have access to a dedicated technology room.

This workshop is an opportunity for a hands-on experience of how iPod Touches can be used in your classroom to enhance your mathematics lessons and develop your pupils’ learning. You will have the opportunity to experience the use of different applications specifically designed for mathematics learning and provided with ideas of how you can use them in a classroom environment. You will also discover how to set up your own interactive quiz using a Gmail account and have pupils send in live data recorded directly onto your screen, providing opportunities for instant feedback, discussion and assessment.

Bring interaction and flexibility to your classroom!

Jane Woods, The Dame Judith Professional Centre, Portsmouth, UK

In this presentation you will see how one department has developed the use of handheld technology to engage all pupils in their classes.

Engaging all learners in your classroom is every teacher’s problem; see how one school is approaching the problem using an audience response voting system. The use of handheld technology has helped assist with Assessment for Learning and Assessing Pupils’ Progress and is a fun way for pupils to be tested on what they have learnt. You will see some of the engaging activities that can be created and learn how the inbuilt report function can quickly and effectively provide feedback on individual pupils and whole class progress.
Part B: Papers submitted after the conference
Mathematics and technology: Exploring teacher educators’ professional development

Maha Abboud-Blanchard

LDAR, University Paris-Diderot, France.

This paper reports on an on-going research which tackles the issue of the professional development of mathematics secondary teachers when preparing to become teacher educators. In order to explore the relationship between research and realistic practice, I developed a training course for teacher educators taking into account, on the one hand, their own experiences/habits as secondary teachers in the use of digital technologies, and on the other hand, the potentialities/difficulties of the integration of ICT into mathematics learning and teaching identified by the research in this domain. A principal task within this course was to design a resource based on the educator’s own experience about a technology-based lesson and to reflect on its eventual use in teacher education. The paper discusses the impact of this course on teacher educators’ understandings of what could be taught in teacher education programmes in order to support teachers in implementing technology into their teaching practices.

Introduction

In recent years an increasing interest has been paid by educational research to teachers’ practices in ICT environments, going along with a global reflection on the barriers to the everyday integration of ICT by teachers and the changes necessary to make this integration a reality in classrooms. Research studies addressing this issue have revealed that integrating technology is not an easy task for teachers who must cope with an increasing complexity of preparing lessons and managing the classroom, simultaneously taking into account several features going beyond familiar formats and routines in a paper-and-pencil environment. They show also that this complexity may constrain the potential benefits of technology to mathematical learning (Laborde, 2008) and affect the emergent goals of the teacher during ICT-based lessons (Monaghan, 2004). Other research has investigated practitioner thinking and professional learning surrounding lessons incorporating the use of digital technologies (Ruthven, 2007).

In spite of an evident body of research which allows us today to better understand how ICT can modify the teacher’s professional work, the knowledge and strategies for teacher training are still inadequate and not well understood (Artigue, 2010).

In the latest ICMI study (Hoyles & Lagrange, 2010), several papers analysed views and options of teacher education courses in mathematics and technology and acknowledged that the quality of teacher professional development is a key factor of any possible evolution of ICT integration in educational systems.

After having focused my own research on issues related firstly to teachers practices in ICT environments (see for example Abboud-Blanchard, 2010 or Abboud-Blanchard & Lagrange, 2006) and secondly to the kinds of knowledge developed in training courses and to training strategies used by in-service teacher educators (Abboud-Blanchard & Emprin, 2009), this new research tackles the issue of interactions between research outcomes and educators’ professional development. This approach is situated within the perspective of Burkhardt & Schoenfeld (2003) who claim that “educational research does not often lead directly to practical advances, although it provides useful information, insights, and ideas for improvement”. They therefore argue for a much closer coordination between research, development, design and practice. Moreover, by drawing on research that acknowledges the role of selecting and managing available resources for facilitating the use of technologies in mathematics teaching (Gueudet & Trouche, 2009), I intend to go further in this direction and explore the role of designing such resources within courses for teacher educators. I therefore developed an educators’ training course taking into account, on the one hand, their own experiences/habits as secondary teachers in the use of digital technologies, and, on the other hand, the potentialities/difficulties of the integration of technological tools into mathematics learning and teaching identified by the research in this domain.
Context and objectives of the research

Like many other researchers (see the outcomes of the 17th ICMI study, op. cit.), I believe that preparing teachers for the challenges of appropriating and integrating technologies into pedagogical practices can be enhanced by forging co-learning partnerships between researchers and practitioners. This could take place in a collaborative environment based on a double movement: a bottom-up movement, where teachers share their teaching practices, experiences and views of technology-based-lessons; and a top-down one, where researchers help teachers to have access to, and to reflect critically upon, a robust body of research issues and outcomes focusing on learning and teaching activities involving digital technology. In the case of the study presented in this paper, a training course for teacher educators was chosen as a suitable place for this type of collaboration. Indeed, by exploring teacher educators’ professional development, the study addresses an issue not so much considered in research until now. The main idea that underlies this exploration, in this collaborative environment, is that improving the knowledge and attitudes of educators regarding ICT integration might encourage new dynamic teacher training practices which, in turn, could help bridging the existing gap between research and ordinary ICT practice.

The educators training course takes place in a master degree program preparing mathematics secondary teachers to become teacher educators. The first courses within this program are dedicated to preparing student-educators to develop ways to analyze what happens in the classroom based on corresponding methodological tools inspired by research into teaching. One of the main ideas is the need for a teacher educator to be able to analyze students’ tasks and to use this analysis to understand the difference between what the teacher expects and what actually happens in the classroom, including students’ mathematical activity (Robert, 2011).

The course devoted to ICT generally follows these first courses; it runs over three months (one session a week). The starting point of the course is for the participants to acknowledge the complexity of introducing ICT into teachers’ practices and that this introduction affects all levels of classroom activity. The work within the course is organized into topics related to the use of specific technology tools. The latter are chosen during the first session of the course and are related to the actual experience of participants: dynamic geometry; on-line resources (e-exercises bases); algorithmic software, IWB etc. For each topic, one or two sessions are organized based on the same model which takes into account the two movements developed above. First, a group of student-educators prepare a synthesis of their own experiences of the use of the technology tool in their classrooms: institutional constraints, material conditions, possibilities for doing mathematics offered by the technology tool, range of tasks proposed to students, difficulties encountered by the teacher and solutions found etc. Secondly, the researcher presents an overview of the relevant research, organized to consider a variety of issues: potentialities and limits of the technology tool, working environments, teaching strategies, classroom management, forms of interactions in the class, eventual changes in the teacher’s role and activity, interactions with established routines etc. Finally, the group moves on to more global issues and works on identifying questions and factors considered important for the training of teachers in the uses of the technology tool. This work is sometimes based on reading and discussing papers or on observing (video recordings) practices of other teachers in ICT environments.

At the end of the course, the participants had to design a resource, in pairs, based on an actual class-activity to be used in ICT teacher education courses. They are free to choose the technology tool to be used. They can also choose to use an available ICT-class-activity and to adapt it to their own needs. They must write a paper presenting this resource and discuss it with the whole group. Indeed, designing a resource in this context and writing a paper explaining the reflexive dimension accompanying this work requires student-educators to clarify the implicit elements encountered during the creative phase.

The objective of the research is to understand how the specific approach chosen for this ICT course, and in particular the activity of designing resources, influences the professional development of teacher educators. Analysing the resources designed allows, on the one hand, emphasizing their characteristics and their affordances to be used in teacher education and, on the other hand, to capture throughout the written paper the connections with the issues raised during the course. At the same time, it permits to observe features of the professional development of the student-educators...
A case study

The course dedicated to dynamic geometry, which had the format explained previously: a group of student-educators presented and confronted their own practices concerning the topic, and then the researcher-teacher made an inventory/synthesis of the outcomes of relevant research literatures. Following this course, two groups of two student-educators decided to focus their project of resource design on this topic. The case I present in this paper is one of the two case studies related to the use of dynamic geometry. The two student-educators concerned in this study, Andrew and Tony, both have extensive experience in the use of technology in their classes. However, Tony has a more reflective approach to his practice and often exhibits a range of aspects of practitioner thinking, while Andrew is more impulsive, constantly trying new ideas about using ICT and eager to learn more about how to improve his experience.

They have conceived a geometric activity which aims to introduce the concept of perpendicular bisector to their students (aged 11-12 years). The activity starts in a dynamic geometry environment and continues in a paper-and-pencil working environment. The aim is to identify the perpendicular bisector of a line segment [MN] as the locus of points that are equidistant from M and N and then to draw this straight line using compasses and ruler. The activity concludes by the enunciation of the definition and properties of the perpendicular bisector.

In the ICT-activity, two fixed points M and N are already drawn on the computer screen along with a set of seven other points; at the top of the screen are posted the distances from each one of these points to M and N. The students are asked to drag each one of these seven points so that it becomes at equal distance from M and N. In the paper-and-pencil activity, the students have a worksheet with two drawn points, M and N (in the same positions as those of the ICT-activity); they have to find the method to construct, with classical tools, seven points equidistant from M and N. Andrew and Tony conducted the session in their classes and compared the two experiences. The resource that they designed is based on this work.

When analysing this resource we can notice that it has two main objectives: to report on the ICT-based-lesson to a fellow teacher and to highlight the elements of analysis and determinants that might be used in ICT teacher training courses. Indeed, the paper presenting the resource is structured in 3 sections:

The analysis of the students’ tasks and the choices related to the design of the ICT-activity

Andrew and Tony explain that the focus of the ICT-activity is to help students create a mental image of what a set of equidistant points from two fixed points might be. This first activity is supposed to prepare students for the paper-and-pencil one. Thus, students could quickly discover that instead of trying to draw seven points, it’ll be enough to draw only two of them, to draw the line and then to place the five others on the line. The ICT-activity was carefully prepared so that students would not be disrupted by the different tools available in the software and focus only on the task to be accomplished. Several other choices are also highlighted such as the line segment was not drawn in either of the two activities in order to prevent students quickly locating the midpoint. Another choice was that [MN] was not horizontal to avoid the prototypical representation of the perpendicular bisector as a vertical line intersecting the segment at its midpoint. This section illustrates that Andrew and Tony conceived their activity having both in mind their own routines and knowledge for action and the discussions in the course about the effective contribution of dynamic geometry to learning. Indeed, Tony and Andrew are used, in their teaching routines, to introducing every mathematical topic by an exploration task; here the software was used to support this activity. Furthermore, while presenting the resource to the whole group, Tony insisted that their aim was not to design an “extraordinary” activity, but rather to show how dynamic geometry can be complementary to established geometry-activity involving construction by hand.
The “storytelling” of the sessions

Tony and Andrew choose two different ways to report on what actually happened during the sessions. Andrew opted to “tell the story” of his session. He transcribed some dialogues between students which explain their understanding or misunderstanding of the tasks such as: Student 1: “Well I must put the point on the same distance, 15cm, from M & N”; Student 2: “No, not necessarily 15cm, just the same distance”. He also expresses his feeling towards what happened: “I was disappointed because even though the students were two to a computer, they didn’t interact, nor collaborate to accomplish the ICT-Task”. While telling his story, he sometimes adds comments to suggest some improvements: “the length of MN [in the paper-and-pencil activity] was not a multiple of 2, so students who found that the straight line to be drawn must pass through the midpoint of the segment have had some difficulties in finding the exact location of the midpoint by using the ruler, so I think that this length must be a multiple of two to facilitate the construction for these students”.

Tony preferred to extract some elements relative to his own choices of conducting the session, to analyse them and sometimes to compare with what happened in Andrew’s session. For example, in order firstly to maintain the attention of the students and secondly to let them be aware of the continuity between the two activities, he chose to use a normal classroom equipped with laptops, instead of changing classrooms when changing activities (which was the case of Andrew). Furthermore, he explicitly told the students that the two activities were related (Andrew didn’t make the same choice). Tony notes that this was a successful choice; his students, better than those of Andrew, had succeeded in making the transition, anticipated when designing the activities, from a dynamic geometry exploration to a conjecture of construction by hand.

However, in both cases, the aim of this section was clearly to share the individual experience with colleagues who might read the document thus designed. For Tony, it is in line with his habit to reflect on his practice but goes beyond to “find the right words to say it” and to transmit to others his own experiences and reflections. Whereas for Andrew, the storytelling was a sort of “need” to communicate on an experience where for the first time he had to explicit his choices, to analyse the procedures and difficulties of his students and to think about possible improvements. Thus, Tony and Andrew have both professionally developed but through different paths.

The key factors and alternatives to be developed by teacher educators.

In this section, Andrew and Tony first propose changes in the ICT-activity in a way to improve the readability on computer screen. For instance, gathering the seven points to be moved in a corner of the screen and thus making M and N more prominent. This improvement enables students to create a mental image more effectively. In the perspective of a teacher training course, Andrew and Tony suggest to invite trainees to find other kinds of such improvements. Secondly, they reflect on classroom management, particularly the aspects related to changing working environments within the same session. The choices to be made when starting and ending the ICT-activity are crucial for certain students: Must we show examples of what to do on the IWB during a whole-class moment? Must we demand that students to turn off the computers when starting the paper and pencil activity? Must we leave on the IWB the solution of the ICT-task while students are doing the construction with classic tools? At first sight, these elements and questions do not seem important, but through the experience carried out, they turn out to be determinant for the success of the activity. Andrew and Tony suggest that a teacher educator could ask trainees to reflect on these elements, and then show them some video recordings of the corresponding moments of a lesson with the effect on students’ activity. For example, if the answer to the last question above is yes, one can see, on the video, that some students tried to place the points on the worksheet at exactly the same distances as on the computer screen! Finally, Andrew and Tony tackle issues beyond the lesson they have experienced, which should be debated in teacher education, namely: what type of application exercises are available in textbooks? Are these exercises in line with the proposed activity? If not, the ICT-based-lesson risks being less worthy compared to the remainder of the course! This last concern shows once more that Andrew and Tony want to highlight in teacher education that ICT-activities must be planned and conducted in a coherent way with paper-and-pencil activities and that a successful integration of ICT into teaching relies on the complementarity of the two types of activities. This latter issue was largely developed and discussed during the course.
Conclusion

The introduction of digital technologies into mathematics classrooms has been accompanied by a demand for teacher education to prepare teachers to make use of these new resources. Initial visions of the outcomes that might be expected of this education were often extremely ambitious and the use of technology in mathematics classrooms across the world has progressed rather slowly (Hoyles & Lagrange, 2010). In light of the complexity, attested by research, of introducing new technologies into existing practices, and regarding the issues on the articulation between research and professional development that have been raised recently, I attempted to develop ways to help teacher educators to become more involved in resource design and to come up with training strategies adjusted to the particularities and complexity of implementing ICT into teaching/learning activity. I presented in this paper an on-going research aiming at studying the impact of such an approach on teacher education.

The major objective of the first phase of data analysis was to investigate if and how teacher educators developed professionally. The first result that is taking shape is related to how the participants invested the collaborative course and adhered to its objectives. At the beginning, most of them expected that the focus of the course is to expand their teachers’ repertoires of well-defined classroom practices regarding ICT integration. As the course progressed, they took fully part of it in interactive way; the course evolved thus constantly by taking into accounts their own contributions. At the end of the course, they were sharing a set of common methodological and theoretical tools and a common professional vocabulary enabling them to have a common approach of the professional aspects related to ICT integration. This first result can also be seen as an indicator of professional development of the participants. Indeed, by constructing together these shared elements, teacher educators moved from an individual view of what happen in classroom when using ICT tools to a common decontextualized way to analyse and to talk about ICT teaching practices.

Other results, regarding how teacher educators developed professionally, have emerged from early analyses but still need to be confirmed. In fact, it is difficult to determine what an indicator of professional development would be or even to verify whether this development is due only to the participation in the course or to factors not previously considered or reachable by the study. Among these results, one seems to be significant at this stage of the study. Despite some regularities in the characteristics of professional development, the latter remains after all an individual process specific to the history, knowledge, skills and conceptions of each teacher. The case study of Tony and Andrew illustrates this result by showing how they both developed but through different paths. Furthermore, the teachers participating in this course are experienced ones and it was difficult for some of them to move, in such a short time, from the position of teacher to the position of teacher educator. Thus, some evolved only regarding their teaching practices and views of implementing technology in their classrooms. Others, in addition, began a process of constructing a new culture and craft knowledge of teacher education. However, in either case, they all believe that the experience that they lived has a real potential permitting them to refine afterwards their professional knowledge and craft skills.

The analysis of the data is not completed and global findings are not yet available. The work will pursue on the model of qualitative case studies, considering on the one hand, the analysis of the characteristics of the training-resources that were designed and, on the other hand, the professional development of teacher educators and its determinants.

Coming back to the general issues evoked in the introduction, new approaches in teacher educators courses have certainly had an impact on improving teacher education which has hopefully, in its turn, repercussions on a wider implementation of technology in everyday teaching practices. Still, a limited-time course is not enough, long-term projects and new and various models of teacher educators courses are recommended. It can though not be the task of individuals, nor left in the sole hands of researchers (Artigue, 2009). It requires specific research and structures able to organize and evaluate the effects of such approaches on teacher educators’ professional development, on teachers’ professional development and on students’ learning.
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Effect of Using Cabri II Environment by Prospective Teachers on Fractal Geometry Problem Posing

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The use of Cabri II in mathematics education has considerably spread in the last few years. Nevertheless, many teachers haven’t yet completely overcome their fears and suspicions about using it in geometry teaching. Furthermore, there is a generational gap which raises further difficulties: while most of the in-service teachers in Oman don’t have confidence in the new technologies or with their application in educational activities. This study investigates how Cabri II might be effective in developing prospective teacher’s skills in developing new fractal problems for school geometry level. The research was designed to introduce prospective mathematics teachers into a learning experience with a Dynamic Geometry Environment (Cabri II); making them work in small groups on developing fractals problems based on ‘Cabri II’ dynamic geometry. Twenty of mathematics prospective teachers participated in six activity sessions in topics of Circles, triangles fractals problem posing. The experience showed that the participants, after getting used to Cabri II, were able to apply their competence in the construction of interactive educational materials for a classroom situation. All of the creative fractals shapes in proposed problems and images were developed by the participants, for that purpose, the Cabri II was used.

Introduction

Technology in mathematics has become an important factor in mathematics teachers' preparation programs. Research indicates that teachers who are effective at integrating technology possess specialized knowledge about technology, pedagogy and content, and the intersections of those three domains (Koehler, Mishra, & Yahwa, 2007; Suhrwoto & Lee, 2005). Educational technology researchers have advanced the construct of technological, pedagogical and content knowledge (Mishra & Koehler, 2006) as a theoretical and empirically-based framework about teachers’ knowledge needed to effectively integrate technology in their teaching. In Sultan Qaboos University; Technology in mathematics and teaching considered to be a main dimension of Mathematics teacher's preparation program.

Part of Methods of teaching secondary mathematics course is to use Cabri II environment for the learning of teaching geometry and Fractal geometry is a new topic in the same course. Fractal geometry is a new language for the complex forms and patterns found in nature. It represents a change in the way that scientists “do science”. It provides new tools to describe, model, analyze, and measure the natural world, and wonderful new connections within the world of mathematics. Fractal geometry is exciting, visual, relevant to many disciplines, and lends itself naturally to technology supported activities. Both students and teachers with relatively little mathematical background can approach a large number of current research problems in this area and appreciate the integration across traditional disciplines.

Students enrolled of this course should create new fractal problems using Cabri II environment as a main project. This study showed that students have the abilities to create new fractals problems based on Euclidean geometry and Cabri II software.

Background

Geometry has always been a rich area in which students can discover patterns and formulate conjectures. The use of Cabri II environment enables students to examine many cases, thus extending their ability to formulate and explore conjectures (NCTM, 2000; 309) as well as developing new questions for a given geometry problem. In Principles and Standards for School Mathematics (NCTM, 2000), NCTM presents its technology principle (pp. 24-27). The technology principle has three components: 1) technology enhance mathematics learning, 2) technology supports effective mathematics learning, 3) technology influences what mathematics is taught. The range of accessible problems is extended by technological tools as students are able to execute routine procedures quickly and accurately, allowing additional time for conceptualizing and modeling. Learning is assisted by feedback, which is supplied immediately by Cabri II.
Problem posing in mathematics

“Problem posing and problem solving is obviously closely related. On the one hand, problem posing draws heavily on the processes of problem solving, such as identifying the key elements of a problem and how they relate to one another and to the goal of the problem. On the other hand, problem posing takes children beyond the parameters of the solution process” (English, 1997, 173). Problem posing instruction involves instruction with student generation or formulation of problems to solve (Silver, 1994). It involves the creation of original problems which may be associated with particular conditions. This study dealt only with the generation of new problems.

The following questions may be asked to guide additional inquiries into fractal construction:

- What other geometric process lead to “interesting” fractals?
- Can inquiry into the patterns of any generated fractal relate to your fractal?
- Will a fractal be formed if the dimensions of the construction matrix are changed?
- Could you have used these ideas in a different way to solve the problem?
- How might you change some of these ideas to make a different problem?
- What if you not given all these ideas? What might the problem become then?
- What if we were adding some new ideas? What ideas might we add? What new questions might we ask then?

Fractal geometry problems

Fractal geometry is a relatively new and important area of mathematics which differs significantly from traditional geometry. Fractals have many applications in a variety of fields of study aside from mathematics, such as art, engineering, physics and computer science. Unlike classical geometric shapes, which are linear or continuous curves (straight lines, triangles, smooth curves and circles) alone do not represent the world in which we live; the shapes of fractal geometry (clouds, trees, mountains, coastlines and snowflakes) are nonlinear.

Fractals are more representative of the natural world we live because they appear to exist within it. However fractal geometry does not appear in many traditional curricula even in Oman mathematics curricula. Teaching Fractal Geometry is consistent with implementation of the guidelines and the recommendations of The National Council of Teachers of Mathematics, NCTM. In the process of creating fractals students will be able to examine the following geometric concepts and skills recommended by NCTM (1989):

- For grades K-4, topics in geometry and spatial sense such as describing, modeling, drawing, classifying, combining, dividing, and changing shapes.
- For grades 5-8, identifying and comparing geometric figures in one, two, and three dimensions and the applications of geometric properties and relationships in problem solving and real life situations.
- For grades 9-12, such topics in geometry as representing problem situations with geometric models, classifying figures in terms of congruence and similarity.

Fractals can be divided into two categories: Natural fractals and mathematically structured fractals. Natural Fractals One does not have to look very far to find examples of natural fractals in the everyday environment of students. Examples of natural fractals include coastlines and curvy rivers (as illustrated on wall maps), flowers such as a rose or carnation, tree branches, rock formations, mountain ranges, seaweed and other aquatic plants, coral, and parts of the human anatomy such as curly hair, veins, and intestines. Mathematically Structured Fractals Included in this category is simulations of fractal patterns that are computer generated; which are a class of fractals created consistent with some mathematical rules and principles such as Cantor Dust, Mandelbrot Set, Sierpinski Triangle, and Koch Curve. This study dealt with Mathematically Structured Fractals as an environment of problem posing.

These fractals are generated by iteration of an event or shape repeatedly. Popular examples of these simulated fractals are the Mandelbrot set, Koch snowflake, and Julia sets. These and other fractals are

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available through different sites on the World Wide Web (Ex. www.yale.edu & www.fractalfoundation.org ). Mandelbort was the first to uncover the beauty of the computer-generated images. This can only be seen because of computer technology, since these images are the result of millions of iterations. Within a study of fractal geometry, students can make conjectures about relationships between figures or number patterns and they can then form their own generalizations and problems. This study is focused on that structured fractals created through Cabri II environment.

**Cabri II Environment**

National Council of Teachers of Mathematics (NCTM) suggests, in Principals and Standards for School Mathematics, that interactive Geometry and Geometer’s Sketchpad. This dynamic geometry software offer opportunities for the users to manipulate and, precisely speaking, to act directly on, geometrical diagrams – particularly by grabbing and dragging certain geometrical objects (e.g. points) with the mouse. These new opportunities of direct access to and interactions with geometric diagrams open new opportunities of experimentation (Balacheff & Kaput, 1996). One aspect of the well powerful of dynamic geometry is that in the new experimental field it granted, the geometric drawings, as opposed to figures by virtue of the distinction made by, Laborde (1993), preserve the invariant properties salient to the geometric configurations. Whilst grabbing and dragging the geometric objects (e.g. a point that in turn changes the shape of a triangle). The “dragging” facilitates the reasoning process in helping student to move backward and forward between particular instances of geometric relations and general theories about invariant relationships. Holzl said that “The drage mode alerts the relational character of geometric objects” (Holzl, 1996, p. 171), for example; if one constructs an equilateral triangle ABC in which points A and B are given, then C cannot be dragged whereas A and B can. From a relational point of view there is no need to distinguish the points A, B and C, as each pair of them determines the original equilateral triangle. From a functional viewpoint (essentially Cabri) the situation looks different: A and B determine the position of C but in return C dose not determine the position of A and B. As Whiteley (2000) spoke about his reflection on his experiences in both studying and teaching geometry, the whole point about using dynamic geometry programs is learning to see differently and therefore think differently.

Cabri presents the learner with two worlds (Sutherland & Balacheff, 1999): a theoretical world, which is that of geometry, and a mechanical, manipulative world, which is the phenomenological domain of Cabri. Cabri objects are part of a computer environment which can be considered "half world" (Noss & Hoyles, 1996; 6) in between theory and practice. With this respect Cabri figures are a midway between empirical and genetic objects. On one hand, as empirical objects they can be manipulated and the effect of this manipulation can be seen on the screen as it happens. On the other hand, dragging figures in Cabri allows one "to see the one as a multitude, other than one among others" (Pimm, 1995; 59).

The researcher has observed in his own courses the power of dynamic geometry (Cabri II) to explore many examples and help student-teachers makes generalizations in geometry. The power of dragging is an important factor to generate new conditions of a figure on a screen, in addition to other features of Cabri II.

**Objectives and Research Questions**

Research question of this study is:

"Does experience with Cabri II environment enhance student-teachers' abilities to formulate new fractal geometry problems?"

In this study I define "formulate" as "to generate a new questions based on a fractal geometry figures developed by student-teachers".

**Instruments:**

- To investigate student-teachers abilities of using Cabri II environment in problem posing, I use "Task Analysis" as a tool for that purpose; it showed how dose student could construct fractal geometry figure and what kind of formulated problem he developed.
• To investigate student-teachers skills of problem posing in fractals, I use "Interview" with each group of them, asking them about their work (procedures, the role of dragging to construct new situation for a fractal figure, and what they can do to modify ill formulated problem?.

Method

Several educational tasks were designed and final generated problems have been analyzed regarding the quality of well problem posing skills criteria.

Objectives of the educational tasks were:

• To construct a fractal figure, based on fractals properties of fractal dimensions and self similarity.
• To develop new questions for that constructed fractal figure,
• To explain how Cabri II would help to solve the posed problem.

Subjects and contexts

The subjects were 20 student-teachers enrolled of methods of teaching mathematics course, each two student-teachers working together to create new fractal figure based on their knowledge of: Fractal geometry principles, their skills of using Cabri II software utilities, and problem posing strategies. Working in cooperative groups allows students to check their results with other group members. After the students finished their individual fractals, they can collect data about the patterns they observe. They can use these data to make conjectures and test them in their group before presenting their fractals to the whole class. Initially, the teacher’s role is to explain how to construct fractals. Training session of using Cabri II in 2 hours a week of 4 times, while Problem Posing Strategies and examples was 4 hours, Fractal geometry topics taught in 6 hours. All teaching process and training has been done by researcher. The phase of constructing new fractal geometry problems using Cabri II has been done in the final stage of the course. All problems about fractal geometry necessarily imply designing and creating macros. They stimulate pupils’ algorithmically thinking and they can be a basis for beginning programming activities as well. All these problems involve pupils actively in the phase of the construction of the fractal, allowing them to invent new ones by themselves. Drawing fractals on a computer has several advantages over drawing fractals by hand: (1) students can create more fractals in the allotted time, and (2) correcting mistakes is easier and less frustrating.

Actually, students rarely make mistakes when they construct fractals with a computer. Technology allows them to copy stages without much backtracking, whereas when they manually build a fractal with transformations, copying requires going through all the steps from stage 1 on. Utilizing the Euclidean rules of construction such as angle bisectors, parallel and perpendicular lines, midpoints, and perpendicular bisectors the students examined the properties and construction of some of the elementary figures of fractal geometry.

Accepted fractal problem criteria were: Using Cabri II environment, context of the formulated problem should be related to Fractal geometry topics, and new generated problems are solvable.

Results and Discussion

The following tasks showed some of the problems of the basic construction that student-teachers have drawn using Cabri II:

Task #1:

Students were asked to use circles properties to construct "fractal Carpet". They started drawing a single circle with specific radius, then using Cabri II environment to connect three of that circle tangent, using circle center of the three tangent circles to construct the main unit as in figure (3), coloring is option as in fig. (5), using millions of iterations to construct fig. (7).
Generated questions like: Is it possible to complete Carpet in fig. 7? Led to formulate a problem done by "Ali and Murshed": based of the given initial circle, could you expect numbers of circles needed to complete the 10th stage of the figure, General formula for n circles?

This task showed that both Ali and Murshed tried to generate a figure of fractal carpet using the properties of circles, and iteration geometry for a basic generator of one single circle, as I asked them; about fig. 6? They said that constructing new question to complete the square carpet need a new conditions, that Cabri allowed them to use it.

![Generated questions like: Is it possible to complete Carpet in fig. 7? Led to formulate a problem done by "Ali and Murshed": based of the given initial circle, could you expect numbers of circles needed to complete the 10th stage of the figure, General formula for n circles?](image)

**Task#2:**

This task done in fig. 8 by Muna and Badria, they tried to construct an iterated figure using equal sides square, dividing each side in 1:2 ratio, then iterate that some stages, formulating a question related to generalization. But they formulate another question “what if we divide sides in a ratio of 2:3, does the generalization will differ?”

In questioning them about fractal properties in that figure, they failed to prove the existing of similarity on the screen, it considered a good idea, but not fractal geometry problem posing.

![Task#2:](image)

**Task#3:**

This task in fig. 9 done by Hamad, showed his tray to construct a fractal triangle, using the idea of iteration of a two triangle with one diamond, this unit was the basic of the whole triangle in fig. 10.

Hammad generate a problem based of that figure: “what if the initial triangle would be right triangle how could be the 5th stage of constructed triangle?” another question based of the original figure (10) was” use dragging properties to formulate other figures by same fractal pattern.
The context of fractal geometry provided an environment which encouraged problem posing. The role of Cabri II in the problem posing skills is highlighted. It is a tool for extending investigations. Exploring figures with Cabri is effective in encouraging creativity. Students were very engaged with geometric construction of the fractals utilizing the principles and rules of Euclidian construction, as well as using Cabri dynamic geometry software. Also, several students went beyond what was required and created their own fractals.

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Risk-based decision-making by 15 years old students

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The judgement of risk is a key factor in our decision-making. Pratt et al. (in press) worked with teachers to explore the nature of their risk-based decision-making in a personal dilemma. They proposed a new mapping tool to support co-ordination between impact and likelihood. This tool was designed at the end of their study and so to date has not been systematically tested. In the study reported here, I test the conjecture that the use of this mapping tool might facilitate students’ (as opposed to teachers’) risk-based decision-making. This paper supports the prior work with teachers that students also drew on personal experiences and values and that, prior to use of the mapping tool, they referred to impact separately from likelihood even though their discussions were thoughtful and rational. Subsequently, the mapping tool provided a resource through which one pair expressed ideas for the co-ordination of likelihood and impact in terms of the situated notion of a trade-off, articulated as ‘balance’ and ‘cancelling’.

Risk

In this paper, we consider how 15 years old students engage with risk-based decision making within a carefully designed computer-based scenario. In doing so, we have an opportunity to analyse how those tools affect inference-making during a risk-based decision making process.

Nowadays, risk is becoming a major focus both for researchers and policy-makers. Therefore, risk is an important socio-scientific concept, which covers a range of disciplines from mathematics to psychology. However, neither of these sciences can explain risk by themselves; a combination of both sciences is needed, because risk lies at the socio-scientific intersection of the two. Adams (1996) proposed that risk is the product of the probability (as a measure of likelihood) and impact (as a measure of disutility) of some future event. However, there is no agreed definition of risk in the literature suggesting an epistemological plurality. In some circumstances risk is expressed as identical to likelihood; in other circumstances risk is expressed as identical to the impact of the future event. In more complex situations where both likelihood and impact seem to vary between compared events, risk may need to be understood as a co-ordination of probability and impact (Akyuzlu, 2011 submitted).

The complex nature of risk means that people find challenging the trade-offs involved in co-ordinating impact, likelihood and the many ethical and value-based judgements entailed (Brandstätter et al., 2006). An important starting point is that, in some important instances, perceptions of risk do not appear to correlate with measurable probabilities of risk, and therefore other factors are clearly important in how people understand risk (Slovic, 2000). In this study, we set out to consider students’ understanding of risk in a computer-based scenario as that understanding focussed on impact, likelihood, ethical issues and their co-ordination.

Method

We aim to design computer-based modelling tools which provide students with the opportunity to explore the context, to build their own model and to express their own understanding in that context. Furthermore, these computer-based modelling tools would make students’ thinking visible and give me the opportunity to observe and analyse students’ thinking. To achieve this aim, we decided to use Deborah’s Dilemma (DD), which has been designed for teachers to explore and interrogate their knowledge of risk. According to the developed scenario, DD, teachers have been asked to advise a fictitious person, Deborah, on whether to have an operation or not, to cure a spinal condition. She has a serious problem, which is causing her considerable pain. On the one hand, the operation entails the possibility of both minor and major hazardous outcomes which are provided in a descriptive scenario. On the other hand choosing not to have the operation entails restrictions on her life style, such as routine daily work and sport, and also the potential hazard of increasing pain in the future. Three computer-based modelling tools, Operation Outcomes (Figure 1), Painometer (Figure 2) and Risk Mapping (Figure 3), have been developed for users to analyse the given information and to use their results to advise Deborah. Having given a brief generic introduction about DD, we explain these three tools in the reminder of this section.
‘Operation Outcomes’, is a probability simulator, in which users model the likely outcome of having the operation. In this case, the probabilities for various complications, such as side effects of the surgery, or death through general anaesthetic, are given. However, there is conflicting information which is deliberately given to provoke discussion; for example consultation is requested from two different surgeon specialists; the first one is from a regional hospital and the second one is from London. There is a conflict in these opinions, to be resolved by the user.

‘Painometer’, is developed to give a quantified experience of Deborah’s pain, which she would feel during everyday activity, in the case of her not having operation. Pain experience is a subjective matter and an interesting context through which to explore people’s models of risk. Painometer provides an insight into how different activities may cause different levels of pain and which combination of activities she should undertake in order to cope with the pain to a “tolerable” level, which can be controlled by the user. The user is able to add different activities such as shopping, playing sport, yoga and jogging to model her daily life and the consequent pain. The outcomes can be analysed through the graphical representation of pain, which dynamically varies to represent the level of Deborah’s pain resulting from the chosen activities hour by hour.

‘Risk mapping’, is developed to support coordination of ideas about impact and likelihood. Users enter information that they have gleaned into colour-coded hazard boxes, associated with decisions to have or not have the operation, represented by decision boxes. There is a ‘Show/Hide risk’ button, which, when pressed, changes the colours of the boxes according to the horizontal position of the boxes. The users are told that darker boxes represent riskier undertakings, so that risk increases when the box is positioned further to the left. As a result, users are encouraged to re-position the boxes according to their estimation of the risk and they are therefore likely to think about risk as a single co-ordinated entity.

We present data that was collected during two pairs of 15 year-old students’ clinical interviews while they were exploring DD, outside the classroom but in school. Data were collected during students’ on-screen interactions and discussions, using screen-capture audio-visual software. This software is able to provide audio and video recording of both the screen activity and the faces of the students. My role as a participant observer was interacting with the students in order to examine and expose the reason behind their actions. In the findings reported below, the students’ names are pseudonyms. All recordings are transcribed and analysed manually after the interviews. We have assessed the students’ attempts to compare and consider information about ongoing pain against the risks of the operation through minor or major complications, with particular reference to trading-off of likelihood and impact.

Findings

We present two sets of data in chronological order: in the first case students’ have difficulties of trading-off between likelihood and impact of the situation whereas in the second case, after similar difficulties to begin with, there is evidence that students’ start to co-ordinate risk. The researcher (Res) is the author.

Case 1: Students’ have difficulties trading-off

C and S read the information about DD and formed an initial reaction:

C: It depends on side effects.
S: Yes.
Res: What kind of side effects?
S: She likes sport, if she has the operation, she cannot do sports again, then, maybe she might not have the operation, and she can carry on doing her sports...she just gets tablets or something.
C: It would be good if she has the operation but ... side effects ..., it depends on what kind of side effects it has. If there is a really serious side effect then she should not to do it. She should stop sport.
Then the researcher explained to them how to use the mapping tool to record their initial thoughts. They read all the information available on DD and stated:

C: I think she should have the operation. After research ... it says, like that operation might have good results in the end.

S: All the research ... there are things like, people who had the operation, they have had a better life and pain has gone and they do not have unnecessary side effects. I think she should have it.

They started to use the Operation Outcomes tool to create their own model. They set the operation success rate as 85% and introduced what they saw as the most significant complications. They ran the tool 1000 times as shown in figure 1.

They created their own model and based their discussion primarily on likelihood, as below.

C & S entered “Paralysis” with 15 in 100 rate, as first complication, and “permanent non serious nerve function” with 2 in 100 rate, as second complication.

S: You should type this one “Death” as a third complication... it is 2 in 100000... it could end your life.

C added “Death” as a third complication and “Hospital infection” with 2 in 100000 as a fourth complication

S: “Transient problems with impaired nerve function” is so boring.

Res: But it is 5%

C: “Death” is better because it could end your life.

C & S: There is a more chance of her surviving with no pain... she should have the operation.

C: There are 863 times success rate.

S: Think positive instead of negative... if there was 863 fail rates, you should say that she should not have it.

Figure 1: Having the operation (Probability simulator) tool
Afterwards, students started to use Painometer (Figure 2) tool to model Deborah’s life if she were to not have the operation. In this case, students’ discussion was based primarily on the impact of the activities and their life experiences.

S: “first activity (cycling)” helps her a little bit. “second activity (swimming)” helps more but this one “third activity (higher desk)” … she has a little bit pain left.

C: And then “cycling” no extra pain. So when you put them altogether, it gives little pain.

S: She does activities as well that help her… cycling, swimming, yoga…

Res: When you compare those tools “having the operation” and “not having the operation”, what do you recommend to her?

C & S: She should have the operation.

S: Because if she does not have the operation, she will be still in pain but if she has the operation there is more chance the pain has gone.

Res: Assume that she has an operation and one of the complications could happen or operation could be unsuccessful. So, what will happen?

C: Yes, but it depends what the doctors are… if the doctors are really good like all the patients have successfully gone through the operation, then, she should go to someone like that …

C & S: The one who has more experience …

S: …The one that has more experience is more expensive and one is not is less. So, she can afford one of the less. But I still think she should have the operation.

C: Yes. I still think she should. But on the other hand if she cannot really afford to pay for the one who has a more experience, then she should not have the operation.

S: … when she does not have the operation and she has done cycling, yoga and swimming stuff, how about when she gets old, she might not able to do that…
C: When she has the operation, she could still continue to do sport. But still think that she should have it.

**Case 2: Second pair students start to co-ordinate the dimension of risk.**

The second pair of students went through a very similar process as the first pair, drawing on personal experiences and switching in between impact and likelihood in a similar way. Finally though, when students began using mapping tool (Figure 3), something interestingly different happened; they appeared to start to co-ordinate judgements of likelihood and judgements of impact, as below.

Res: Regarding those tools, what is your decision? Do you recommend her to have the operation or not?

C&A: Yes, she should have the operation.

A: For her family sake and for her sake.

C: Yes, the higher chance for getting better then she might take a risk.

Res: What is risk for you?

A: 50 per cent chance of her living or dying.

C: No, 2 per cent of her dying... being paralysed is a risk but it is not a high risk, so, she should have the operation.

The researcher introduced the “Show Risk” button and their reactions were as follows:

C: That one (paralysis) and death have a big risk

Res: In case of big risks you recommend her to have an operation?

C: Yes, because that’s only like 2 in 100000 chances... it is a big risk but it is less likely to happen.

Res: Ok. So, you consider consequences with likelihood.

C & A: Yes.

C: If this box, mapping tool would be bigger, I will put them all in the middle.

Res: What? Which one?

C: All of them.

Res: You mean hazard ones or decision ones?

C: All of them.

Res: If you put them in the middle... all will be the same colour which means that all will have same risk.

C: Yes, but listen, this one is unlikely (death), this one is likely (more stress and pain) but this one is very serious (paralysis), this is not serious (she cannot sit more than 5 minutes). It is kinds of balances are...like equally.
C moved the hazard boxes as shown in figure 3.

![Diagram of hazard boxes](image)

Figure 3: Mapping tool

**Discussion**

I separate the discussion section in three sub-headings to illustrate my findings as below.

**Switching between impact and likelihood.**

Despite the complicated details of DD, students engaged successfully and created their own models. First instance, they decided that Deborah should not have the operation, and, after reading given information they shifted their decision to Deborah having the operation. While they were using the Probability simulator, they chose complications, by considering either likelihood or impact (lines 8-15). On the one hand students chose “Death” as a complication because it had serious impact but they disregarded the likelihood of “Death”, which is very low. On the other hand students chose “Hospital infection” as a complication because of its likelihood but they disregarded its impact. In this sense, this finding study offers support to the prior work with teachers who also referred to impact separately from likelihood even though their discussions were thoughtful and rational.

**Discussion of students’ personal experiences and values**

The software triggers students’ personal experiences and they empathise with Deborah’s condition. While they were using the software, they supported their ideas using their personal experiences. At the beginning of the clinical interview in case 1, student S (line 4) recommended to her not to have the operation because she could take tablets to endure the pain. They chose low impact activities for Painometer tool because of their personal experiences (lines 15-17). They regarded the experience of the doctor as trustworthy, which is based on operation success. Having said that, understanding of experience led student to change decision in case 1 (line 25) but student S convinced her by giving a counter example (lines 26-27). In addition, students also considered Deborah’s family condition (lines 29-30) in case 2. As shown in the findings, students like the teachers, used their personal experiences and values when they were working with the software.

**Discussion of students’ co-ordination of the dimension of risk**

In contrast to the first case students, in the second case the students’ started to co-ordinate risk despite its complexities. On the one hand, student C (line 37) considered “Death” as a big risk
(meaning impact) but less likely. On the other hand, subsequently, the mapping tool provided a resource through which students expressed ideas for the co-ordination of likelihood and impact in terms of the situated notion of a trade-off, articulated as ‘balance’ and ‘cancelling’ (line 40-59). As a result, this study shows that mapping tool can help students to make co-ordination between judgements of likelihood and judgements of impact.

References


Mathematical Modelling with Technology: the Role of Dynamic Representations

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In this research we present the use of some technologies in problem solving activities (at different secondary school grades), aimed at finding a model for a geometric configuration, and representing this model in various ways: through a construction, through a Cartesian graph, etc. The task is part of a teaching experiment, in which students used paper and pencil, and technological tools: a sensor and a calculator (at a lower grade), GeoGebra and TI-Nspire (at a higher grade). We show results in terms of the passage from static to dynamic representations and back, to observe how technology may foster dynamic thinking for students solving mathematical problems. Data suggests that the dynamic features of technology support the genesis of conjectures, and their validation or refutation, along with the choice of independent and dependent variables. Results are used to prepare materials for teacher training in an e-learning Moodle platform (Comenius EdUmatics Project).

Introduction

Representations in mathematics education have been studied since a long time, but particularly in the last decades, when technology offers many kinds of representations (graphic, symbolic, numeric, and so on). The representations available in the technological tools are different from some years ago, when a prevalence of static characters constituted the scene, as final products of specific computer processing (e.g.: a table in a spreadsheet, a graph in a programming language like Pascal; see Ferrara et al., 2006).

Nowadays, more advanced forms of representation have been created. Dynamicity is an intrinsic feature of tools, and is also present on the screen of a computer or a calculator. Examples are given by: a geometric figure constructed in Dynamic Geometry Software (DGS); a graph of a surface traced through a symbolic software like Mathematica; a function represented using an i-Pad application. All of these mathematical objects can be dragged, enlarged, turned, viewed from different perspectives. The new representational infrastructures have consequences on learning too (Kaput et al., 2002; Hegedus & Moreno-Armella, 2009; Sträßer, 2009), giving a fundamental support for exploring, discovering, conjecturing in mathematical problems where processes changing over time (dynamic representations) must be considered. In such cases, dynamic representations allow cognitive affordances that may lighten solvers’ cognitive load according to the specific type of representation used (see for example Ainsworth & Van Labeke, 2004).

Since conjecturing using dynamic thinking is also possible without technology (Simon, 1996), this does not mean that poorer materials (like paper and pencil) just permit for static representations, which of course may also have dynamic features. Sketches by learners are a case: they are powerful given their iconic character, and acquire dynamic nature in the moment students interact with and use them to communicate thoughts. Simply, the modern infrastructures give students new possibilities that previous generations students did not have, for developing a type of transformational (dynamic) reasoning at primary, secondary, and university level.

Furthermore, the use of dynamic representations in modelling situations enhances the dialectic between the empirical side and the theoretical side of the mathematical objects. For example, Data Capture function in TI-Nspire (used to model a situation) allows data collections in a way similar to physical samples (Arzarello, 2009). On the other hand, DGS enable students to choose various quantities as independent variable, supplying different dynamic representations of one situation, and the exploration of invariants in the corresponding mathematical objects. As R. Noss said in his plenary lecture at ICTMT10, technology may help students in their activity, “re-representing ideas of change with a medium that supports change”.

The project

EdUmatics is a Comenius Project aimed at preparing materials of lifelong learning for in-service teachers across Europe, and using them in presence (by means of congresses, seminars, workshops,
and at a distance (through a Moodle platform for e-learning: forum, activities, wiki, ...). Our part in
the project is to construct, with a French partner (University of Lyon), a section devoted to static and
dynamic representations (Aldon, 2010), used in a fruitful way to solve problems and model situations.
In this section, we prepared a problem on sequences of natural numbers and a problem on geometric
modelling, while the French partner prepared a problem on statistics. The activities were carried out
in Italian and French schools to experience the role of technology in real classroom contexts. We
focus here on the geometric activity. “The walker problem” proposes some situations about the
motion of a constant speed walker moving along the perimeter of a geometric figure. It asks students
to set up a model describing the variation of the dependent variable (the walker distance from the
centre of the figure) as a function of an independent variable, given or not, according to the school
grade. Learners are first invited to imagine, recognize and describe quantities in the geometric
situation individually, then in small groups, just with paper and pencil, before making calculations or
using technology. Starting from the observation in the particular case of the circle (Figure 1), where
the dependent variable (the radius) is invariant to changes of the independent variable (the angle or
the arc), students come to consider the more complex case of a square (Figure 2). When working in
groups, students have to fill in a paper-sheet, and then they participate to a classroom discussion
coordinated by the teacher.

**The walker problem in the case of the circle (Figure 1)**

Mr Bean moves at a constant speed along a circle with centre O and given radius r, starting from a point A on the
circle.

Mr Bean wants to describe how his distance from the centre O of the circle changes, during his moving along the
circle. How can you help him?

**The walker problem in the case of the square (Figure 2)**

Mr Bean moves at a constant speed along a square ABCD with centre O and given side, starting from a point P on
the square.

Mr Bean wants to describe how his distance from the centre O of the square changes, during his moving along the
square. How can you help him?

“The walker problem” has been implemented at different secondary grades, and technological tools
of various kinds were also used. At lower secondary school (grade 8), students first utilise a motion
sensor and a graphic calculator to observe the graph of the situation, then they construct and
represent the model in GeoGebra. At higher secondary school, grade 10 students use GeoGebra with
a free choice of the independent variable; grade 11 students use TI-Nspire to find the model
(although the choice of the independent variable is suggested by the teacher). Different tools give
different possibilities to represent and to work with the model of the situation, but they also have
some common features. We show here some excerpts from the videotaping of the classroom
activities carried out during plain school time. The methodology was that of *mathematics laboratory*
(Arzarello & Robutti, 2010). The filmed students attend different classes and schools, their teachers
participated in the project as experimenters, and a master degree student was the observer.

**Dynamic representations**

The first excerpt is taken from the 8th grade classroom activity. Students are thinking of the problem
in the case of the square, and discussing in their group (before using the motion sensor), while the
observer films them sometimes interacting with them.
Observer: You told me that distances change. Why?
Paolo: From the centre to A (distance) will be 1,5, but from P to C since it is not a line anymore … say, it is always a straight line, but the angle is not 90° anymore, and it changes, now here it is not proper at the half (between A and B).
Gabriele: Hm, no, ‘cause here you are not anymore … here he is going away (his finger follows the segment AB and goes over, passing B).
Paolo: He’s going far and far.
Observer: Ok, here he’s going away, far and far. Then what distance does he measure?
Paolo: It is always … greater.
Gabriele: Greater till arriving at B (with his pen he follows the segment AB and he stops in B).
Paolo: Here (at point B) will be the greatest point.
Observer: And then?
Paolo: In the four angles…
Gabriele: And then it will decrease till arriving at the middle point of the side (with his pen he follows the side subsequent to AB), and he will find the same distance as PA.

The students come to understand that the function is not constant as in the case of the circle, but it changes its values from a maximum, at each vertex of the square, to a minimum, at the middle point of every side. At this moment, they are not aware of the shape of the function graph, and they sketch it with a piecewise graph: four straight lines going up and down connected with each other. Only after the experiment with sensor and calculator (Figure 3), they are able to understand the actual shape of the graph, made of the same curved piece repeated 4 times (Figure 4).

The second excerpt comes from 10th grade students working with GeoGebra. They conjecture on and construct the figure relative to the circle, then they approach the square problem: some groups choose as independent variable the angle, other groups the path covered along the square. In both cases, the model is a periodic function (with period given by 90° or one side), whose graph is a decreasing and increasing curve that can resemble a parabola (they find it through the locus function of the software). In order to investigate what kind of curve it is, the students select five points on the first piece of the curve (with the angle as variable). Asking the DGS to find a conic passing through these points, it gives an ellipse (Figure 5). However, trying to calculate the solution symbolically, the students discover that the model is the inverse of a cosine, instead of an ellipse.

![Figure 4: graph by the sensor](image1)

![Figure 5: angle as variable](image2)

![Figure 6: path as variable](image3)

This mismatch offers the teacher a chance for discussing on the approximation by the software to look for a conic interpolating the five points, and on the necessity to always check the solution the software provides, comparing it with that theoretically found in paper and pencil. By the way, the approximation is easily revealed by zooming on the graph, and by the fact that moving the points the symbolic expression of the ellipse changes.

Teacher: Moving the points (on the locus), instead of those points, you take other five points, on the same arc, the ellipse modifies … This already tells us something. What does it happen? What does it mean that given five points I get an ellipse, moving the five points in another place, again on that locus, I get a new ellipse, with another equation? It means I’m having approximations, it means this arc here is not exactly the first ellipse, nor the second, nor the third one … maybe, it’s not even an ellipse, but the approximations of GeoGebra, to find a curve through those five points we chose, makes it to become an ellipse.

Choosing the path as independent variable, students obtain another periodic function, and again they select five points on the curve to get a model. GeoGebra gives them a piece of a hyperbola (Figure 6), which can be calculated applying Pythagoras’ theorem. In this case, the symbolic manipulation furnishes a model matching that obtained with the software.
The third excerpt refers to 11th grade students, who solve the same problem as 10th grade students, using TI-Nspire and TI-Navigator. In the following, a part of group activity in paper and pencil, before using technology, is presented.

| Emanuela: Then, practically it is ... then ... | Alessandro: A zig-zag (miming it in the air) ... let’s fix the side equal to 1. |
| Alex: It starts from the maximum ... | Alessandro: Yeah ... \( \frac{\sqrt{2}}{2} \) ... ’cause it is at the middle of the diagonal ... and it arrives at \( \frac{1}{2} \).

The students’ reasoning on the activity in paper and pencil is, corroborated by imagination, more complex than the one of the 8th grade students, and makes also use of measures and quantitative relations among the square’s sides. Nevertheless, at their first approach they come to sketch a zig-zag function as well as 8th grade students did (Figure 7).

![Figure 7: graph in paper and pencil](image1.png) ![Figure 8: graph on the calculator](image2.png)

Students are used to analyse a function in terms of variation of its values, and ways of variation, because they study the mathematics of change since grade 9, using different representations of functions and different technologies. The intuition on this problem guides them to infer that the model of the situation is made of linear functions connected together. Using then the TI-Nspire handheld technology, students want to check if their conjecture is true or not. With TI-Nspire, they construct a square and connect a point on the square with its centre through a segment, calculating this distance. In the spreadsheet environment, they collect data for this distance capturing it as variable from the geometric figure, and automatically receive data from the figure in function of a counter that has the invisible meaning of time (a natural sequence from 1 to 400). Simultaneously, in the graphic environment, they have the possibility to see the model animating the independent variable (Figure 8).

| Teacher: What do you want to verify? | Emanuela: We would like to verify if PO (namely the distance from the centre) constantly decreases. |
| Teacher: When you say constantly ... that is, are you thinking linearly? | Emanuela: Yeah. |
| Teacher: So, pass to use TI-Nspire. | |

The students calculate first and second differences and observe that the last ones are not constant. So they come to the conclusion that it cannot be a parabola, and they decide to go on finding differences, in order to discover if, at a certain point, they are constant. The teacher, through the
discussion, outlines that it could happen that they do not find constant differences, as in the case of a non-polynomial function: hence, they are finally able to acknowledge that this is the case.

Discussion

To approach the task, all the students move from the personal senses (Leontiev, 1994) they attach to the situation to making conjectures on a possible mathematical model. Regardless of the different kinds of technology used, most students (at all levels) first conjecture that in the case of the square the model is linear. This is not in contrast with the usual manner we all, included secondary school students, think in everyday life, since, at a first approximation, the world is interpreted as linear (De Bock et al., 2007). Then technology is used to test the conjectures: these may be more or less effective for properly modelling the situation. Hence, different interventions of the teacher are necessary to support the evolution of students’ personal senses towards a shared scientific one. Such behaviour is generalised throughout the grades. The feedback of the tools and the modulation of the teacher’s interventions also depend on the classrooms’ mathematical knowledge and culture. All these components (technology, teacher, classroom culture) play a fundamental role in supporting the students to elaborate new conjectures about the mathematical nature of the graph.

The case of the finite differences for the 11th grade students working with TI-Nspire is emblematic. They are used to calculate finite differences, in the classroom practice, as a means for understanding the degree of a polynomial function. This practice constitutes a background competence for their mathematical work on functions. During the activity on “The walker problem”, they recover and apply it to the particular case of the square, in order to validate or confute their conjectures. In this manner, the students go back and forth between static and dynamic representations (the zig-zag shape mimed by Alessandro’s gestures in the air becomes the piecewise graph traced on paper; then, such a graph is compared to that obtained on the calculator through the animation, according to which the finite differences are founded). However, this kind of behaviour points out a limit of students’ reasoning: indeed, they implicitly assume that the finite difference strategy will work in any case (only knowing that it does for polynomials). There is a contrast here between the personal sense and the scientific one, according to which the right model is not a polynomial. To overcome the contrast, the teacher’s intervention is essential: he is able to prompt the students that not all functions in the end have a constant n-th difference table and thus to accept within their conjecturing horizon the possibility of a non polynomial function as a model of the problem. In a Vygotskian perspective, he triggers the evolution of students’ personal senses towards a shared scientific sense: the mathematical discussion he orchestrates in the classroom is the means by which he is able to manage such mediation.

Another aspect of the teacher’s eventual interventions is shown by the grade 10 students using GeoGebra. Once the groups have got the graph that models the walker problem, they conjecture that it is made of arcs of hyperbola. Now they use the software to validate this finding. They know that given five points, the software is able to draw the conic passing through them. Hence, the students choose five points on the graph: but in one case (when the independent variable is the angle) they get an ellipse, against their expectation. Of course this is just an approximation of the actual model. Again, in this situation the intervention of the teacher is relevant to help the students to find the algebraic expression of the functions describing the geometrical situation (respectively, an hyperbola and a cos⁻¹, depending on the independent variable chosen) and to share the work of the filmed small group with the whole classroom. The classroom discussion opens then a cognitive space where all the strategies and the solutions are shared, and students can discuss the comparison between the choices of one independent variable in place of another one in terms of the model. Anew, the teacher makes apparent a bridge between students’ personal senses and the scientific sense. It is not the case that the personal senses of the students are cancelled; on the contrary, they may evolve in new senses closer and closer to the mathematical ones.

In relation to the choice of the independent variable, an interesting point may be outlined with respect to producing a deeper cognitive bridge. In fact, we involved some University students attending a course in Mathematics Education (for prospective mathematics teachers) in “The walker problem” (without filming them). We could orchestrate a very rich discussion on the different types of the phenomenon described by one choice or another of the independent variable. Taking the angle as independent variable gives rise to the model relative to a constant angular velocity motion, while
the path as independent variable corresponds to the model for a constant linear velocity motion. Namely, the two models concern different physical motions. This is relevant regarding the expected shape of the graph, and as such may affect students’ conjectures and their checking.

In the course of the project, another activity was given to the students, in the arithmetic context of sequences of natural numbers. The aim of the activity was again to find a model for different sequences starting from some of their numbers, and to represent the model in various ways: using a table, a recursive function, a close formula. Despite the difference of the tool used (in this case, a spreadsheet), we noticed that a continuous passage from static to dynamic representations and back is still present. Moreover, the spreadsheet fosters students to think dynamically, adopting dynamic strategies in the search for the formula.

References


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Using multiple representations in the classroom - The EdUmatics-Project

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Abstract. The EdUmatics-project (European Development for the Use of Mathematics Technology in Classrooms) aims to increase the integration of ICT in European mathematics classrooms. An online training course is constructed to provide learning and teaching material for in-service and pre-service secondary teachers. In this talk, we will give a short overview of the aims and methods of this project and we will present two activities. The first one illustrates what we call "teachers activities". It explores, using appropriate technological environments, the limits of a method used to determine the position of a curve with regard to its tangents. The other activity, which is to be conducted in the classroom, compares exponential and linear growth processes using interactive Geogebra applets or handheld devices. Results of empirical investigations carried out by French and German teachers about the later activities will also be examined.

Recent studies in Mathematics education show that despite many national and institutional actions within the EU aiming to integrate ICT into mathematics classrooms, such integration in secondary schools remains weak (e.g. Hoyles & Lagrange 2010). The rate of this integration increases slowly compared to the evolution of the technology. The huge diversity of ICT resources leaves teachers often unsure of which to use and when and how to use them. Studies also reveal that reasons for the slow integration of ICT in mathematics into classroom practice are deeply linked to the training strategies used. Approaches to training are sometimes unrelated to teachers’ current classroom practices, being essentially based on the transmission of technological rather than pedagogical skills. Thus, there has been little impact on supporting teachers to make best use of new opportunities created by digital educational content and services.

The EdUmatics-Project

The European Development for the Use of Mathematics Technology in Classrooms (EdUmatics) project aims to increase thoughtful integration of ICT in European mathematics classrooms by building and disseminating an online training course for in-service and pre-service secondary teachers, in particular by providing high quality teaching material based on research and experience of the 20 partners involved (Drijvers & Weigand 2010). In using ICT, the partners are experts from ten universities and research institutes, together with ten secondary level schools across six European countries.

The project consists of five different "chapters" (called modules): 1. Starting to work with ICT, 2. From static to dynamic representations, 3. Constructing functions and models, 4. Using ICT in the classroom and 5. Multiple representations.

The Wuerzburg Group develops module 5 together with the IREM of Montpellier. This module deals with the use of multiple external representations (MER) in the classroom, the interrelationships between employed software and how to wisely use them in class. The module includes didactic considerations about the use of MER, methodical reflections on how to make thoughtful use of ICT, discussions concerning the theoretical background of MER in the learning of mathematics and ready-to-use classroom activities. Advantages and disadvantages, goals and difficulties of the use of multiple representations are also discussed.

The project will develop a research-based course, which aims to educate in-service and pre-service teachers to use new technologies in their mathematics classroom to maximise students’ learning.

The course resources will become available through a multilingual European collaborative internet-based platform to include videos of classroom case studies, interactive applets, teaching materials, etc. This course also integrates face-to-face meetings (within each country), online-work, individual tasks and practical work in the classroom.
This paper will focus on the fifth module of the course: "Module 5: Multiple Representations". In the following, after having presented the outline of the module, we will examine two trends underlying the content of the module, namely the "teachers' activities" and the classroom activities. An illustrative example of both activities will be presented: "limits of a method" for the first trend and "growth processes" for the second one.

Multiple Representations (Module 5)

The structure of Module 5 is similar to the other four modules of the course, paying particular attention to the use of ICT to display mathematical objects in various ways. After an introductory part to show the possibilities of spreadsheets, dynamic geometry, graphing and CAS for mathematical representation, the module revolves around a three main topics: growth sequences, data and statistics and the derivative. These are primarily discussed in two parts, namely teachers' activities and classroom activities, which will be described below.

Teachers' activities

Based on the large experience in working with teachers and in developing professional developments that aim to enhance mathematics education through technology, the Module 5 team believes that providing teachers with classroom activities that illustrate the use of environments supporting multiple representations, even when accompanied by substantial didactical and methodological elements, is not sufficient. Teachers also need to reflect upon their practice before they practice. For this purpose, several "teachers' activities" were included in the module, where teachers' autonomy, considering their ICT choices, increases as they progress in the course.

Two kinds of activities were designed. Activities meant to familiarise teachers with one representation at the time (four in total, referred to as "basic competencies" in the course) were introduced at the earliest stages of the course. For most of them, the idea was to provide teachers with mathematical problems devised such that one particular representation would be especially helpful to make conjectures. The second kind of activities aimed to gradually introduce more than one representation that would potentially enrich the answer to the given mathematical question. The interplay between different representations, at the core of Module 5, is at the forefront of the later activities.

All teachers' activities are based upon the idea of an a priori analysis of classroom situations. Throughout the activities, teachers are asked to predict students' strategies used for solving the problem, reflect upon the technological environment they would use and argue on the advantages and disadvantages of different representations on offer. Hints on expected answers are given on teachers' demand (accessing the "expected answer" is possible through clicking on parts of the activities).

In order to illustrate the underlying principles of the teachers' activities, let us focus on the one called "Limits of a method", in which three representations (graphical, numerical and algebraic) are involved.

The aim of this activity was two-folded: not only has it been designed to put the limits of specific representations to solve the problem into evidence, but it also reveals the limits of a widely taught mathematical method used for examining the position of a curve with regard to its tangents, namely the method of studying the sign of an expression by factoring. This activity, as all teachers' activities, revolves around student tasks that begin with the following question:

(C) designates the graphical representation of a function f chosen amongst common functions (such as “square”, “cubic”, “inverse”, “square root”, “polynomial”, “homographic”, “rational”) and $T_0$ the tangent to (C) at point $M_0$ of x-coordinate $x_0$.

1. What conjecture can you give for the position of (C) with regard to $T_0$?
2. Clarify the methods you have chosen in order to answer the previous question. What technological tools have you utilized?
Tasks addressed to teachers included naming the different environments and methods they think students would use to make conjectures and compare each of the environments. For the later question, for example, expected answers when analysing the advantages of the graphical environment are such as: "the visualization is more spontaneous and enhances possible conjectures. The graphic enables a more global view of the position of the two curves, while the spreadsheet limits the visualization to some rows only. Furthermore, the graphic enables one to make conjectures by anticipating the positions of the two curves for the portions not seen in the window".

In the following question, which was meant to investigate the status students attributed to the different environments, teachers were asked to predict the different environments and methods students would use to prove their conjectures.

<table>
<thead>
<tr>
<th>Question 3</th>
<th>Are the technological tools utilized to answer question 1 sufficient to prove the conjecture? If not, what would you suggest?</th>
</tr>
</thead>
</table>

In fact, some students may realize that neither the graphical nor spreadsheet environment are sufficient to prove their conjectures about the position of \( C \) and \( T_0 \). In this case, it is likely they will think of using an algebraic method and therefore use CAS, the only environment that allows developing a rigorous proof. A possible method used for comparing two expressions consists in examining the sign of their difference, which may be done by factoring (when the sign is not evident). This method is particularly efficient for the common functions quoted in the subject, but not for other functions such as exponential. Question 4 of this activity (see below) was designed to reveal the limits of such a method.

Interestingly, an experiment conducted in a French grade 12 class, draws the attention on difficulties that have not been anticipated. In fact, some students, even if they have correctly envisaged to use an algebraic method and the CAS environment to prove their conjectures (which has been thought as the eventual main difficulty of this question), had great difficulties to interpret answers provided by CAS in order to match the conjecture, which was established from the graphical environment.

When considering the cubic function for this question, students frequently established the conjecture: "the graph of the cubic function is above the tangents on \( \mathbb{R}^+ \) and below on \( \mathbb{R}^- \). With the help of CAS, students then find the factored expression and try to identify the positive and negative sign in such expression.

![Fig. 1: Screenshot of the “Limits of a method” work sheet](image)

While students easily recognised the positive sign of, they had trouble with interpreting the one of. And long discussions with the classroom teacher do not seem to suffice to students, whose final answers provide evidence that the interrelationship of both environments (graphical and algebraic) is far from been grasped by students: if and . The same difficulty arose in question 4, where the abscissa of the tangent point is numerically fixed.
Later in the activity, tasks addressed to the teachers relate to the following question:

**4. Are the environments and methods utilized in the previous questions still relevant to answer questions 1 and 2 when the function is defined on \( \mathbb{R} \) by?**

Teachers are asked to predict students' answers and imagine a helpful action for students whose analysis may remain a bit shallow. This question is the occasion to alert teachers from some bias of the graphical environment. In fact, depending on the window chosen, the graphic visualization may hide some subtle characteristics of the function. In a "standard" window, for example, the curve seems to be always on the top of its tangents. Appropriate zooming is necessary, as shown in the figures below.

![Fig. 2: Standard window](image1)

![Fig. 3: Zoomed window](image2)

Issues about the interpretation of spreadsheets are also raised. In fact, when using "standards values" for the -value of the tangent point and, for this step, the spreadsheet provide a table of values for where all numbers have the same sign. To make the change of sign explicit, a specific value for (between 0 and 0.1) and for the step (strictly smaller than 0.1) is necessary.

The task ends with the analysis of, for which the method of factoring in order to examine the sign of the difference is not helpful anymore (the use of CAS shows that it is not possible to factor the expression); another method is needed.

**Classroom activities - Growth processes**

The overall aim of Module 5 is to foster the thoughtful use of multiple representations with ICT and to let teachers experience the relevant use of technologies in their classrooms on their own. To lower the entrance hurdle, we offer a number of pre-built classroom activities that are to be used directly in the participating teacher’s classroom. These activities contain work sheets and suggestions for student tasks, but also methodical and didactic considerations on how to use these activities and especially ICT as a valuable supplement to the common classroom practice.

The aforementioned didactic considerations include a short overview of the mathematical background, a discussion of the topic’s educational relevance, its prerequisites to be used in class and why this subject matter was chosen for multiple representations with ICT in the classroom. These student tasks constitute the heart of the classroom activities. The following example is taken from the activity “Quiz show”, where students are to compare linear and exponential growth.

**In a quiz show, a candidate is to be asked up to ten questions. If he or she gives the wrong answer, the candidate must leave the show, but may keep the prize he or she won up to that question. Prior to being asked the first question, the candidate is given two alternative prize options from which he or she must choose one:**

**Type 1:** Each correctly answered question adds 100 €.

**Type 2:** The first question correctly answered yields 20 €, each additional correct answer doubles the prize.

a) Consider these two options! Which one would you choose spontaneously? Discuss with...
Growth processes and their model as growth sequences are good examples for classroom activities with multiple representations and the use of ICT. There are some competencies which are related to this topic: dynamic experience of mathematics, working with different representations, emphasizing the relationship between algebra and geometry, etc. Depending on their complexity, growth sequences can also be used in any grade of secondary education, with linear growth in the lower and polynomial, exponential or logistic growth in the higher grades.

While symbolic explicit and recursive notations, discrete graphs and value tables are the sequences’ basic mathematical representations, it is not easy to handle sequences as discrete objects on a symbolic level when using only paper and pencil. High school students are not confident with algorithmic knowledge concerning sequences. ICT is a tool to work with representations of discrete processes. We especially think about the use of spreadsheets, because the analogy between cells in spreadsheet programs and sequences’ elements can be used in a beneficial way. When participating in the EdUmatics course, it is the teacher’s task to integrate the classroom activity into his or her lessons, e.g. by using pre-built programmes or applets. In our course, we use TI-Nspire programmes (see Fig. 3) and Geogebra applets. Both provide possibilities to use dynamically linked multiple representations (see also Guin et al 2005, Hegedus & Moreno-Armella 2008).

We intend to show teachers the benefit of certain representations, but also the inappropriateness of others and the use of multiple representations. This especially means to know about the role of these representations in the learning process and – as a consequence – in the teaching process. So, we first did a pilot investigation with university teacher students and students of grade 10 about the use of multiple representations with the “Quiz show” problem. We have been especially interested in the following research questions concerning the use of multiple representations:

1. Which representations do students use when solving the problem?
2. How do students use representations in their arguments?
3. How do the students use multiple representations and when do they switch representations?
The representations which students chose to solve the task, are highly dependent on their previous knowledge, in particular whether they used self-defined functions to model the situation or the spreadsheet. First classroom trials using the TI-Nspire – in which students created their own representations – have shown that, as expected, most students preferred graphical representations for globally comparing the two types of growth, since it gives a more global view of the depicted growth sequences, especially – visually – giving a good impression of the exponential growth process. While value table and graph do not yield any additional information that cannot be read from the other representation as well, the representations’ global or local views on the growth sequence support global or local approaches that complement and support each other (see Ainsworth 2006). Experimenting with different starting values of the sequences in the task’s part c) emphasizes the link between the two representations, with ICT automatically translating changes in the spreadsheet view to the graph.

The idea of the project is that teachers, after trialing an activity in their own class, have the possibility to discuss their experiences with other participants of the EdUmatics course. Especially, they should reflect on the use of ICT of their students, the advantages and disadvantages they see and also on how to change the teaching units. To support this reflective process, methodical reflections at the end of the module give hints in advance and for the follow-up of the lesson. Moreover, the teachers are to discuss their results of the trials in relation with relevant research results, comparing them to pre-recorded, exemplary classroom episodes.

Conclusion

The study of a mathematical problem can often be enriched by investigating it from different points of views (e.g. Weigand & Bichler 2010). Multiple representations promote the emergence of conjectures at the same time it enriches them, assuring flowing links between them. However, using multiple representations requires teachers to work with their students in order to help them interpret information from different representations – see the comments on students’ difficulties encountered in the ”Limits of method” activity. The interrelation between representations is not self-evident, but takes a significant effort by the students. The use of interrelated software is a great way to help them with this task, but it is not sufficient if it does not come with competent guidance and instruction. The EdUmatics project provides training for teachers to qualify for this task, offering teacher activities to sensitize for important issues of using ICT, classroom activities that provide an easier access to new technologies in their own classrooms, underpinned by the current research in this area and with the possibility for reflection on their progress by communicating with other teachers participating in the course.

References


Effects of feedback conditions for an online algebra tool

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Procedural skills and conceptual understanding have been widely debated, especially with regard to algebra. In the mean while the use of ICT in education has increased. In this article we report on one of the design principles for and results of a digital intervention for algebraic expertise. The intervention aimed at improving algebraic expertise and was deployed in 15 grade 12 mathematics classes in 9 secondary schools. In this paper we focus on the implementation of IDEAS feedback and report on the effects of two feedback conditions. Preliminary results show that relevant feedback aids students in learning algebra by decreasing the number of attempts needed for an algebraic task, whilst improving the scores. We conclude there is potential in the use of feedback in an online algebra tool but that further development is needed.

Introduction

During the last decade the dichotomy between procedural skills and conceptual understanding has been widely debated. It has been a focal point in the so-called ‘Math War’ (Schoenfeld, 2004) discussion. The debate also influenced the realm of algebraic expertise: should students focus on practising algorithms or on reasoning and problem solving strategies? One approach stresses the fact that computational skills are an essential ingredient for understanding mathematical concepts (US Department of Education, 2007). Another approach starts off with more focus on conceptual understanding (ibid.). Although most experts seem to agree that essentially both are needed, there is no clear agreement on the relationships and priorities among the two. Another development in recent years involves the advent of the use of technology use in mathematics education. The National Council of Teachers of Mathematics (2008) formulated the potential of Information and Communication Technology (ICT) for learning in their position statement. Our research combines the aforementioned elements: we want to use the potential of ICT to address algebraic skills, on both a procedural and a conceptual level, and aim to design and test an intervention doing just that. In this paper we will focus on one of the design principles behind the intervention, the use of item feedback.

Conceptual framework

For the purpose of this paper we will focus on one of three topics involved in the general framework of our study: theoretical notions of formative assessment and feedback.

Formative assessment

Black and William (2004) distinguish three functions for assessment: supporting learning (formative), certifying the achievements or potential of individuals (summative), and evaluating the quality of educational programs or institutions (evaluative). Summative assessment is also characterised as assessment of learning and is contrasted with formative assessment, which is assessment for learning. Black and William (1998) define assessment as being ‘formative’ only when the feedback from learning activities is actually used to modify teaching to meet the learner’s needs. From this it is clear that feedback plays a pivotal role in the process of formative assessment.

Feedback

Both Hattie and Timperley (2007) and Vasilyeva et al. (2007) conducted an extensive meta-review of the effectiveness of different types of feedback. The feedback effects of cues and corrective feedback are deemed best. Seeking feedback is governed by a cost/benefit ratio. In general, feedback is psychologically reassuring, and people like to obtain feedback about their performance, even if it has no impact on their performance. The model provided by Hattie and Timperley (ibid.) distinguishes three questions that effective feedback answers:
Where am I going? (the goals) 
How am I going? 
Where to next? 
FeedUp 
FeedBack 
Feed Forward

Each feedback question works at four levels (focus of the feedback): the task level: how well tasks are understood/performed (FT), the process level: the main process needed to understand/perform tasks (FP), the self-regulation level: self-monitoring, directing and regulating of actions (FR), and the self-level: personal evaluations and affect (usually positive) about the learner (FS). Hattie and Timperley (2007) also provide some statements on the effectiveness of (combinations of) feedback types, including that FS feedback is least effective, simple FT feedback is more effective than complex FT feedback, FT and FS do not mix well (“Well done, that is correct” is worse than “Correct” only), and that FT is more powerful when it’s about faulty interpretations, not lack of information. Furthermore they state that we should be attentive to the varying importance of the feedback information during study of the task. The three main design principles of our digital intervention involve feedback. In this paper we will not focus on two of these principles, formative scenarios and crises, but only on feedback at the item level. Here, both custom feedback and so-called IDEAS feedback are used to provide more “intelligent” feedback. The accompanying research question, therefore, is: does a variation in feedback type influence scores, attempts and student behaviour and in what way?

Method

An intervention called “Algebra met Inzicht” (“Algebra with Insight”) is designed in the Digital Mathematical Environment (DME, www.algebrametinzicht.nl). It was field tested in a pilot lesson series by the end of 2010 for 15 groups of grade 12 mathematics students at nine secondary schools throughout the Netherlands (N=334). Schools were randomly allocated to two feedback conditions c1 (N=133) and c2 (N=178). The collected data included results from a pre- and post-test, and the scores and log files of the digital activities. The log files record information on item scores, feedback, answers, and number of attempts. The DME has a provision for feedback by connecting to the IDEAS web service (Heeren & Jeuring, 2010), as well as the feature of providing custom feedback, the latter which is described by Bokhove (2010). Custom feedback consists of feedback that teachers can program themselves, IDEAS feedback consists of a web service that provides feedback automatically. The IDEAS web service is also implemented for other online mathematical environments.

![Figure 1: Screenshot of DME's authoring environment for IDEAS feedback](image)

Figure 1 shows the essential characteristics of the IDEAS implementation in the DME. Firstly the general characteristics of IDEAS feedback, which includes what feedback is shown when and where. These settings were used to create the two conditions c1 and c2. Secondly IDEAS implements a block of diagnostic messages, which concerns feedback on strategy, the ‘correct step’ but also possible
‘detours’. The third block of feedback concerns rewrite rules, rules that can be applied to an expression. Finally there are buggy rules, which describe the feedback that appears when a mistake is made. Using the authoring environment we implemented two series of tasks with both custom and IDEAS feedback we will refer to as d1 and d3. Two versions of these series were made: one for condition c1, and one for condition c2. The first feedback condition c1 consists of IDEAS and custom feedback without buttons in the interface. Feedback is only provided in the stepwise approach.

![Figure 2: Stepwise custom feedback](image)

To illustrate this, figure 2 shows the solution process for a polynomial equation. The student loses solutions for the equation along the way, and appropriate feedback warns the student that this is happening: “You are about to lose two solutions. Keep in mind that the expression also yields two solutions. Please complete”. This feedback is along the lines of ‘feedback about the task’ (FT). The second feedback condition c2 is essentially the same as c1, but additionally provides several buttons on the screen that could be used for getting hints and solutions of the exercises.

![Figure 3: Feedback condition c2, including buttons](image)

Figure 3 shows these buttons for (i) tip, which provides a hint for the next step, (ii) stap, which provides the next step in the solution process, and (iii) los op, which solves the whole equation and thus provides a ‘worked example’ (Sweller & Cooper, 1985). These buttons can be used by the student at will, providing self-regulatory tools (feedback type FR). In the case of figure the student used the ‘stap’ button to obtain the next step in the solution process, and feedback “A*B=A*C gives A=0 or B=C”. After deploying the intervention we will look at scores, attempts and case examples to formulate an answer to our research question.

**Results**

First the quantitative findings will be presented and then these will be illustrated by some case examples. As a Kolmogorov-Smirnov test shows that both d1 and d3 scores are not distributed normally (Z=5.408, p<0.001; Z=6.768, p<0.001 respectively), we apply a non-parametric Mann-Whitney test. This test shows that there is a significant difference between the feedback conditions when we look at the score for d1 (U=7680.00, p<0.001, r=-.321) with condition c2 scoring higher than condition c1. According to Cohen (1992) this accounts for a medium effect size. The second series of tasks d3, however, did not show a significant difference (U=10560.00, p=.531)

When considering the number of attempts, this was significantly higher for feedback condition c1 (Md=126.00) (without extra buttons) than for feedback condition c2 (Md=105.00), U=9904.50,
p<.001, r=.202. Although we can classify this as just a small effect, it suggests that the addition of buttons for hints and solutions results in fewer attempts.

Apart from these quantitative results there also is a substantial body of case examples where students have successfully or unsuccessfully used feedback options. We now illustrate the use of IDEAS feedback with three examples from both feedback conditions. The examples show the subsequent steps of a student when confronted with a task (step 0), and the accompanying feedback. Note that feedback was translated from Dutch into English for the purpose of this paper, and that randomization of the tasks means that students received different equations to solve. The symbols \( \times \), \( \checkmark \) and \( \& \) designate incorrect answers, correct but not final answers and correct answers, respectively.

<table>
<thead>
<tr>
<th>Step</th>
<th>Student</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Solve:</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>( \checkmark ) You are rewriting correctly</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>( \times ) Hint: rewrite to [expression]=0</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( \checkmark ) You are rewriting correctly</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>( &amp; ) You have solved the equation correctly</td>
</tr>
</tbody>
</table>

Table 1: Feedback example from condition c1 (stepwise feedback)

In the example in table 1 the student starts of by expanding the left and right hand side of the equation. As there are no feedback-buttons available the system evaluates the expression in step 1 as correct (but not the final solution). The student makes a calculation error in rewriting in the form \[expression\]=0. Now the system hints that the expression is incorrect and gives feedback. The difficulty of judging what mistake was made instantly becomes apparent: in this case the student already understood he/she should rewrite to \[expression\]=0 but makes a calculation error. In step 3 this error is corrected. The student then remembers to apply the Quadratic formula and solves the equation correctly. We see that the system does still have difficulties with judging what format of the expression is “good enough” and what isn’t. The evaluation is quite ‘liberal’, giving the notation in step 4 full marks.

The second example in table 2 shows that the addition of buttons helps a student to overcome initial difficulties with the given equation.

<table>
<thead>
<tr>
<th>Step</th>
<th>Student</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Solve:</td>
<td>Hint: (AB=AC) =&gt; A=0 or B=C</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>( \times )</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>( \checkmark ) You are rewriting correctly</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( \checkmark ) This is not quite the exact format</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>( &amp; ) You have solved the equation correctly</td>
</tr>
</tbody>
</table>

Table 2: Feedback example from condition c2 (hints)

After being given a new equation the student uses the ‘tip’ button to get a hint. The student uses the hint to apply the correct strategy, first making a notational error, but correcting this in step 2. After this the student concludes the task in steps 3 and 4. In step 3 the system prompts that the expressions can be simplified.
The third and last example concerns a student that uses the ‘los op’ button to automatically solve the given tasks, with the system adding the strategies as feedback for every step in the solution process.

<table>
<thead>
<tr>
<th>Step</th>
<th>Student</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Solve:</td>
<td>AB=AC =&gt; A=0 or B=C</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>✓ Rewrite in form [expression]=0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>✓ Move constants to the right</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>✓ Free up variable by dividing on both sides</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>✓ Simplify by factoring</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>✓ Use quadratic formula</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>✓ Simplify roots</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>❌ You have solved the equation correctly</td>
</tr>
</tbody>
</table>

Table 3: Feedback example from condition c2 (solve)

This example was added because it shows the difficulties of evaluating student answers. Most solutions from students combined several steps into one. For example, steps 2, 3 and 4 could easily be combined in one step. The notational issue mentioned earlier also crops up: many teachers would perhaps have given full marks for the solution in step 6, but because the square root can be simplified this is not seen as the final solution.

**Conclusion**

In this paper we set out to see whether variation in feedback type influences scores, number of attempts and student behaviour. When observing the scores students obtained in the two conditions, we can see that there is a medium effect for the feedback condition including self-regulatory feedback (condition c2). This effect, however, only was apparent in one of the series of tasks. We think we can explain the difference between d1 and d3 in the fact that both series address the category of polynomial equations (Bokhove & Drijvers, 2010). Having solved polynomial equations with feedback in d1 meant that similar types of equations (in d3, which followed after d1) could already be solved, and subsequently no additional form of self-regulation was needed any more in d3. The addition of buttons for feedback also had a, albeit small, effect on the number of attempts. This makes sense as the additional feedback that can be requested discourages more attempts. When looking in more detail at the use of the feedback in the three case examples it is clear that both task-related and self-regulatory (FT and FR) feedback can be used in a formative way for the learning of algebra. Students can use the feedback to overcome difficulties and check whether they are on the correct solution path or not.

However, a word of caution is needed. Although research suggests that worked examples are effective (Sweller & Cooper, 1985) students could easily be tempted to ‘just push the button’ (this is an actual statement from a student) to get full marks. To address this pitfall, a design principle involving fading (Renkl, Atkinson, Maier & Staley, 2002) and formative scenarios (Bokhove, 2008), whereby the amount feedback and worked examples are decreased during the course of the intervention, was applied to the intervention. This approach shows promising results which, however, lie outside of the scope of this paper. The same holds for a third design principle involving crises: non-standard tasks that can’t be solved with the standard algorithms. The idea is that these tasks force students to think ‘out of the box’. The role of feedback is to provide enough support to students so they won’t just give up when confronted with such a crisis.
Although the results of using custom and IDEAS feedback for algebraic expertise are promising, there still are many improvements to be made. These improvements should firstly focus on notational aspects as shown in the case examples. Student motivation declines when they do not get full marks just because the system says so. Secondly feedback should be adapted to the target audience and math curriculum. Clearly, the mathematical language of higher education is different from that of secondary education. It is not a viable option to let all teachers author their own set of feedback comments. One goal in the near future will be to try and provide default values for feedback that applies to the most common student errors and behaviours, resulting in feedback ‘out-of-the-box’. It is imperative that the appropriateness and quality of ‘intelligent’ feedback is improved before we can reap the benefits.

References


Using Cryptology to Teach Fundamental Ideas of Mathematics

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Cryptology is a very old science and until a few decades it was a science for government, military, secret services and spies. Nowadays, cryptology is almost everywhere in our lives. This paper reports on a research, an epistemological analysis of the question: “Is it possible to teach fundamental ideas of mathematics by using cryptography?”

In a first step fundamental ideas of mathematics, which are the basic guidelines for mathematical education are discussed. For the analysis a set of fundamental ideas of mathematics is developed e.g. algorithm, functional dependence, modelling, number, measuring and ordering. In a second step connections between the set of fundamental ideas and various techniques of cryptology are shown. Some outstanding examples with high invitingly character for students for this part of the analysis are the Fleissner grille or the Diffie-Hellman key exchange.

1 Why is cryptology so important?

Since the existence of humanity people have always the need to communicate confidentially with each other. Nobody should understand the communication or even don’t know that the communication is in progress. So cryptology is a very old science. In the history of cryptology you find many funny examples of secret communications. For example, Herodotus tells one case. Histiaeus (520-493 BC), also called the tyrant of Miletus was captured and imprisoned in Susa. He wanted to send a signal to his son in law Aristagoras for rebellion and freeing himself. So he cut the hair of a slave. Then he pricked a tattoo on the head of the slave. Afterwards the slave has to wait till his hair was re-grown. Then he sent the slave to his son in law. Now the slave went to Aristagoras and nobody was able to see his secret message. All the hair covered the important secret. When the slave arrived at Aristagoras his hair was cut a second time. So Aristagoras read the message, made the rebellion, freed his father in law, but made himself king.

This example shows that Cryptology is a very old science. Until a few decades it was a science for government, military, secret services and spies. Nowadays, cryptology is almost everywhere in our daily life. For example:

- login at the E-mail Account,
- working on a https-Server, for example online banking,
- all cards in our pockets are full of cryptology, credit cards, calling cards and so on,
- mobile phones need many applications of cryptology, for example the GSM-Standard for ciphering phone calls
- car keys for opening the car
- or RFID’s (Radio-frequency identification), for example books in the library are signed with this tags.

These examples show the importance of cryptology in our daily life. So we see cryptology is nearby everywhere. All modern cytological applications are working so well because they use a lot of mathematics. But Mathematics is hidden by the technology. So for a modern math-education we should recover mathematics in the technology and show that mathematics is a high-tech science. But in which way should we do this? For an epistemological answer of this question we use the fundamental ideas of mathematics.

2 Fundamental Ideas

The principle that math education should be led by fundamental ideas goes back to the beginning of the 20th century. Alfred North Whitehead was one of its famous devotees. In his “INTRODUCTION TO MATHEMATICS” he states: “The study of mathematics is apt to commence in disappointment.” (Whitehead, 1911, 1924, p. 8) Whitehead notice the cause that “the pupils are bewildered by a
multiplicity of detail, without apparent relevance either to great ideas or to ordinary thoughts. The extension of this sort of training in the direction of acquiring more detail is the last measure to be desired in the interest of education.” (Whitehead, 1970, p. 119)

Thus he proclaims: “The science as presented to young pupils must lose its aspect of reconditeness. It must, on the face of it, deal directly and simply with a few general ideas of far-reaching importance. ... For the purposes of education, mathematics consists of the relations of number, the relations of quantity and the relation of space.” (Whitehead, 1932, 1970, p. 119) In another section he added the fundamental idea of functionality (Whitehead, 1932, 1970, p. 125).

The American psychologist Jérôme Seymour Bruner (born 1915) is another very important person in this context. In his book “The Process of Education” from 1960, he writes: “It is simple enough to proclaim, of course, that school curricula and methods of teaching should be geared to the teaching of fundamental ideas in whatever subject is being taught.” (Bruner, 1960, p. 18) He does not define the term of fundamental ideas because he uses different terms: basic and general ideas (Bruner, 1970, p. 17), fundamental structure of a discipline (Bruner, 1970, p. 25), fundamental principles and ideas (Bruner, 1970, p. 25). All terms have essentially the same meaning. Bruner concentrates the following items of fundamental ideas for math education, which are not complete. ”If the understanding of number, measure and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child’s forms of thought.” (Bruner, 1966, p. 53)

The aim of this paper is to examine if the contents of cryptology allows teaching fundamental ideas of mathematics. So sets of fundamental ideas of mathematics are needed. In the German mathematics didactics sets of fundamental ideas are of particular interest. Therefore, the researchers draw up many catalogues of fundamental ideas (Schweiger, 2006). In table one some important catalogues are listed for overlooking many years of discussion. The catalogues of Schreiber, Tietze/Klika/Wolpers and Heymann are chosen because they contain the entire mathematics. The catalog of Humenberger/Reichel is chosen because this catalog is special for the applied mathematics.

<table>
<thead>
<tr>
<th>Schreiber</th>
<th>Tietze/Klika/Wolpers</th>
<th>Humenberger/Reichel</th>
<th>Heymann</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. algorithm</td>
<td>1. algorithm</td>
<td>1. models, language and translation processes</td>
<td></td>
</tr>
<tr>
<td>2. exhaustion</td>
<td>2. approximation</td>
<td>2. Approximation method, approximate values and error control</td>
<td></td>
</tr>
<tr>
<td>3. invariance</td>
<td>3. function</td>
<td>3. stochastic</td>
<td></td>
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<tr>
<td>4. optimality</td>
<td>4. modelling</td>
<td>4. optimize</td>
<td></td>
</tr>
<tr>
<td>5. function</td>
<td>5. geometrization</td>
<td>5. algorithms</td>
<td></td>
</tr>
<tr>
<td>6. characterisation</td>
<td>6. linearization</td>
<td>6. represent situations with a mathematical view</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>7. networking of mathematical facts</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 1: different catalogues of fundamental ideas (Borys, 2011, p. 165)

The fundamental idea of algorithm is common in all sets. Modelling or model and function or functional dependence are parts of three of four sets. So you see the importance of these fundamental ideas. For the purpose, to examine whether cryptology carries fundamental ideas of mathematics, the following ideas are selected: algorithm, functional dependence, modelling, number, measuring, ordering (means the geometrical classification and the logical ordering). Number, measuring and ordering are chosen, because they cover the other ideas (see Borys, 2011, p. 165).
3 Cryptology in the view of the fundamental ideas of mathematics

What is cryptology? The Encyclopædia Britannica (2002) writes: “cryptology, science concerned with communication in secure and usually secret form. It encompasses both cryptography and cryptanalysis. The former involves the study and application of the principles and techniques by which information is rendered unintelligible to all but the intended receiver, while the latter is the science and art of solving cryptosystems to recover such information.” Thus cryptology is a very wide field so I will focus only on cryptography so you don’t lose the track.

The plaintext is the message that will be put into a secret form (Kahn, 1996, p. xv). The message maybe hidden in two basic ways, the methods of steganography and or cryptography. The methods of steganography conceal the very existence of the message, like invisible ink, microdots, secret compartment or the example of the slave. The methods of cryptography don’t conceal the presence of the message but render it unintelligible to outsiders by various transformations of the plaintext. In the cryptography two basic methods of transformations exist, the transposition and the substitution. For math education cryptographic systems which work with transpositions are very interesting because with them are a lot of geometric skills trainable. So my first example is the Fleissner grille which works with transpositions and uses the mathematical concept of rotation. My second example is the Diffie-Hellman key exchange because it is a system which we use every day, it is pure math and it is the easiest public key cryptosystem.

Why is cryptography so interesting for teaching? Cryptology is a science with a lot of mysteries, so students are very interested in these themes and they are very excited when they encrypt or decrypt a plain- or cipher text. This motivation and interest should we use for math education to make the lessons more exciting.

**Fleissner Grille**

Eduard Fleissner of Wostrowitz describes in his manual to cryptography by 1881 (original title: “Handbuch zur Kryptographie”) turning encryption grilles. With these grilles he creates a permutation of letters of a plaintext. Even Archduke Rudolf (1858-1899) son of Franz Joseph I (1830 – 1916) Emperor of Austria encrypted his letters because he didn’t want that his father get notice of his liberal views (for an example see Borys, 2011, p. 272). The Fleissner grille was really in use, with this example we teach real history. Fleissner works in his book with 5x5, 7x7 and mostly with 15x15 grids. For pupils it is easier to work with grids which have an even number of rows and columns because then the centre of the grid is formed by the crossing of the centre lines which is the turning centre. A suitable example is published from the novelist Jules Verne in his novel “Matthias Sandorf”. This grille is made by a 6x6 grid with 9 holes. Some cells are covered. They are black in figure one; the white one´s are holes of the grille.

![Fig. 1: 6x6 turning encryption grille published from Jules Verne](image)

Which fundamental ideas of mathematics may you teach with Fleissner grilles?

You can easy illustrate the fundamental idea of algorithm with it. A nice model for showing this is to describe the process of encryption and decryption with the Input-Process-Output Model.
Encryption

Input: Plaintext

Process:
1. Lay the grille onto a sheet of paper and fill in sequentially the blanks of the grille with letters of the plaintext.
2. If the blanks are filled in the template so rotated it around its centre by 90° and new free fields will appear. Fill in the free fields.
3. Step 2 is twice more repeatable till the entire square is filled. If the plaintext is too short so fill in the remaining blanks with a meaningless string of letters. If the plaintext is too long you have to use a second square and repeat step 1 to 3.

Output: The cipher text is read row by row.

Decryption

Input: Cipher text

Process:
1. Fill in the cipher text letter by letter and row by row in a table (mind its size).
2. Lay the grille over the filled table (mind the starting position of the grille).
3. Read all letters which appear in the blanks of the grille.
4. Turn it around its centre by 90° (mind the direction of rotation). Finally read again.
5. Step 4 is twice more repeatable then you have all letters of the square. If some letters of the cipher text remaining, repeat step 1 to 4 until all letters are deciphered.

Output: Plaintext

With the Fleissner grille it is also possible to illustrate the fundamental idea of functional dependence. The idea of encryption with turning grilles is to make a permutation of all letters of the plaintext. This permutation is set by four parameters: size of the grille, position of the holes, position of laying up the grille and direction of rotation (clockwise or anticlockwise), e.g. for this grille see figure two.

<table>
<thead>
<tr>
<th>Position in plaintext</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in cipher text</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>11</td>
<td>15</td>
<td>20</td>
<td>23</td>
<td>30</td>
<td>34</td>
<td>1</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Position in plaintext.</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>Position in cipher text</td>
<td>10</td>
<td>13</td>
<td>18</td>
<td>21</td>
<td>25</td>
<td>28</td>
<td>3</td>
<td>7</td>
<td>14</td>
<td>17</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>Position in plaintext</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>Position in cipher text</td>
<td>31</td>
<td>33</td>
<td>35</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td>19</td>
<td>24</td>
<td>27</td>
<td>29</td>
<td>32</td>
<td>36</td>
</tr>
</tbody>
</table>

Fig. 2: Permutation of the grille of figure one

The fundamental idea of ordering can be illustrated in different ways with the Fleissner grille. The essential idea of turning grilles is the rotation of the grille and rotations are mathematical concepts. Figure one shows a grille which is working with three rotations. Figure three shows a grille which is working only with one rotation by its centre by 180°.
What is therefore interesting for teaching?

It is very well possible to train mathematical problem solving skills within this context. For example by solving the following problems:

- Produce a working Fleissner grille.
- In which way do you build up a working Fleissner Grilles?
- How many different 4x4 or 6x6 or (2n)x(2n) Fleissner templates exist?

Answers of these questions are easy when you enumerate the grille of figure one by 1 to 9 (see figure four). For a working grille only one cell of each number is taken out. For every cell you have 4 different opportunities. Thus for the 4x4 type $4^9=256$ different templates are existing, for the 6x6 type $4^9=262,144$ and for the (2n)x(2n) type exist $4^n$.

For teaching at the elementary school, I recommend the 4x4 grille, because the handling is easier and the students have no problems to overlook all cells, letters, turns and so on. Some gifted students maybe work with the 6x6 grille.

I worked with children in the secondary schools of different ages. Figure five shows some grilles which are made by a girl at the age of 13 years. The left picture shows a grille, which doesn’t work, because there are some holes in the wrong cell. The grille on the right is working, because every hole has a unique number (see figure four).
In this stadium of my studies the children produced the holes with scissors or cutters. Producing the holes in this way is very exhausting. A better way is to work with a stamp. With a stamp, a hammer and a wood pad you only need seconds to cut out the holes (see figure 6). Another solution for the cutting problem is to use a punch.

![Fig. 6: Tools for producing quickly Fleissner grilles](image)

### 3.2 Diffie-Hellman key exchange

Diffie-Hellman key exchange is invented by Whitefield Diffie and Martin Hellman. They published it 1976 with their paper “New Directions in Cryptography”. It allows two parties that have no prior knowledge of each other to share a secret key over an insecure communication channel. Which fundamental ideas of mathematics may you teach with the Diffie-Hellman key exchange?

You can illustrate the fundamental idea of algorithm with it. For example we call the two parties who like to exchange a key Alice and Bob.

#### Input:
Over an insecure canal Alice and Bob communicate two numbers:

1. a prime number $q$
2. a generator $g$ ($2 \leq g \leq q - 2$) which is a prime root of $q$.

#### Process:
Alice chooses $a \in \{1, ..., q-1\}$, computes $\alpha = g^a \mod q$ and sends $\alpha$ to Bob.

Bob chooses $b \in \{1, ..., q-1\}$, computes $\beta = g^b \mod q$ and sends $\beta$ to Alice.

#### Output:
Alice computes $K = \beta^a \mod q$.

Bob computes $K = \alpha^b \mod q$.

Both get the same key.

The key $K$ is the shared secret of Alice and Bob, only they know it. An observing third person is not able to compute the key, if Alice and Bob choose $q$ as a large prime number about a length of 1000 bit and more. Mathematics guarantee that nobody is able to calculate $K$, if he has only the information about $(q, g, \alpha, \beta)$. For computing $a$ or $b$ you have to solve the equation $\alpha = g^a \mod q$ or $\beta = g^b \mod q$, but this problem is known as the discrete logarithm problem. For this problem no efficient algorithm is known. Exactly this point is the core of mathematical modelling, because the mathematics ensures that the communication is secret.

So with the Diffie-Hellman key exchange it is easy to illustrate the fundamental idea of mathematical modelling. This method of key exchange is applicable in all types of client-server environments. Thus, client and server use the Diffie-Hellman key exchange to arrange a secret key. After that client and server encrypt information by using the exchange key. The SSL protocol which is used on the Internet at the pages marked with HTTPS works in this manner.

Fundamental idea of number has a centre role in the Diffie-Hellman key exchange. Let’s make an example by using small numbers for illustration:

Alice and Bob arrange
\[ q = 97 \quad g = 23 \]

Alice chooses \( a = 20 \) and computes:
\[ \alpha = 23^{20} \mod 97 = 43 \]

Bob chooses \( b = 31 \) and computes
\[ \beta = 23^{31} \mod 97 = 87 \]

Exchange of \( \alpha \) and \( \beta \)

Alice computes:
\[ K = 87^{20} \mod 97 = 73 \]

Bob computes:
\[ K = 43^{31} \mod 97 = 73 \]

The common secret key is \( K = 73 \).

Some aspects about the needed numbers:

- For the first number \( q \) you need a prime number.
- For the second number you can choose any \( g \), such that \( g \) is primitive to mod \( n \) and there is no reason why not choosing the smallest \( g \) you can – generally one-digit number (Schneier, 1996, p. 514). In school you do not need a prime root of \( q \) to work with the Diffie-Hellman key exchange because \( g \) does not have to be primitive, it just has to generate a large subgroup of the multiplicative group mod \( n \) (Schneier, 1996, p. 514).
- With a standard school calculator you may get problems to compute some exponents because it is not possible to type this in with one step. You need several steps by using exponentiation by squaring and the modular arithmetic. If you would like to avoid this just work e.g. with the calculator of windows or CAS.

The two examples Fleissner grille and Diffie-Hellman key exchange show that it is very interesting to examine the illustration capability of cryptology concerning the fundamental ideas of math. Furthermore other useable examples are: the Greek skytale, the Roman Caesar code, the Vigenère code, the RSA code, etc.

References


Using live online tutoring to provide access to higher level Mathematics for pre-university students

Tom Button and Richard Lissaman

The Further Mathematics Support Programme/ Mathematics in Education and Industry, UK

A level Further Mathematics is an additional, higher-level, qualification taken by many students in England. The Further Mathematics Support Programme (FMSP), which is managed by Mathematics in Education and Industry, has been working to ensure universal access to this qualification since 2005. Integral to his work has been providing tuition to students in schools and colleges that are unable to offer it themselves. This is delivered remotely through the use of Elluminate (shared whiteboard and audio-conferencing technology that runs from a desktop). The FMSP has been delivering live online tuition in mathematics for over 4 years and, through experience, has developed much good practice in the teaching and learning of mathematics online.

Introduction

In England the main pre-university qualification is A level Mathematics which is taken by approximately 70,000 students. Around 15% of these take a second, higher qualification: A level Further Mathematics. The Further Mathematics Support Programme (FMSP), managed by Mathematics in Education and Industry, has been working to ensure universal access to this qualification since 2005 (prior to 2009 the FMSP was known as the Further Mathematics Network). Integral to his work has been providing tuition to students in schools and colleges that are unable to offer it themselves. In cases where the students and tutor are not able to meet for face-to-face lessons this tuition is delivered using Elluminate (shared whiteboard and audio-conferencing technology that runs from a desktop). This enables communication between tutor and students by audio, instant messaging and via handwritten mathematics and text.

This paper will identify these experiences, sharing the good practice developed as a consequence of them, as well as suggesting some possible areas for future development.

Background

Since 2005 A level Further Mathematics has seen a rapid growth in entries, from 5,627 students in 2005 to 11,312 students in 2010, and is now widely encouraged by universities for students wishing to apply for STEM (Science, Technology, Engineering and Mathematics) subjects. To ensure universal access to Further Mathematics the FMSP provides tuition in cases where schools and colleges are not able to offer it themselves. Initially much of this tuition was face-to-face but more recently there has been a move towards live online tuition.

The live online tuition usually takes the form of weekly, early-evening sessions in which a small group of students meet online with an experienced tutor to learn new mathematics and solve mathematical problems. There are also extensive online resources to support the students’ learning. The students are drawn from across England and many of them would not be able to access this qualification without the live online tuition provide by the FMSP. In the academic year 2010-11 there are 125 receiving live online tuition through in this way the FMSP.

In addition to this the FMSP has widened the scope of its live online tuition to include other ways of supporting students and giving them access to higher mathematics. The FMSP has also developed the use of live online sessions for delivering professional development for mathematics teachers.

Our experiences

The Further Mathematics Support Programme has been offering live online tuition since 2006. A report on the experiences up to 2009 was presented at ICTMT 9 (Lissaman, de Pomerai and Tripconey, 2009).
**Current range of tuition supported**

The tuition supports students in a number of situations:

- Weekly online sessions;
- University entry examinations, such as the Sixth Term Examinations Paper (STEP) and the Advanced Extension Award (AEA);
- Online revision sessions;

The FMSP is currently tutoring 125 students for some or all of their A level Further Mathematics. These students meet online for 1-hour weekly sessions where there will typically be some coverage of content followed by an opportunity to work through problems. The mean group size is 3.75 and there is a notional maximum of 6 students per group; however, this is very occasionally exceeded.

In addition the FMSP currently provides online support for over 190 students preparing for STEP/AEA examinations. Success in STEP/AEA exams requires students to develop advanced problem-solving skill. Online sessions focus on students attempting question as a group, discussing strategies, carrying out calculations and setting out solutions together. These sessions probably have the least pre-planned structure of all sessions, the tutor acts to guide and steer students’ discussions and only contributes when necessary.

The FMSP provides online revision sessions. Sessions leaders cover key points and produce solutions to examination questions, emphasising mathematical and examination techniques. The number of students in such sessions can be as many as 150 and so these sessions are necessarily much less interactive than many of the others described here. Interaction is limited to the use of polling and instant messaging. Instant messaging does make an important contribution to the sense of a live session and frequently reveals misconceptions held by many students. The session leader can use this to increase the overall impact of the sessions. For the most recent examinations session 40 revision sessions were provided. These were attended live by over 1,500 students with a further 5,000 students watching recordings of them.

**Range of students supported**

Through using live online tuition the FMSP is able to provide access to higher level mathematics to a wide range of students, many of whom might not have been able to access it otherwise. Many of these students could be supported using a traditional distance-learning model, i.e. access to courses notes and some form of telephone/email support; however, the inclusion of live online sessions results in a much more effective package. The majority of students supported are pre-university students aged 16-19 years who will have had very little experience studying academic courses outside a conventional school setting. Consequently they are unlikely to have developed sufficient independent study skills to be successful using traditional distance learning. Live online tuition, with regular ‘lessons’ and contact with a tutor, is closer to the learning experiences that these students will be familiar with and can help them progress to a more independent style of learning.

Live online tuition is a very effective method of offering access to higher level mathematics to as many students as possible. This is reflected in the comments about using Elluminate for distance-learning mathematics students at university level: “Conducting online tutorials in the mathematical sciences using a synchrononous communication software tool such as Elluminate Live! can be an effective means of distance learning in mathematics and statistics at university level ... both students and tutors liked the medium, particularly for the interaction it offers and the convenience of not having to travel to tutorial” Mestel et al (2011) pp16-17.

**Good practice that has developed**

The FMSP has been delivering live online tuition in mathematics for over four years and in this time developed a range of good practice as a result of experience. This practice can be categorised as good practice for communicating during the sessions and good practice for supporting learners outside the sessions.
Communicating during the sessions

Elluminate offers a variety of communication tools: audio through a headset, a shared whiteboard, instant messaging and polling/emoticons for quick feedback. However, the practice as a tutor is different from that experienced in a face-to-face setting as one doesn’t have the normal level of visual feedback. This absence of social cues was identified by Tanis and Postmes (2003). To counter this it is necessary to build in as many opportunities for students to communicate as possible.

Elluminate offers audio-conferencing, which is an effective way for students to communicate in sessions; however, in practice many students can be reluctant to use the microphone themselves. This may be because they are uncomfortable talking in front of a group of students that they do not know very well. Alternatively it may be related to the sound quality on Elluminate which can deteriorate when there are simultaneous talkers. This can be overcome by using short introductory problems at the beginning of a course that require students to interact, especially through using the microphone.

The important factor is for the students to communicate during the session through whatever format they feel comfortable with. The other forms of communication on Elluminate can be used as an alternative to encouraging students to use the microphone. Many students seem to prefer instant message to talking and consequently questions where students can type short answers can be used. Similarly there are various polling options such as Yes/No or A/B/C/D response that can be used.

As well as typing answers, students can engage with sessions by writing on the shared whiteboard: a graphics tablet is very useful for this. As the whiteboard is shared it adds an additional level of interaction beyond what would be available using a face-to-face lesson: it becomes a shared space on which all students can write during the sessions. To effectively manage this it is necessary to limit the group sizes to 5 or 6 students it is possible to keep track of the comments. One useful tool for this is for each student to contribute using a different colour.

In addition Elluminate features the opportunity for breakout rooms where a single student or subgroup of students can work on a problem. One strategy that works particularly well is for each student to be assigned a separate screen with a copy of the same problem on for them to be working through. The tutor is then able to move between the screens giving extra help to students who need it. This ensures that all students are engaged and involved and individual difficulties are picked-up.

Support outside the sessions

Although the weekly online lessons offer greater support than a student would get through traditional online learning it is still necessary to support their studies outside these sessions. This is achieved through a combination of an extensive website of support materials, offering recordings of sessions, support from a local contact in the students’ school/college and opportunities to communicate with the tutor and other students outside the sessions.

FMSP students have access to the Integral Mathematics Resources website: http://integralmaths.org/. This contains thousands of pages of resources, including interactive resources, and formative assessments for students to complete during the course. The resources are structured to match the course that the students are taking and the online sessions are delivered to complement learning using these resources. In particular there are online multiple choice tests which are set at the end of every session for the student to complete before the following session. This gives the students a focus for their work outside the sessions ensuring that they stay up-to-date and also allows the tutor to monitor their progress.

As stated, the online sessions require students to be able to work more independently than they previously might have experienced. There is a reliance on students to be well-motivated and to aid this it is important that they have a local contact in their school or college who can support them. For example, one early-warning sign that a student is in difficulty is when they miss sessions and a rapid intervention by the local contact can prevent these difficulties worsening. Often there is a single lesson each week so consequently a single missed session can result in a two week break; to counter this recordings of sessions are made available to all students.
There are also opportunities to communicate with the tutor or other students outside the sessions either through email or forums. Many students take advantage of the opportunity to email the tutor for support outside the sessions. There are forums on the Integral Mathematics Resources website that the students can use; however, these have not proved to be especially popular. One possible reason for this could be that asynchronous discussion forums are not as useful in mathematics as they are in other subjects. This difficulty with the use of forums for mathematics, and science, is identified by Reid (2009) and could be a result of the way that questions are posed in mathematics does not lend itself to discussion and that the essential use of symbols and diagrams is harder to replicate in many text-based online forums.

**Potential for future development**

As discussed, for students to be successful in learning in this way requires them to be relatively conscientious. However, learning in this way may have a positive impact on their learning skills and this would be interesting to investigate further. Golden et al (2006, p vi) reported this in their research into the impact of e-learning in Further Education: “Use of e-learning had a positive impact on some aspects of learners’ ability to independently manage their own learning”.

In addition to this there is potential to use live online tuition to support other students who would benefit from access to higher-level qualifications. This could include mature students and others who are outside traditional school/college environments.

**References**


Live, Online Professional Development for Mathematics Teachers

Sue de Pomerai and Sharon Tripconey.

Mathematics in Education and Industry, the Further Mathematics Support Programme, UK

In recent years, the demand for a pre-university qualification in Further Mathematics has been a national success story in England. Numbers of students taking the qualification have rapidly increased from 5315 in 2003 to 12287 in 2011. As a consequence, there is a growing demand for more teachers to acquire the necessary skills and experience to teach mathematics at this level. The Further Mathematics Support Programme has developed a portfolio of cost-effective online courses to increase access to specialised training for mathematics teachers. The online courses are delivered in real-time using a web-based package. Participants have the opportunity to interact and engage using a range of features that include audio communication and a shared whiteboard. To date over 200 teachers have taken part in these online courses and the level of participation is expected to rise.

Introduction

The Further Mathematics Support Programme (FMSP) aims to provide universal access to A level Further Mathematics for students aged 16-18 years in England. An Advanced Level General Certificate of Education (GCE), commonly referred to as an A-level, is a pre-university qualification offered by schools and colleges and are typically required for a student to gain entry to a university. Mathematics is unique in that there are two A levels available for students; A level Mathematics and A level Further Mathematics. The former is designed to be accessible to a broad range of students and is a pre-requisite for A level Further Mathematics which is aimed at those wishing to study mathematics or related courses at university. During the 1990s the number of students taking A level Further Mathematics was in decline in state-funded schools and colleges. This decline was attributable to two fundamental issues; a shortage of suitable teachers and relatively small group sizes, making classes too expensive to schedule. The problems in offering Further Mathematics faced by schools meant that universities felt unable to make A level Further Mathematics an entrance requirement because it was not available to many prospective students. However, this had the effect of accelerating the decline in student numbers as it removed an important incentive to those state-funded schools still striving to teach it.

In 2005, in response to this spiralling decline, Mathematics in Education and Industry (MEI), an independent educational charity, was awarded government funding to set up a national programme called the Further Mathematics Network, with the aim of giving every A level Mathematics student in England the opportunity to study A level Further Mathematics. The number of students taking A level Further Mathematics has increased from 5315 in 2003 to 12287 in 2011. Moreover, during this period, the proportion of state-funded schools and colleges offering A level Further Mathematics has risen from 33 per cent to 63 per cent. Many universities now encourage or even require students to take a Further Mathematics qualification to improve their preparation for degree courses in mathematics-dependent subjects such as engineering and the sciences, as well as mathematics itself.

The rapid growth in the number of students studying Further Mathematics has increased the demand for more teachers to acquire the necessary skills and experience to teach mathematics at this level. In recognition of this, the FMSP has placed greater emphasis on providing professional development opportunities for mathematics teachers. In this paper we specifically discuss the ways in which we have used online technology to provide teachers with subject knowledge and pedagogical support. We summarise our findings, share our experiences and suggest some possible areas for future development.

Background to online professional development

In 2006, the Further Mathematics Network ran a successful pilot using online distance-tutoring with GCE A Level students using Elluminate Live! a synchronous communication software. This software was selected because of its interactive features that include the facility to upload MS PowerPoint and PDF files which are easily converted to the shared whiteboard display; allowing pre-prepared...
mathematical material to be viewed without loss of format. Another useful facility is the ability to share a software application which, despite only being installed on the host computer, allows all participants to interact with it. The online tuition programme has made significant impact in the FMSP’s aim to ensure universal access to Further Mathematics for Post-16 students aged 16-18. Following the initial success of using online tutoring with students, it was realised that online provision could be extended to facilitate professional development support for teachers. The FMSP recognised that many mathematics teachers encounter difficulties accessing courses with a focus at Post-16 level. Lack of funds and time constraints associated with being released from teaching commitments during school hours are the two main reasons given by teachers who cannot access training courses. The development of a comprehensive programme of online support was felt to be one possible solution to increase professional development opportunities. Online courses are relatively inexpensive compared to the total cost of attending a traditional one-day course. Furthermore online courses are scheduled to start after school hours so teachers involved do not need to miss lessons with their students and incur expense for travel and supply cover. A pilot programme of online courses for teachers was conducted in 2007/08, the results of which were reported in de Pomerai and Tripconey (2009). Since its inception the Live Online Professional Development (LOPD) programme has continued to grow and currently offer 13 different LOPD courses, covering pure and applied mathematics content from England’s current GCE A level exam specifications, delivered by a team of 12 fully trained tutors.

**Live Online Professional Development (LOPD) courses**

The LOPD courses are primarily aimed at teachers who would benefit from mathematics subject knowledge support. Course participants meet weekly with a tutor in small groups of four to eight to jointly work through content, tasks and problems. Group numbers are purposefully small so that the live sessions can be an interactive experience for all participants. In addition to the weekly online tutorials, course participants are given access to Integral.org, extensive online resources for teaching and learning provided by Mathematics in Education and Industry (MEI) and online forums for tutor and peer support.

**Course structure**

Each course consists of an introductory session outlining the features of Elluminate Live! This first session is to familiarise participants with the online classroom environment in preparation for the rest of the course and to resolve any connectivity or audio issues. This is followed by either five or ten hours of mathematics tutorials delivered ‘live’ online in real-time. The material in the longer courses is divided up in such a way as to preserve the coherence of the subject matter and as result there may be between seven and ten sessions in total, each lasting for between 60 and 90 minutes. The sessions are scheduled shortly after the end of the school day with the intention of making it feasible for teachers to take part either from school or from home. Teachers on the longer courses are also invited to take part in an optional session in which each teacher presents a learning resource which they have produced to other members of the group and there is opportunity for collective discussion about how the resource could be used to promote effective learning.

**Session structure**

Each session includes several main elements. Key mathematical ideas and techniques are introduced and presented by the tutor; this may include sharing applications where appropriate. Participants are encouraged to communicate by speaking or typing messages and will have opportunities to work on problems together using the shared whiteboard as well as having opportunities to work individually on problems using their own designated whiteboard. At the end of each session the annotated whiteboard screens can be saved as a PDF file. Each live session is recorded and a hyperlink to the recorded session is sent by the tutor to all participants. Recordings capture all interaction during the live online session and can be viewed at a later date.
Enrolment and participation trends

To date, 205 teachers have enrolled on one or more of the LOPD courses. 171 teachers (83%) have completed one course and 34 teachers (17%) have taken two or more courses. Table 1 outlines the growth in the number of courses offered, teacher places and tutors over the last four years.

<table>
<thead>
<tr>
<th>Academic year</th>
<th>Number of different topic courses scheduled</th>
<th>Number of teachers enrolled</th>
<th>Number of tutors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007/08</td>
<td>3</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>2008/09</td>
<td>10</td>
<td>60</td>
<td>4</td>
</tr>
<tr>
<td>2009/10</td>
<td>13</td>
<td>54</td>
<td>5</td>
</tr>
<tr>
<td>2010/11</td>
<td>13</td>
<td>113</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 1 - Participation trends

Following the success of the pilot in 2007/8, we had expected enrolment numbers to grow year on year but in reality, the numbers in 2009/10 decreased slightly. One possible reason for this may have been poorly targeted publicity. It was also suggested that some teachers may perceive online learning as ‘self-study’ of static online resources and they may not appreciate the live and interactive nature of our courses. In an attempt to overcome these issues, online ‘live’ taster sessions were advertised and offered free of charge to teachers in September 2010 and February 2011. The taster sessions intentionally focused on demonstrating the interactive features of Elluminate Live! rather than mathematical content. Over 110 teachers signed up for sessions however, as is often the case when there is no financial penalty, only 55% of those who booked actually attended the sessions.

Following the taster session teachers were asked to submit a questionnaire about their online experience. A total of 46 teachers provided a response. Teachers were asked to rate the likelihood of enrolling on a future course using a five point Likert scale. Responses are summarised in Table 2.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>(online courses are not for me)</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4%</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>9%</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>43%</td>
</tr>
<tr>
<td>(very likely to apply for an online course)</td>
<td>19</td>
<td>41%</td>
</tr>
</tbody>
</table>

Table 2 - Likelihood of enrolling on an online course after attending a taster session

Teachers were also asked if their online experience had met their expectations and results are summarised in Table 3.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Better than expected</td>
<td>26</td>
<td>57%</td>
</tr>
<tr>
<td>What I expected</td>
<td>16</td>
<td>35%</td>
</tr>
<tr>
<td>Less good than I expected</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
<td>7%</td>
</tr>
</tbody>
</table>

Table 3 - Rating of teachers’ expectations

Over half of the teachers reported that their online experience had been better than expected. In 2010/11 and 28% of these teachers have subsequently enrolled on a course. We have concluded that the taster sessions were generally successful in convincing some teachers to overcome their initial reluctance to engage with online learning.
Developing a model for good practice

Throughout the LOPD programme, teacher feedback has informed the continuing development of the online courses. When teachers apply for an LOPD course, it is made clear to that online courses are essentially for teachers who have never covered the course material before or wish to refresh their subject knowledge. The course application form specifically asks teachers to give reasons for enrolling on the course and outline their previous teaching experience. Table 4 summarises reasons for enrolment given by 113 teachers during the academic year 2010-2011. More than one reason could be provided so the percentages total more than 100%.

<table>
<thead>
<tr>
<th>Reason for enrolment</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain confidence</td>
<td>19%</td>
</tr>
<tr>
<td>I am teaching the content for the first time</td>
<td>62%</td>
</tr>
<tr>
<td>Refresh my subject knowledge</td>
<td>22%</td>
</tr>
<tr>
<td>Other</td>
<td>8%</td>
</tr>
</tbody>
</table>

Table 4 – Reasons for enrolment

This information is available to LOPD tutors for each member of the group before a course starts. During the course, tutors regularly give opportunities for participants to feedback on their experience and understanding. As group sizes are small, tutors can easily adjust their delivery, pace and session content to respond to the group’s needs. At the end of the course, teachers are asked to complete a questionnaire to reflect on their experience. This summative feedback is submitted online and includes a rating for the quality of the course, perceptions of online learning and technical performance and accessibility issues associated with the online environment. Although teachers are generally very responsive when asked to feedback to tutors during online sessions we have had relatively low submissions of summative course feedback. The rate of response is approximately 60%.

Summative feedback results

Summative feedback has been submitted by 70 teachers. These teachers completed a course between September 2009 and February 2011. From this group, 80% had completed an online course for the first time and 20% had taken part in an FMSP online course previously. Table 5 summarises course participants’ response to being asked if the online course had met their expectations.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>better than expected</td>
<td>25</td>
<td>36%</td>
</tr>
<tr>
<td>what I expected</td>
<td>41</td>
<td>59%</td>
</tr>
<tr>
<td>less good than expected</td>
<td>4</td>
<td>6%</td>
</tr>
</tbody>
</table>

Table 5 – Rating of teachers’ expectations

It is interesting to compare the expectations of the teachers who enrolled on a taster session (Refer to Table 3) with that of those who completed online courses (Refer to Table 5). Of the teachers who attended the taster sessions, 57% reported that their experience was better than expected and 35% reported that it was as expected, where the figures for those who completed courses were 36% and 59% respectively. However, this difference needs to take into account the fact that one fifth of the teachers in the second group had done an online course before and were better informed than those who had done a course for the first time or had only attended a taster session. That said, it is important to state that we did not determine the exact nature of teachers’ previous experience or perceptions of online learning before embarking on a taster or a course for the first time.

We are aware that the numbers summarised in Tables 3 & 5 only report on the teachers that are willing to experiment with online learning and thus the samples are self-selecting. It is widely accepted that some teachers lack confidence in embracing new technologies and we recognise that token taster sessions are unlikely to persuade the most reluctant techno-phobic teachers to engage with online learning. Furthermore, our sample does not include teachers who face practical
difficulties with regard to accessing the online sessions such as inconvenient timing or poor internet access. One willing and interested teacher reported, “It looks like the live sessions won’t work for me - the computers at school struggle with simple tasks and the earliest I can get home is 4:45 pm”. Without further enquiry, the extent to which teachers are unable to access the courses and their reasons for not participating cannot be established.

The opportunity for teachers to actively participate is a key feature of our online courses and as part of the course feedback teachers are asked to reflect on their preference for engaging with the group and the tutor. Teachers were asked, “When a question / activity was presented by the tutor which of the following scenarios did you prefer? Responses are summarised in Table 6.

<table>
<thead>
<tr>
<th>Preference for online engagement</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Follow the tutor working through the problem while I watched and listened</td>
<td>15%</td>
</tr>
<tr>
<td>Work in my own space to try the problem / activity myself</td>
<td>20%</td>
</tr>
<tr>
<td>Jointly work through the problem with the tutor and the group</td>
<td>2%</td>
</tr>
<tr>
<td>Have an opportunity to do a mixture of some of the above</td>
<td>63%</td>
</tr>
</tbody>
</table>

Table 6 - Summary of teachers’ responses

The majority favour a variety of ways to engage and get involved during a session though there is a significant minority who prefer working on their own or simply observing the tutor’s demonstration. It is also worth noting teachers’ confidence in using the different practical ways to communicate online. Our summative feedback informs us that 98% of participants feel very confident to use the text chat facility; only 73% of teachers feel very confident with the microphone. Many teachers made comments that related to feeling comfortable such as this comment, which is typical of the feedback received, “Having a small group of us so we felt we could ask questions, make mistakes etc without worrying about what other people would think”.

This summative feedback from teachers, reinforces that course structure needs to be sufficiently varied and flexible to address the learning preferences of the majority of participants and to take into account the differing preferences for ways to communicate, allowing opportunities for teachers to build confidence with online interaction.

Advantages, barriers and difficulties

One difficulty with more traditional CPD is that the learning can be transient with teachers having little time for reflection and consolidation when they return to school. Online courses are delivered over a period of weeks and as one participant said, “A great opportunity to try things out in the classroom between the weekly sessions and to report back to the tutor how it went. Sometimes on day courses that never happens.” Panda (2004) remarks “Professional development is a continuing process and much of this development takes place offline; this suggests that there is the necessity of offline individual reflection and collaborative negotiation in the community of professional practice”. In this respect, the online courses offer something that traditional one day courses do not, the extended timescale gives more opportunity for both consolidation and collaboration with colleagues and the tutor. As one participant remarked “I found that I forgot quite a lot of the material from week to week so in an ideal world I would have found some of my own time to look back over it and prepare - I think this does show the value of breaking it up and doing it over several weeks as if it had been done all at once I would most likely have forgotten far more!”

Access to additional support outside scheduled online sessions has been developed recently to include a dedicated online forum where course participants can post issues, problems and comments. Teachers continue to have access to this forum after they have completed the course. To date this forum has not been widely used but we hope that, over time, teachers will come to utilise this facility more. As one teacher said, ”It was great to have instant access to an experienced teacher of the subject, not only during the sessions but also via the online forum.”

The facility to record live sessions is a valued by teachers. Table 7 summarises teachers’ responses.
 Teachers have reported that recordings are particularly useful when they have been unable to attend a live session; allowing them to fully observe both the verbal and written interaction that took place in the live session. In addition, some teachers have explained that they will use recorded sessions to revisit the content at a time when it is most relevant to their teaching in school.

In an institution where PD is valued, it is possible to develop an ethos and environment that views online learning positively (Golden et al, 2006) however it is a new concept for many teachers and their managers so there can be considerable barriers to be overcome in trying to engage with those in a variety of institutions and geographical locations. One of these is a reluctance to accept that an online environment can equally provide the interaction and support that can be given in face to face learning. The taster sessions have helped to overcome this barrier for some. Golden et al report that there appears to be a close inter-relationship between confidence in, and attitudes towards, e-learning. (Golden et al, 2006). From feedback, one teacher reported that, “My confidence in using the technology (microphone, text writing, emotions and writing on the electronic whiteboard) has increased tremendously over the course." Another said “ We improved on a few glitches with the online classroom as we went along, particularly as [the tutor ] seemed well skilled in putting right one or two problems.” We suggest that it is possible to overcome both technical difficulties and improve teachers’ confidence by having skilled and patient tutors.

**Potential for future development**

Treacey et al suggest that it is valuable to connect online PD with face-to-face professional development activities (Treacey et al, 2002). This suggestion is being incorporated into existing professional development provision where we have integrated online tutorials into the established Teaching Further Mathematics (TFM) course and the Teaching Advanced Mathematics (TAM) course run by MEI. The group size for these extended courses varies between 30 and 100 people and as a result the interactive nature of the LOPD courses cannot easily be replicated in the online sessions for these courses. They are often more akin to online lectures allowing only limited interaction, typically using the polling facility and text chat. However, many teachers enrolled on these larger courses are unable to attend all of the live online sessions and the recordings enable them to view the online session afterwards and follow up any ambiguity with tutors. This additional online provision has been well received by participants on these courses and has considerably enhanced the support offered.

The current model for LOPD offers distinct courses at published times. However, following a couple of requests from teachers for training to teach Mechanics 3, a module not commonly offered in schools, we were able to negotiate a flexible programme of online support to meet their specific needs. We believe that this type of course, tailored to the needs of a single or small group of schools, may be an area of increasing demand and development.

We would also like to support existing and new teacher networks by possibly setting up discussion groups to share ideas and facilitate collaborative working. How precisely this could be done has yet to be devised.

**Conclusion**

It is difficult to see how professional development in Further Mathematics could be made widely accessible to teachers across the whole country without incurring great cost in terms of time and travel expense. Providing online professional development in mathematics using a synchronous communication software tool such as Elluminate Live! can be an effective means of enabling teachers
to access specialised training courses at an appropriate time for them and facilitate collaborative work. Despite some technical problems (audio, connectivity, whiteboard design), both teachers and tutors like the medium, particularly for the interaction it offers. Not having to take time out of school and having the convenience of being able to access sessions from school or home is a bonus. We have deemed the LOPD programme a success and this is certainly supported by the simple fact that 99% of teachers who provided feedback said that they would recommend their course to other teachers. Generally we have found that participating teachers agree that the online environment facilitates their development opportunities. One teacher reported, "The concept and use of the virtual classroom is a fantastic way to make valuable resources available to both teachers and students".

References


Kleiman, G., Peterson, K., Treacy, B. (2002). Successful Online Professional Development International Society for Technology in Education.

The Impact on Student Achievement of When CAS Technology is Introduced

David Driver
Brisbane State High School, Australia

When a Computer Algebra System (CAS) is used as a pedagogical and functional tool in class and as a functional tool in exams, its effect on student achievement can be quite profound.

The timing of when students are first introduced to a CAS has an impact on gains in student achievement.

In this action research project, the CAS calculator was introduced to students and used to varying degrees in years 10, 11 & 12. The effects on students’ final year results were examined in terms of the timing of the introduction of the CAS device and the extent of its use in the classroom.

There is some evidence of greater improvement in learning by introducing the calculator in year 10 rather than year 11. This gain is evident for low, average and above average mathematics achievers.

Introduction

When graphical calculators were introduced into Queensland senior secondary mathematics classrooms, many experienced teachers believed student algebraic manipulation skills would be negatively impacted. It was thought that the perceived negative impacts, such as reduced algebraic manipulation skills and the time required to master the technology would outweigh the positive impacts such as the ease of visualising a function graphically and the availability of alternative approaches to teaching and learning. Teachers did not think that graphical calculators would assist student learning any more than direct teaching and practice.

Since the early 1990s, most senior mathematics teachers in Queensland have become familiar (and comfortable) with the graphical calculator. Few would choose to go back to the “good old days” when the graphing of a function relied on the use of algebraic manipulation to find the x- and y-intercepts, when differential calculus was used to find the location and nature of any turning points, and the laws of limits had to be applied to locate any discontinuities or the behaviour of the graph, either near a discontinuity, or at the extremes of the domain.

However, either because the Queensland syllabus has not kept up to date with the capability of the technology or because there is intrinsic value in performing the necessary algebra and calculus, teachers require students to do “on paper” what the calculator can do (at least approximately) in a fraction of the time. Paul White’s claim that “advances in technology and mathematical competence of students studying calculus have brought under fire the traditional place of calculus courses”, has been proved baseless. (White, 1990, p. 105).

These concerns were not unlike those expressed several decades earlier when the scientific calculator largely replaced the need for mathematical tables and the use of logarithms. (Etlinger, L., 1974) However, the author remembers vividly his early studies in statistics when the calculations required to determine the correlation coefficient (including finding the square root of the variance) were performed on a Burroughs machine which could add and subtract (by cranking a handle forward or backward). The exercise (literally) of calculating the correlation did not add anything to my understanding of what a correlation coefficient means.

Conversely, the optimism expressed at the Australian Association of Mathematics Teachers conference Students, Mathematics & Graphics Calculators: Into the New Millennium in 2000, (Morony & Stephens, 2000) has been realised to a considerable degree.

Students in some schools now have access to technology which will perform algebraic manipulation (including differential and integral calculus) to find the exact location of the intercepts, turning points, points of inflection and limits. There is the potential for loss of “by-paper” skills, when students use technology to do algebra and calculus.

In addition to the potential advantages of graphical calculators, an algebraic calculator allows:
• quick comparison of approximate graphical solutions and exact algebraic solutions to problems, and
• development of algebraic sense.

The Research Setting

The study took place at Brisbane State High School (BSHS) - a large selective state (public) school in Queensland, Australia. Students gain entry to the school on one of four criteria: local entry, academic merit, cultural merit, or sporting merit. In each year level of about 400 students, typically: 40% are local entry, 25% qualify for enrolment on academic merit, 25% on sporting merit, and 10% on cultural merit.

Queensland students in their final year of schooling can study Mathematics at four levels: Mathematics B (essentially a functions and calculus course), Mathematics A (general maths), or Prevocational maths (basic maths). Accelerated students who have already completed Mathematics B undertake a university level course. Accelerated students and Mathematics B students may also study Mathematics C (applied maths). In 2010, of the 409 students in year 12, the numbers studying maths at each level were as shown in the table below. The selective nature of the school is also demonstrated by the high proportions of students studying Mathematics B and C as shown in the following tables:

<table>
<thead>
<tr>
<th>Course</th>
<th>Number of Students at BSHS</th>
<th>Percentage of Students at BSHS</th>
<th>Percentage of Students in all Queensland Schools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics C</td>
<td>82</td>
<td>20.0</td>
<td>6.6</td>
</tr>
<tr>
<td>Mathematics B or University Mathematics</td>
<td>218</td>
<td>53.3</td>
<td>27.4</td>
</tr>
<tr>
<td>Mathematics A</td>
<td>138</td>
<td>33.7</td>
<td>37.0</td>
</tr>
<tr>
<td>Prevocational Mathematics</td>
<td>31</td>
<td>7.6</td>
<td>24.2</td>
</tr>
</tbody>
</table>

Table 1: Student subject choices at BSHS and in Queensland

<table>
<thead>
<tr>
<th>Level of Achievement</th>
<th>Brisbane SHS</th>
<th>Queensland</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Percentage</td>
</tr>
<tr>
<td>A</td>
<td>100</td>
<td>37.6</td>
</tr>
<tr>
<td>B</td>
<td>84</td>
<td>31.6</td>
</tr>
<tr>
<td>C</td>
<td>53</td>
<td>19.9</td>
</tr>
<tr>
<td>D</td>
<td>24</td>
<td>9.0</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>1.9</td>
</tr>
<tr>
<td>Total</td>
<td>266</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Student Achievement Levels of Mathematics B Students at BSHS and All Queensland Schools in 2010

The Research Methodology

The research was undertaken as an action research project. Consequently it was not possible to control for any variables other than the focus of the project which, in the initial stages, was the impact of CAS technology on student achievement. When it was decided on the bases of the initial research to extend the use of CAS from an experimental group to an entire cohort, the focus became how the timing of the introduction of the technology impacted the effects of the technology on student achievement.
The research extended over three years and there was consequently some attenuation of the group sizes due to changes in school enrolment and subject selection in particular. To maximise the size of the experimental group, the membership of the control group varied throughout the project.

**Initial Research**

At the beginning of 2008, each of the students in the author’s year 10 Mathematics B class was issued with a Casio *Classpad*™ calculator. This calculator has all of the functionality of a graphical calculator plus a Computer Algebra System (CAS) which enables it to perform symbolic manipulation.

The calculators were used extensively in class during semester one both as a functional tool and as a pedagogical tool as by (Stacey, 2007):

- Functional – to find answers to problems beyond expected by-hand skills
- Pedagogical – to explore and support learning of new mathematical ideas.

Students were permitted to take the calculators home each night and most, but not all, opted to do this. The calculators were not permitted during any assessment tasks throughout this semester. The content of the curriculum and assessment tasks were identical to that followed by the other eight classes studying this subject.

The teaching approach, however, was different in order to take advantage of the calculator’s ability to assist students’ development of mathematical understanding. The calculator was used as a pedagogical tool in a variety of structured investigations of procedures such as:

- pattern searching to inductively develop the laws of indices;
- factorising trinomials; and
- completing the square leading to the development of the quadratic formula.

All nine classes were of similar mixed ability, and in total comprised 241 students of the total cohort of 400 students. (There were 110 students studying mathematics at a less advanced level, and 49 students studying mathematics at a more advanced level.)

At the end of semester one, the learning outcomes of the students in the author’s class were compared on two criteria (Knowledge & Procedures, and Modelling & Problem Solving) with matched pairs from the remaining 8 classes. The matching was on the basis of the students’ Working Mathematically and Thinking Mathematically results at the end of year 9.

A comparison of the results on both Knowledge & Procedures, and Modelling & Problem Solving (output) against Working Mathematically and Thinking Mathematically (input) showed that there were no significant differences between the year 9 and year 10 semester 1 mathematics results across the range of abilities of the students. (Driver, 2008)

<table>
<thead>
<tr>
<th></th>
<th>Knowledge &amp; Procedures (gain/loss)</th>
<th>Modelling &amp; Problem Solving (gain/loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experimental Group</strong></td>
<td>mean -3.05</td>
<td>mean -13.95</td>
</tr>
<tr>
<td></td>
<td>standard deviation 8.56</td>
<td>standard deviation 22.80</td>
</tr>
<tr>
<td><strong>Control Group</strong></td>
<td>mean 0.05</td>
<td>mean -14.81</td>
</tr>
<tr>
<td></td>
<td>standard deviation 6.78</td>
<td>standard deviation 19.55</td>
</tr>
<tr>
<td></td>
<td>t-statistic 1.68</td>
<td>t-statistic 0.23</td>
</tr>
<tr>
<td></td>
<td>probability 0.109</td>
<td>probability 0.822</td>
</tr>
<tr>
<td></td>
<td>correlation coefficient 0.41</td>
<td>correlation coefficient 0.68</td>
</tr>
</tbody>
</table>

Table 3: Comparison of Year 9 and Year 10 Achievement

In summary, the students in both the experimental and control groups, on average, had almost identical marks on Working Mathematically / Knowledge & Procedures between years 9 and 10, but a little over a grade point lower on Thinking Mathematically / Modelling & Problem Solving. The latter
result may be due to a difference in meaning of the two terms or the application of a higher standard in year 10. It is very unlikely that there is a statistically significant difference in gains or losses on either criteria between the experimental and control groups. The feared loss of algebraic manipulation skills is not clearly evident.

Given that the students who had been using a Classpad were not permitted to use their calculators during assessment in year 10, it is possible that any positive impact of the calculator may have been masked.

At the beginning of semester two, classes were rearranged. Students with Classpads™ were spread amongst all nine classes. They were still able to use their Classpads in class, but not for assessment. All students were told that if they continued to study Mathematics B the following year, they would be required to purchase a Classpad. Some other students chose to purchase a Classpad at that time rather than wait until the beginning of year 11.

The subsequent study

The students in the author’s semester 1 year 10 mathematics B class of 2008 were matched with students who did not have a Classpad until either semester 2 of year 10 or semester 1 of year 11. (Although taught by a variety of teachers, students from the semester 1 year 10 class were still considered as the experimental group.) The matching was on the basis of students’ results at the end of semester 1 year 10 on both Knowledge & Procedures, and Modelling & Problem Solving. (The matched students formed the control group.)

The experimental group included only the sixteen students who remained at the school and continued to study Mathematics B in year 11. Most students whose year 10 result was less than a High Achievement (a B grade) studied the lower level (Mathematics A) course in year 11.

Three of the sixteen students in the experimental group were in the author’s year 11 Mathematics B class. The other thirteen students were taught by one of the other six Mathematics B teachers.

Knowledge & Procedures

![Figure 1 Comparison of Year 10 and Year 11 Knowledge & Procedures Results](image-url)
In summary, while the experimental group had very similar scores on both Knowledge & Procedures and Modelling & Problem Solving in years 10 and 11, the control group, on average, was almost a grade point lower on both criteria in year 11 compared to their respective year 10 result. The differences between the experimental group and control group were statistically significant, i.e. they are unlikely to have been a chance phenomenon. These differences were fairly consistent across the range of ability levels (as determined by their year 10 result).

**Preliminary Findings**

Heid & Blume (2008) have suggested that: “The greater facility in solving problems ... may be related to students developing new representation strategies” i.e. students experienced an increased ability and facility in moving between and seeing relationships between numerical, graphic and symbolic representations when they were freed from some of the cognitive load associated with symbolic manipulation, which is the case when students are required to do on-paper procedures.

There are several likely causes of the apparent improved performance (relatively speaking) of the experimental group compared to the control group in year 11 (given comparable achievement in year 10).

1. The use of the Classpad in year 10 may have resulted in an increase in students’ understanding of the mathematics they were learning (in year 10),
2. This stronger foundation may have led to improved outcomes in year 11.
3. The familiarity with the Classpad when they commenced year 11 may have resulted in an increase in students’ understanding of the mathematics they were learning in year 11.
4. Students in the experimental group may have been able to take full advantage of the availability of the Classpad during assessment in year 11.
It may be possible to distinguish between these hypotheses by considering the achievements in year 11 of those students, not in the author’s class, who chose to purchase a Classpad at the beginning of semester 2, and therefore were more familiar with its use when they commenced year 11 than those who delayed their purchase until the beginning of year 11.

During year 12, students were asked to complete a simple survey to gauge their usage of the Classpad in each semester of years 10 & 11 on a four point scale: not at all; a little; somewhat; a lot. The most significant result was the level of reported usage of their Classpad in year 11 if students had used it in either one or both semesters of year 10.

<table>
<thead>
<tr>
<th>Group</th>
<th>usage (year 11 semester 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not at all</td>
</tr>
<tr>
<td>Experimental (all of year 10)</td>
<td>0</td>
</tr>
<tr>
<td>Extended (semester 2 of year 10)</td>
<td>1</td>
</tr>
<tr>
<td>Neither semester of year 10</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5: Effect of Timing of Introduction of Technology on its Usage

A Chi square analysis of this result (and of the levels of usage in semester 2 of year 11) indicates that with 6 degrees of freedom, the χ² values of 29.11 (with a p-value of 0.000058) and 21.79 (with a p-value of 0.0013) indicate a highly significant increase in technology usage with increased familiarity.

The “advantage” gained by students in year 12 from being introduced to the Classpad in year 10 rather than year 11 is still evident, although reduced, particularly for more able students. The students who voluntarily obtained a Classpad in Semester 2 of year 10 also had an advantage in year 12, for the more able students.

Although there were small differences in the performance of students in the three groups in year 9 on knowledge and procedures, and slightly larger differences in problem solving, by the end of year 12 these differences had increased, most noticeably in problem solving.

Using linear regression to predict year 12 results from year 9 results and then fitting trend lines to the residuals indicate that when prior achievement is kept constant, if a student uses their Classpad “a lot” for an additional two semesters, then their year 12 result for Knowledge and Procedures will be raised by 5.2% and for Modelling and Problem Solving by 6.2%.

It could be claimed that the observed results were due to the selective nature of the school. However a comparison of the proportion of students studying each of the alternative courses in mathematics indicates that although there is a significant difference in proportions for each enrolment category, when the local entry students are compared to all other students, the difference is not statistically significant. In other words, the academic, sporting and cultural students combined are similar in mathematical ability to the students who live in the local catchment of the school. So, by extrapolation, in any school, the students from the local area would benefit from the early introduction of CAS technology. The number of students in the experimental group was too small for a comparison of their enrolment categories.
Conclusions

The earlier introduction of the Classpad calculator in the experimental group and its voluntary usage by students in the extended trial group had a significant, positive impact on these students subsequent utilisation of the technology and consequently on their knowledge and procedures and modelling and problem solving results.


Inspired Connections in Maths Lessons – New Pedagogy for New Technology

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University of Aberdeen, Scotland

Does the use of multiple representation technology impact upon the dynamics of classrooms? Do teachers change the way they teach? What is the impact of changes in pedagogy on students’ motivation, interest and ways of working? The findings of the research, published in 2010, are described. I then focus on particularly successful lessons that demonstrate the benefits to be gained by using both new technology and new teaching approaches. Links between different representations of maths concepts and different areas of mathematics are highlighted. The use of the technology, together with a more investigative teaching approach, with more opportunities for questioning and discussion, both between teacher and students and among students themselves, led to a deepening in understanding, an increased pace of learning and a surprising increase in motivation and engagement across all ability levels.

Research Background

This paper considers the use of multiple representations of mathematics concepts, relationships between these representations and the pedagogical approaches which highlight and emphasise links between representations. The underlying mathematical concepts may be represented in different ways and a deep understanding of the concept includes the ability to fully appreciate each representation, its connection to the concept itself and also the links that exist between the representations. Duval (2006) argues that a characteristic feature of mathematical activity is the simultaneous use of at least two representations, or possibly the changing from one representation to another and that ‘standard teaching’ never focuses on this recognition of the mathematical object across representations. He sees this as a possible cause of lack of understanding among students. In contrast, the present study looks at teaching which does indeed focus on the use of at least two representations and which forced students to constantly change from one representation to another. In some lessons, students were using linguistic, algebraic, geometrical, numerical and graphical representations, noting and discussing the inter-relationships between these. For a detailed review of related research literature, see Duncan (2010a).

A group of 12 teachers in 6 schools in Scotland were supplied with TI-nspire™ software and handhelds for use with students for an initial period of one academic session. Schools were chosen to represent a range of types from a range of geographical locations. They are fully comprehensive, range from rural to city and cover a variety of socio-economic backgrounds. The roles range from 500 to 1200. The teachers had a range of years of teaching experience. There was also a considerable range of experience with ICT in general and with mathematics software, especially in its use in classrooms. Each school was supplied with 30 (non-CAS) handhelds and teachers had the computer software. Teachers all had some facility for projecting an image of their work with the software. The research method involved the use of a variety of instruments including questionnaires, interviews, lesson observations and lesson evaluations. Each teacher undertook to write up 6 lesson evaluations relating to lessons specifically designed to make use of multiple representations. A total of 66 detailed lesson evaluations were received (92% response rate).

Research Findings

The full research report was published in the spring of 2010 (Duncan, 2010b). What follows is a summary of the relevant findings.

The teachers involved in the study, no matter what their background, length of experience as a teacher or extent of experience with ICT were convinced that the use of multiple representations of mathematical concepts generally enhances their students’ relational understanding of these concepts and were willing to provide extensive evidence to support their arguments. In response to the question ‘Did the use of multiple representations enhance the students’ relational understanding of the mathematics involved?’ in an overwhelming 80% of the 66 evaluations they indicated Yes.

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As evidence for their conclusions teachers provided a variety of reasons as indicated in Figure 1. (In all tables, items with response rates of less than 5% are ignored.)

<table>
<thead>
<tr>
<th>Comments (N = 105)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evidence detailing specific use of multiple representations</td>
<td>33</td>
</tr>
<tr>
<td>Evidence detailing verbal or written responses from pupils</td>
<td>13</td>
</tr>
<tr>
<td>Evidence of improved discussion</td>
<td>12</td>
</tr>
<tr>
<td>Evidence of ‘aha’ moments – ‘seeing’ pupils’ understanding</td>
<td>12</td>
</tr>
<tr>
<td>Evidence of improved retention</td>
<td>10</td>
</tr>
<tr>
<td>Evidence detailing increased motivation, engagement/encouragement</td>
<td>7</td>
</tr>
<tr>
<td>Evidence to support a ‘NO’ or inconclusive response</td>
<td>11</td>
</tr>
</tbody>
</table>

Figure 1. Categorisation of teachers’ comments on multiple representations and relational understanding

Teachers were asked if they were conscious of changing the way they teach the mathematics topic featured in the lesson being evaluated. Again an overwhelming 79% responded positively. Examples are provided below in the section describing teachers’ lessons. Teachers’ comments were again categorised as indicated in Figure 2.

<table>
<thead>
<tr>
<th>Comments (N = 86)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Details of changing the way they teach the topic</td>
<td>42</td>
</tr>
<tr>
<td>Evidence of more active involvement from pupils</td>
<td>20</td>
</tr>
<tr>
<td>Evidence of links across maths topics</td>
<td>10</td>
</tr>
<tr>
<td>More opportunity for more open questioning and discussion</td>
<td>6</td>
</tr>
<tr>
<td>Teaching topics earlier than normal</td>
<td>5</td>
</tr>
<tr>
<td>Using TI-Nspire to support my normal teaching methods</td>
<td>16</td>
</tr>
</tbody>
</table>

Figure 2. Categorisation of teachers’ comments on ways of teaching the topic

The teachers were also asked if they were conscious of changing the way they teach in general and if yes, what are these changes and how are they justified? They only had to answer this question in their final evaluation so the total number of comments was significantly less than for other questions. The comments were again categorised as in Figure 3 below. Taking Figures 2 and 3 together, it appears that by being encouraged to think about possible multiple representations of the mathematics involved and by having to use the technology to assist with this, teachers were more inclined to produce a different way of teaching specific topics. Not only did they appear to change the way the mathematics was introduced and developed but also their more general classroom pedagogy was being altered. The move to more investigative teaching approaches consequently led to less teacher exposition and direction from the front of the class, more active involvement and engagement by the students, more discussion between the teacher and the students and also more debate among the students themselves. Interestingly, teachers also admitted to thinking more carefully about how to deepen their students’ mathematical understanding of the topics being taught and also about how best to emphasise connections both between different mathematics topics and among their various representations.
Comments (N = 46)  

<table>
<thead>
<tr>
<th>Comment</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allows students more freedom to investigate possibilities</td>
<td>28</td>
</tr>
<tr>
<td>Conscious of changing classroom dynamics</td>
<td>13</td>
</tr>
<tr>
<td>Allowing/encouraging more discussion with and amongst students</td>
<td>13</td>
</tr>
<tr>
<td>Consciously making an effort to link topics together</td>
<td>11</td>
</tr>
<tr>
<td>Consciously thinking about how best to utilise the facilities of the technology</td>
<td>11</td>
</tr>
<tr>
<td>Consciously aiming to improve/deepen students’ understanding</td>
<td>9</td>
</tr>
</tbody>
</table>

Figure 3. Categorisation of teachers’ comments on ways of teaching in general

In response to another question which focused specifically on the students’ motivation and engagement when using the technology, more than half the teachers’ comments described a positive impact on motivation and almost a sixth described a positive contribution to both the pace of the lessons and the amount of learning taking place.

**Exemplars of Teachers’ Lessons**

During the period of the research I had the opportunity to observe a number of lessons being taught using the TI-nspire technology. A number of these are described here but for a fuller treatment see the section on Lesson Observations in Duncan (2010b). A booklet of case studies of innovative teaching ideas based on the observed lessons has also been published (Duncan, 2010c).

**Lesson 1 – Connecting mathematical language, algebra, geometry and graphs**

In this lesson students, aged 13, were asked to convert sentences such as “add two to n then square the answer” into algebraic expressions, in this case \((n + 2)^2\). The teacher created a TI-Nspire file (tns file) that pictured such expressions as dynamic geometry diagrams, with a split screen which also showed a dot moving along the graph of the correct expression. The correct graph is hidden. Students were asked to drag the point P on the geometry diagram and watch the graphed dot moving. They then had to decide on an appropriate expression and key it into the function entry line. If they used the correct expression, the dot did indeed move along the graph which was then visible. If not, then they had to rethink the entry line and the algebraic expression.

In the example below, the student initially tried \(y = 3x^2\) then later corrected this to \(y = (3x)^2\).

The teacher was very pleased with the students’ work and felt that her learning intentions were met. Students also described connections that the teacher hadn’t intended to make explicit. For example they noticed that if the variable n only appeared on one side of the rectangle then the graph was a straight line but if it appeared on both sides then the graph was a curve.
Lesson 2 – The MaxBox Problem

In this lesson, the teacher used a pre-constructed tns file that consisted of a single page containing three dynamically linked items; the card from which the squares are cut, the resulting box and a graph showing the volume of the box. By dragging a point to vary the value of h, the card can be seen changing, the box changes accordingly and also the volume, indicated by a point on the hidden graph. The screenshots below attempt to put a dynamic (animated) situation onto a fixed image.

The teacher found that the animation of the dynamically linked images was visually very powerful and helped his students to ‘see’ the connections. They dragged the open point to vary h and watched as the box changed shape noting that there was a maximum turning point on the hidden graph. They then analysed the geometry and deduced an algebraic expression for the volume which they then entered in order to see the graph. They produced the graph of the derived function and noted the maximum value for the volume, which was then confirmed using the calculator application as shown below.

Lesson 3 – Area of a Circle

Prior to using the TI-Nspire technology, this teacher had introduced this topic by simply stating the area formula and asking students to find the area for circles with a variety of radii or diameters. The contrast with his new approach is considerable. He created a tns file using a geometry page to create a circle with measures of radius, diameter, circumference and area. He used data capture by dragging the circle and transferring the data to a spreadsheet page. In a previous lesson he got the class to use linear regression to find the relationship between the circumference and the diameter and they found that \( C = \pi D \). In the observed lesson, they looked at the graph of Area v radius and found that it was a curve. The teacher asked them to create a new column in the spreadsheet for radius\(^2\) and they used
linear regression to find the formula $A = \pi r^2$. The interesting aspect of this lesson was that the students were aged 12 and had not yet been introduced to the quadratic function. Notwithstanding this, several of them worked independently of the teacher and went ahead to investigate the various items on the regression menu applying them to the curved graph. They then informed their teacher that both quadratic and power regression may be used and that power was better as it didn’t involve the use of standard form!

![Figure 7. Finding the formula for the area of a circle using regression](image)

**Lesson 4 – Investigating quadratics**

This lesson started with the traditional ‘sheep pen’ problem of maximising area with given perimeter then moved on to a general investigation of graphs of quadratic functions. The ‘standard teaching’ of this subject matter usually involves a purely algebraic exercise of multiplying out brackets, then the reverse process of factorising followed later on by the graphs and quadratic equations. In contrast, this lesson involved all of these processes and emphasised the connections. The most fascinating part of the lesson occurred when the students taught their teacher something he didn’t already know. They were trying to find the axis of symmetry of a graph but had not yet and probably never would be introduced to completing the square (because of their lower ability categorisation!) They explained very clearly to him that if you take an example like $x^2 - 4x + 9$, you can ignore the constant because it only moves the graph up or down and doesn’t affect the line of symmetry. You then factorise $x^2 - 4x$ to get $x(x - 4)$ which crosses the x-axis at 0 and 4 so the line of symmetry is $x = 2$. The students had discovered this quite independently of their teacher and were both proud and pleased with their finding, so much so that they went on to search the internet to find out if anyone else knew about it!

The teacher was greatly impressed by the students’ interest, enthusiasm and engagement. He concluded that they had an improved appreciation of the interconnectedness of maths topics rather than the compartmentalised view that he suspects many end up with.
References


Exploring Impulse and Momentum using handheld technology

Ian Galloway
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Students at secondary school rarely encounter motion graphs beyond displacement or velocity against time. Yet in order to understand Newton’s third law it is useful to be able to graph force against time. This is relatively straightforward using any datalogging system. Most students, and teachers, have a limited understanding of the third law which if not rectified causes problems for the further study of dynamics. Force, and consequently impulse, is an abstract quantity which deserves more attention within the curriculum. As J W Warren writes, “it [force] has long been regarded as a simple concept...and insufficient consideration has been given to ensuring that it is taught correctly.” Using modern digital technologies it is now possible to address the problem in a new way. This is practitioner material supported by anecdotal evidence, food for research.

Previous Research

As recently as this year Rosengrant, 2011, writes that “instructors have limited choices when they want to help their students understand impulse and momentum. One of the only available options is the impulse–momentum bar chart.” Ross, 2008, states that “Children think of forces in terms of movement, not staying still.” And further states that “No matter how many times we refer to Newton’s Second Law, pupils will insist that moving objects have a force driving them and they stop when this ‘runs out’.”

I would assert that little has changed since the earlier literature review of McDermott and Reddish (1999) who cite seventeen pieces of research focusing on student understanding of energy and momentum including Lawson and McDermott (1987) who note that most students are unable to relate algebraic symbolism to motions which they observe, while Boyle and Maloney (1991) find that even after instruction only half of a cohort of one hundred university students were able to apply Newton’s third law correctly. Many problems are linked to naive beliefs (Caramazza, McCloskey and Green, 1981) while others (McDermott, Rosenquist and van Zee, 1987) are related to graphical issues. Such widespread views of problems associated with this area of physics indicates that perhaps a different approach might help students to better understand the concepts of force, impulse and momentum. Lebouttet-Barrell (1976) in Concepts of Mechanics among Young People concludes with insufficient consideration has been given to ensuring that it is taught correctly.”

Far more information is needed concerning the growth of such concepts as force, momentum, work, speed, energy and the like in pupils and students at each age level. We need to know how pupils and students integrate networks of new concepts and structure them. It would be indispensable to explore the area of preconceptions and it would be of extreme importance to make an inventory of the entire range of erroneous concepts elaborated subsequently by pupils in the course of learning and which distort the construction of coherent systems of knowledge. p.465.

The Ontology of the Concept of Momentum

Hooke had already carried out investigations into springs by the time Newton was putting together the Principia. Using the relation $F=kx$ and the fact that the energy stored in the stretched spring is the area under the graph of $F$ against $x$ Newton may well have considered the link between force and momentum as follows.

A graph of $p$ or $mv$ ($p$, the momentum of a moving body, mass $m$ and velocity $v$) against $v$ shows similarity to the graph of $F$ against $x$ in that the area underneath has a similar form and is the same quantity, namely energy, one being elastic energy ($\frac{1}{2}kx^2$) and the other kinetic energy ($\frac{1}{2}mv^2$). Newton ascribed the term vis vitae, force of life, to the quantity $p$ and may have forever associated in people’s minds a connection between motion and force. Force misconceptions cited include that of believing that force produces motion rather than that force results in a change in motion.

Warren (1984) quotes Newton’s second law in the “correct” form as $F=\Delta p/t$ that is to say that force results in a change of momentum in a certain time interval. This leads immediately to the idea that impulse or $Ft$ is simply the change in momentum or $\Delta p$. 

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It would be fair to say that most students encounter the second law in the form \( F=ma \) which is frequently interpreted as meaning that force results in motion. It would also be fair to say that most students encounter dynamics through drawing graphs of displacement and velocity against time. It is just possible that this approach invariably results in the sorts of misconceptions that students hold.

**Graphing Force against time, some anecdotal evidence**

In delivering physics courses to in-service school teachers for whom physics is not their specialist subject I have noted that not one out of all who attended (approximately 50) had encountered the second law in the form described above and so not one had considered that an implied force for a given time results in a change of momentum. Such were the misconceptions held that one teacher insisted that speed was merely another way of measuring force. By and large all of the teachers involved felt that their understanding of the concept of force, using the additional concepts of impulse and momentum, had improved. Indeed, an Institute of Physics network coordinator who encountered the material in a different course declared in a personal communication that “she did not expect to learn anything new at this late stage of her career but now wanted to explore this new approach”.

Using a force sensor and a datalogger it is relatively easy to obtain graphs of force against time for particular motions. The force sensor described here is a force plate, similar to a bathroom balance. I initially question teachers about the force actually being exerted on the plate in order to tease out the idea that it is the push of their feet and not their weight which acts upon the plate. It is astonishing how badly understood is the idea of weight. Recently I put a key stage three question (for fourteen year old children) on weight to a group of fifteen teachers, seven of whom declared themselves to be physicists and all of whom declared themselves to be perfectly confident teaching physics. Only one was able to answer the question correctly, the others having all made the mistake of believing that the weight of an object acted on something other than the object itself!

The force plate is put in an elevator with somebody stood on it and force data are recorded as the elevator travels between floors. This is most easily accomplished using handheld technology. Simply asking for a description of what is happening in figure 1 below usually elicits the response that the elevator has travelled up and then back down. In fact the elevator has only travelled upwards in this case. Students are then asked to estimate the number of floors travelled. A surprisingly large amount of information is revealed in the graph. Assuming an acceleration due to gravity of 10 m/s² one can deduce the mass of the person on the balance. The problem can then be analysed using acceleration or momentum. Assuming 3 metres per floor gives about ten floors in this case. This question alone is sufficient to keep a class occupied for a full lesson.

![Figure 1: force against time for an ascending elevator](image)
The force plate is now used to analyse the motion during a simple vertical jump. Figure 2 shows force against time during the motion and several questions can be asked. Exactly when does the body leave the plate? At what points is the body in equilibrium? What is the height reached during the jump? How does this compare to the elevator data? Most teachers are confused for while because the relation is between force and time not displacement and time. As was done for the elevator one can calculate the impulse given to the body or as this has been captured on a handheld one can draw polygons around the appropriate and compare their areas, figure 3. It can be seen that the impulse delivered to the body is the same as the impulse delivered on returning to the plate. Remembering that the velocity has been reversed shows that there has been no net change in momentum which is of course what one would expect!

Conclusion

While there is nothing to prevent data being collected in other ways, using handheld technology has made the whole task very easy. It has helped to be able to tackle the problem in an investigative way and so enable students to explore for themselves the outcomes of different approaches. For example exactly where should the polygons be drawn in figure 3? Would it help to do the jump a bit at a time and so analyse each part? Data can be viewed quickly and repeated if necessary.

Anecdotally, viewing motion data through looking at the force which causes the change in the motion helps students to understand balanced and unbalanced situations better. Analysis of the motion can be carried out without recourse to acceleration and so help to reinforce the idea that a net force produces a change in the momentum and is not the cause of motion itself. Students at Bacon’s College are convinced that this is a worthwhile approach.

Given the lack of research in this area there is ample opportunity for work to be undertaken with regards to the pedagogy of force impulse and momentum and the use of new technologies for data logging. Little has changed since Lebouttet-Barrell, 1976.

References


Embedding computer-aided assessment of mathematics and statistics for first year economics students

Martin Greenhow and Kinga Zaczek
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This paper seeks to share our experiences from the use of computer-aided assessment (CAA) in a formative/summative mode with first-year economics students in their mathematics module. Five years’ worth of results is presented that demonstrates a positive impact on students’ perception of their learning and on the actuality of their learning as measured by their examination performances. Students (correctly) view the CAA as a learning resource in its own right and spend most of their time studying the very complete feedback screens. Repeating assessments and group work is allowed, even encouraged, since each question realisation uses random parameters that are carried through to all aspects of the question (stem, key, mal-rule based distracters, MathML equations and SVG diagrams). The transferrable nature of this methodology is demonstrated by presenting a new project aimed at statistics for social sciences students.

Introduction

It is a common perception amongst university lecturers that often students’ learning is significantly driven by assessment – put bluntly, students often simply want to accrue marks. Whilst this is not what lecturers want, it is evident that in the short term at least, we should use this driver constructively. When coupled with traditional delivery and assessment methods, we contend that CAA offers an excellent and cost-effective way of allowing students repeated practice, thereby increasing their fluency and understanding. Moreover, we believe good quality material can capitalise on this engagement to get some, but not all, of the important techniques and concepts across to students. This paper presents evidence to support this view.

Consequently, since 2000 the CAA team at Brunel University has been developing a database of question styles called Mathletics, written using an extended form of the CAA package Question Mark Perception version 3.4 (2011). (Note our extensions are currently incompatible with later versions 4 and 5). Each question style is a program that uses random parameters within the Javascript, MathML equation and SVG diagram coding, thus producing from underlying algorithms thousands or millions of realisations. Randomised numbers, words, names and scenarios are carried through to all aspects of the question, especially the very full feedback screens.

An overview of the database coverage and question format and design is given by Greenhow (2008). It is worth noting that although some more advanced topics have been covered, the question design process, where algorithms for visible or hidden distracters are based on mal-rules, lends itself very well to GCSE and A-level topics. These topics are widely taught, not only at school, but in service mathematics modules at university, for example to first-year economics students as discussed here. Many of the questions are in no context or in an everyday context, and hence are widely applicable, whilst others are dressed up in a subject context; see Figures 1 and 2 respectively (which also demonstrate user-determined screen appearances). Given the database of some 2000 question styles at this level, it is a simple matter to produce assessments that cover the intersection of typical mathematics or quantitative methods modules (often as much as 80% of the content); currently Mathletics users at Brunel University comprise students from Economics, Electrical and Electronic Engineering, Financial Computing, Foundations of Engineering, Foundations of IT (FoIT) and Sports Science. The CAA often applies to several modules, for example economics students take 12 assessments in mathematics and 3 in research methods, whilst FoIT students take 2-5 assessments in each of algebra, calculus, discrete maths, programming, statistics and study skills (where an innovative English assessment has been trialled this year using the same question design randomisation methodologies).

All of this activity can be categorised as low-stakes summative assessment, given that students’ best-ever mark from their first 5 attempts counts towards their module mark (typically 20%-30% for the 5-12 assessments required). However, as we will see, students rightly use the assessments as a learning resource, i.e. in formative mode, and spend most of their time studying the feedback screens.
The system has proved fairly robust and easy to manage even with hundreds of users, although network problems can be frustrating. Less of a problem has been students detecting errors within the questions, feedback or marking; perversely a few errors seems to benefit the learning process since students challenge the correctness of the system. Usually they are wrong, but the tutor then discovers the precise nature of the students’ incorrect thinking and can correct it.

![Figure 1](image1.png)

**Figure 1** A decontextualised question with all numbers and variable names (here v and F) randomised.

![Figure 2](image2.png)

**Figure 2** Part of the feedback for an economics question with an accurate SVG diagram realised according to the random parameters in the question. The related material button links to web resources.
Case study: EC1005 Mathematics for Economics – a chronological study

Figure 3 gives hard data on the effect of introducing CAA into this module and, given the teaching (lectures and tutorials) and other assessments (problem sheets and written exam) were identical, are considered to be relatively free from confounding influences. It should be read in conjunction with the following:

2006/7 & 2007/8 In 2006-07 the Department of Mathematical Sciences was asked by our Economics Department to share the teaching of their first year mathematics and statistics/research methods modules (EC1005 & EC1006). Since Greenhow was engaged in writing the CAA material for the FDTL5 Metal Project (2007), from where the assessments, and much else, can be downloaded, he welcomed the chance to teach the group without A-level mathematics (typically about half the cohort of about 340 students aged 18-20 years, 55% male) since he anticipated using the CAA to replace the two class tests.

2008/9 In fact this did not happen until 2008-09 (for the both cohorts); in addition to saving Greenhow about 8 solid days marking, the exam and overall performance of the students then increased substantially, see Figure 3. During 2006/7 to 2008/9, the performance of the two groups (with/without A level) was indistinguishable at the end of the year, and it was felt that the weaker students benefitted relatively more from the CAA, see below.

2009/10 & 2010/11 In 2009-10, Greenhow took over the entire cohort and the admissions policy changed to require at least AS level mathematics. In fact, about half of the cohort had the full A-level at grade B or above, and the qualifications for 2010-11 comprise grades A*(3%), A(12%), B(50%), C(21%), D(4%) with the rest having AS grades A(3%), B(4%) and C(3%). It is gratifying that very few students who took AS level mathematics did not continue to the full A-level. Given the enhanced mathematical preparedness of the students in 2009-10, the low exam average was somewhat disappointing. It can be explained by the fact that Greenhow set the exam for the first time and overestimated the speed with which students could do the questions (corrected in 2010/11). So although the overall average slightly increased, some 70 students had to resit the examination in September 2010 since a genuine pass in the exam component was introduced as a module requirement (in addition to the exam being weighted at 70%). Of the 2009/10 resitters about 50 passed. Results for 2010/11 are encouraging across all grades. The almost complete absence of D grades appears to be an artefact of the exam pass requirement: students that just pass the exam will generally have better CAA marks and hence raise their overall grade to a C, whereas students who fail the exam, even narrowly, simply fail overall, regardless of their CAA mark. Resit results for 2010-2011 are not yet available.

![Figure 3](image_url)  
*Figure 3 Year-on-year overall grades (% of students), and exam and overall averages (% marks).*
We have also looked at two other possible sources of data, questionnaires and answer files. As part of a wider study Zaczek designed, administered and analysed a short questionnaire in 2010/11 lectures with 147 returns out of a possible 280. The object was to understand how students used the CAA and to gauge their perception of their learning from it. Some clear trends can be seen: 79% of students use the CAA for both assessment and as learning material (21% for assessment only) and 94% use all 5 allowed attempts to get the best possible mark. Timelines of student marks show that they often use their first one or two attempts inputting wrong answers simply to read the feedback from which they learn. This is confirmed by the constrained optimisation assessment which lacked detailed feedback screens. This triggered several emails along the lines of How are we supposed to learn this without feedback? The average of the best mark for 5 allowed attempts for this assessment was only 68% compared with over 90% for the other 11 assessments (with the exception of eigenproblems that came right at the end of semester 2 which many students failed to complete satisfactorily by the deadline). Clearly such students do not learn as much as we imagine during lectures or subsequently from lecture notes or books – yet these are not weak students lacking in study skills generally. Rather this seems to point to a preferred method of learning being engagement with the CAA, and if widely true, this has consequences for CAA designers who should spend most time and attention on the feedback.

Most students stated that they would use the assessments, even if not compulsory: as much as now (28%), once or twice (35%), only for the feedback (26%) and not at all (11%). About 90% of students aim to achieve a mark of at least 80% which may indicate some pride in their progression and a sense of achievement rather than simply marks accrual. Moreover, 84% plan to use the CAA as part of their revision even though marks will not then count. This contrasts markedly with their engagement with problem sheets and tutorials. Perhaps tutorials are an expensive waste of time (8 per week were given to groups of nominally 40 with only about 20 turning up regularly) and should be replaced by staffed CAA sessions. Students seem to value the immediacy of the feedback and the chance to work with others.

Curiously, the questionnaire results indicate that substantially more students help others than are helped by others, which shows that their perceptions are not always accurate! The questionnaire results fail to distinguish clearly between their enjoyment of mathematics and that of CAA per se, both being rather positive, maths/CAA results being: Hate it (1/0%), Don’t enjoy (5/4%), Neutral (36/56%), Enjoy (45/29%), Love it (14/11%). This might imply that when doing the CAA they regard themselves as doing mathematics itself and see no distinction despite a difference in medium (paper/CAA). Crosstabs show that the more they enjoy mathematics (or CAA) the more they use the feedback as a learning resource; a reasonable inference is that good quality feedback enhances their enjoyment although we cannot yet claim a causal link. Another interesting crosstabs show that, as suspected, weaker students benefitted relatively more from the CAA: 19/79 ‘good’ students (categorised as having AS grade A or A level grade C or above) use the CAA only for marks (perhaps they do not need the feedback) in contrast to weak students (not in the above category) where this ratio is 6/39. Finally it is worth noting that the mathematics module is perceived as being of comparable difficulty to their other modules (stats and economics modules) maths being: very much easier (4%), easier (33%), about the same (31%), harder (29%), very much harder (3%).

The other data available is from the circa 15000 answer files produced annually. This vast data set certainly requires closer study to discern not just what students get wrong but how exactly they get questions wrong i.e. what mal-rules do they employ. In the long term this might then inform the teaching processes used. In the short term, it is worth noting that multi-choice questions (MCQ) are usually easier than numerical input questions (NI), partly because guessing is possible but also because the form of the options effectively scaffolds the students’ thinking and makes them think again if they end up with an option that is not shown (although none of these can be the correct response). So MCQs can be helpful in building students’ confidence in mathematics in semester 1 before using NIs (and other question types such as True/False, Responsive Numerical Input where mal-rules act behind the scenes to trigger specific feedback if student input implies one has been used, Hot Line, Sequential and Revealed Multi-Choice) later to test real mastery of the content. However this is not the whole story; some multi-operation questions, such as that of Figure 1 are difficult regardless of how asked – in fact rearranging equations was the hardest topic of all. Another
important statistic is discrimination: the most discriminating question was \( \text{differentiate } (1+x^3)^3 \) which only stronger students could do.

Future directions

Given the success of Metal generally, the team (Nottingham Trent, Portsmouth, Brunel and Bristol universities) recently secured JISC funding to develop similar material for statistics for social sciences. The CAA component of this DeStress project (no reference available yet) builds on the Metal question design methodology and experiences with using it. The aim is to release new material by August 2011 but already some assessments spanning the A-level S1 and some of S2 A-level syllabi have been trialled with our FoIT students. Results have been encouraging and student feedback has suggested edits, mostly in the question wording. One immediate problem is that hand calculation with realistic data sets where various statistical concepts have validity is unfeasible. A solution (not yet trialled) is to link with external software such as Excel, see Figure 4. Consequently, the focus will be less on manipulative skills but more about meaning of statistics. This approach has major advantages in testing mathematical concepts since questions can be reverse-engineered to definitely include or preclude data with explicit features, for example with outliers, bimodal, skewed etc. The other possibility is to use real data. Whilst this is superficially appealing from the economics point of view, there are problems with keeping the data and answers current without accessing live web sites (which may pose problems of access and communication with the marking scheme). Another challenging area is interpretation of charts and graphs where data still needs to exhibit certain characteristics after randomisation has taken place.

For both mathematics and statistics, ideally one wants to analyse student mistakes easily using the thousands of answer files. This then implies an outcome metadata must be recorded that captures not the student’s actual answer, but rather the mal-rule(s) they may have used. Such mal-rules then need their own taxonomy of errors, so that one may consider whole sets of questions and act on the information gleaned by modifying the questions, providing new ones and identifying supporting web resources. Although various error taxonomies exist, none are sufficiently detailed to encapsulate the required information; we plan this as an area of future research. In the meantime, the entire database is being translated into a web application for student formative use and for teachers to use to build their own assessments without the need for any software beyond a browser.

Conclusions

Javascript, MathML and SVG provide a rich environment for setting objective questions within mathematics and statistics, and even beyond. To make the most of this, careful question design is required that encodes algorithms for the correct answer and for mal-rule based distracters, with associated metadata. Thus thousands or even millions of realisations are generated, opening up the possibility for repeated attempts at assessments, possibly as group work. Being targeted at the school/university interface level, the database of questions is widely applicable to many modules and across many departments, our largest user group being the economics students that form this case study.
We have discussed an effective way of embedding this resource and shown its positive effects on students’ exam performances and on their perception and enjoyment of their learning. Central to this success seems to be that high-quality CAA must focus on trapping students’ errors (detecting mal-rules) and using this to provide specific, very detailed and immediate feedback to capitalise on the students’ engagement. We conclude that CAA is a practical way of doing this, and moreover one that utilises the natural driver of marks accrual in a constructive way to enhance students’ mathematical fluency and understanding.

References


Acronyms


JISC: Joint Information Services Committee: http://www.jisc.ac.uk/ retrieved July 4, 2011.


A jump forwards with mathematics and physics

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We jump on human body motions such as bouncing on a jumping stick, hopping, and making kangaroo jumps. Students can record the movements with a digital camera and use their video clips to investigate the motions with suitable video analysis and modelling software. We discuss some mathematical models of these motions using basic biomechanical principles and we compare modelling results with experimental data obtained from video measurements. Highlight is the application of the model of a planar inverted spring-mass system: this rather simple model works qualitatively and quantitatively well for the complex motions of hopping, skipping and running at moderate speeds. The examples of video analysis and modelling activities give a good impression of the potential of the subject of human gait for student practical investigations and as a context for applied mathematics and physics at secondary and undergraduate level.

Introduction

Human gait can take many forms such as sauntering, walking, hopping, skipping, jogging, running, sprinting, and so on. In this paper we construct mathematical models of the following bouncing gaits: bouncing on a jumping stick, hopping, and making kangaroo jumps. This seems very ambitious because such vivid motions are at first sight not easily modelled. The pushing-off and landing of a vertically hopping person savour strongly of the motion of an extending and compressing inverted spring-mass system. All sorts of models of this type are used in biomechanical studies. But how simple or complex must such a mathematical model be to describe reality to a reasonable extent? Can students at secondary and undergraduate level with modest knowledge of mathematics and physics actually do such investigations? In an attempt to answer these questions we investigate bouncing gaits and corresponding inverted spring-mass models of increasing complexity.

Vertical bouncing on a jumping stick

In a 2008 nationwide secondary physics examination in the Netherlands, Thomas was put on the scene with his pneumatic jumping stick (See the left part of Figure 1). The main components of this type of jumping stick are: (i) foot and hand supports; (2) an air-filled cylinder; (3) a piston that can slide within the cylinder and forms the bottom of an air chamber; and (4) an elongate shaft coupled to the bottom of the piston and moveable therewith. When Thomas repeatedly jumps in the vertical direction, we can distinguish two phases: (1) the aerial phase, in which the jumping stick is off ground, maximally extended, and assumingly moving as a rigid body under gravity; and (2) the contact phase, in which one end of the jumping stick remains in contact with a fixed point on the ground. At landing, the shaft and piston are forced into the cylinder, the stick length is shortened and consequently the volume of the air chamber decreases and the pressure therein increases. After shortening of the jumping stick, the shaft springs back and the jumping stick elongates again, just like a compressed spring. It turns out that Thomas could comfortably jump when the airborne and contact phase took almost equal time. The motion of Thomas on his jumping stick serves as source of inspiration for describing some natural gaits of humans and animals with an inverted spring-mass model.

This rather clear situation of a periodic motion of a person on a jumping stick can be described well with a model based on simple mathematics and physics. The quality of the chosen model can be evaluated by comparing the model results with data acquired through video analysis of the motion. A schematic drawing of the one-dimensional spring-mass model is shown in the right part of Figure 1. In this model we ignore the mass of the spring and we divide the period of one jump into two phases, viz., the aerial phase (with aerial time $t_a$) and the contact phase (with contact time $t_c$). We assume that Thomas is able to vertically jump on his stick without changing his posture and that his body centre is near the hand supports. At landing, the jumping stick has rest length $L$. We further assume that during aerial phase only the gravitational force $F_g = -mg$, where $m$ is the body mass and $g$ is the acceleration of gravity, plays a role and that during contact phase also the spring force $F_s = C(L – y)$

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must be taken into account, i.e., for heights $y \leq L$, the linear elastic motion of the spring depends on a spring stiffness $C$ (for heights $y > L$ we may take $C = 0$).

Figure 1. Thomas on his jumping stick and the corresponding spring-mass model.

The dynamics of the spring-mass system is now determined by a second order differential equation and two initial conditions:

$$a = \ddot{y} = \left( F_x + F_s \right) / m = F / m, \quad \text{with} \quad y(0) = y_0 \quad \text{and} \quad y'(0) = v_0.$$

This can be rewritten as a system of first-order differential equations:

$$\dot{v} = v' = a = \left( F_x + F_s \right) / m,$$

$$\dot{y} = y' = v = v_0.$$

We have done this because the rewritten equations can then be easily implemented in a system-dynamics based modelling tool. We use the modelling tool of the Coach learning environment (Heck et al., 2009). Figure 2 is a screen shot of a graphical model that implements the spring-mass model expressed by the above equations. In the graphical model, each combination of a rectangle and an inflow double arrow represents the integration of a quantity. In the example to the right, the arrow represents the derivative of the variable $y$ and this quantity is integrated in time during a simulation run. The quality of the mathematical model is determined by comparing the model results with data coming from real experiments. The diagram in the middle of Figure 2 shows the graph of the computed height and the point plot of the vertical heights measured in a digital video recorded while Thomas was jumping on his stick. The measured data suggest that a sinusoidal regression curve would describe the data quite well, and indeed it does from mathematical point of view, but the spring-mass model is considered better because it is based on physics laws. Then a sinusoidal displacement during contact phase is followed by a parabolic aerial phase. This judgment of the quality of a model is what students should learn.

Figure 2. A graphical model implementing the one-dimensional spring-mass model and the results of a simulation run compared with data obtained from a video analysis of a movie clip.

**Hopping upward**

One may truly wonder whether the previous one-dimensional spring-mass model is a good model for human hopping in the upward direction without the use of a device. The proof of the pudding is in the eating. So, for the purpose of data collection, we went with students to the University Sports Centre of the UvA to let them hop in vertical and forward direction on a motorised treadmill. We recorded motions with a high speed camera at a speed of 300 frames per second so that we could observe as many details as needed and work with a high time resolution. We present the results in this section.

In order to get more insight in the motion of the body centre during contact phase of upward hopping we determine the exact solution of the one-dimensional spring-mass model. The stance leg (in this case actually both legs) is modelled as a massless, linear spring with stiffness $C$ and rest length $L$.

Here we imitate models of Blickhan (1989), McMahon & Cheng (1990), and many other.

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biomechanical scientists. For ease of computing we assume time $t = 0$ when the leg makes first contact with the treadmill (in a video measurement we can easily calibrate time in this way) and we assume that landing speed is equal to $-v$. The vertical position of the body centre has been chosen to be the same as the position of the hip joint of the hopping person (See Figure 3). Under the assumption that only gravitational force and spring force play a role, Newton’s second law of motion and Hooke’s law of elasticity lead to the following equation of motion for the height $y$ during contact phase: $my'' = -mg + C(L - y)$, $y(0) = L, y'(0) = -v$. Let $u = y - L$ be the displacement during ground contact. Then the equation of motion can be rewritten as follows:

$$u'' + \omega^2 u = -g, \quad u(0) = 0, \quad u'(0) = -v,$$

where the natural spring frequency $\omega$ is given by $\omega^2 = C/m$. This equation can be solved analytically and the solution is the following sinusoid:

$$u(t) = -\frac{v}{\omega} \sin(\omega t) + \frac{g}{\omega^2} \cos(\omega t) - \frac{g}{\omega^2}.$$

This leads to the following formula for the speed of the body centre: $v(t) = -v \cos(\omega t) - \frac{g}{\omega} \sin(\omega t)$. Halfway contact time the stance leg is maximally bent and the speed of the body centre is equal to zero. This gives the relation:

$$-v \cos\left(\frac{1}{2} \omega t_c\right) - \frac{g}{\omega} \sin\left(\frac{1}{2} \omega t_c\right) = 0.$$

Thus, contact time is related to speed and frequency as follows: $t_c = \frac{2\pi}{\omega} - \frac{2}{\omega} \arctan \left(\frac{g \cdot v}{\omega}ight)$. What we learn from the last formula is that the motion during contact phase depends on three out of the four factors: (1) the acceleration of gravity $g$; (2) the natural frequency $\omega$ of the spring-mass system; (3) the take-off and landing speed $v$; and (4) contact time $t_c$. Because of the definition of the natural frequency one can exchange this factor by the spring constant $C$ (stiffness), provided that the body weight $m$ is known. To conclude, by using exact mathematical methods one can make grounded statements about a bodily motion and investigate the dependencies of determining factors.

![Figure 3. Video analysis of an upward hopping girl on a motorised treadmill that is not turned on.](image)

The exact solution of displacement can also be used to estimate the natural frequency and the landing speed on the basis of measurements. Figure 3 is a screen shot of a video analysis using Coach. The sine curve matching best the data points can be obtained with the data analysis tools. The sinusoidal regression curve $y(t) = 0.12 \sin(21.25t + 2.95) + 0.88$ can be rewritten as

$$u(t) = -0.118 \sin(21.25t) - 0.023 \cos(21.25t) - 0.02.$$

From this formula follow the initial estimates $\omega = 21.25\text{Hz}$, $v = 2.5\text{m/s}$. This landing speed is only a little bit greater than the speed of $2.3\text{m/s}$ obtained by numerical differentiation of the measured data. The estimated values give a contact time of 0.17s, which is also only a little bit less than the measured contact time of 0.19s in the video clip (For this precision one needs a high speed camera). The take-off speed can be used to compute the duration of the aerial phase, under the assumption that the aerial motion depends only on gravity: $t_a = 2v/g$. This gives $t_a = 0.47s$ and the estimated flight time deviates little from the measured flight time of 0.40s.

In short, the one-dimensional spring-mass model applied to a person hopping upward using no special device leads to model results that are in good agreement with results obtained from video analysis measurement on recorded movie clips. The agreement between model results and measured data get even better when we do not find out a sinusoidal regression curve for the measured data,
but instead try to find the best values for parameters in the spring-mass model by the method of trial and improvement. For the hopping girl in Figure 3 we found a very good match between model and video analysis, which holds for nine consecutive hops, using \( C = 19 \text{kN/m}, \nu = 1.95 \text{m/s} \). This value of the stiffness \( C \) is in good agreement with values found in the literature (Farley et al., 1991). From these values we obtain the following results: \( \omega = 18.9 \text{Hz}, t_c = 0.19 \text{s}, t_e = 0.40 \text{s} \). Whereas aerial and contact time were almost equal for hopping on a jumping stick, they differ for human hopping without a device.

### Hopping forward like a kangaroo

What should set the seal on our work is the application of a planar inverted spring-mass model to human double legged forward hopping, i.e., mimicking kangaroo jumping. The model is in this case two-dimensional. So to start with, the one-dimensional spring-mass model of upward hopping is extended. The new model contains besides the kinematical variables \((y, v_y, a_y, F_y)\) also the variables \((x, v_x, a_x, F_x)\), as it were ‘doubled’. Quantities like speed and force are decomposed in the \(x\)- and \(y\)-direction. The planar inverted spring-mass model for bouncing gaits such as hopping and running is schematised in Figure 4.

![Figure 4. Planar inverted spring-mass model for forward hopping and running.](image)

In comparison with the one-dimensional spring-mass model of upward hopping, we have now two new conditions: the leg angle of attack \( \alpha \), when the leg makes ground contact, and the angle of take-off velocity \( \beta \), when the leg looses ground contact. These angles are most easily defined when we select the stance point as the origin of the coordinate system during contact phase, with the positive \(x\)-axis in the direction of motion and the positive \(y\)-axis in the upward direction, and when we assume that the stance leg lands at time \( t = 0 \) : \[ \tan \alpha = \frac{-y(0)}{x(0)} \] and \( \tan \beta = \frac{v}{u} \), where \( u \) is the horizontal landing speed (equal to the speed of the motorised treadmill when the gait is on such device) and \( -v \) is the landing and take-off speed. The leg angle of attack and the angle of take-off velocity are not necessarily equal. It is not difficult to determine these angles in a recorded video clip because Coach, like any professional video analysis tool, provides its user a digital ruler, a digital protractor, and graphs of position and velocity. Notice that one cannot freely change the parameters \( \alpha \) and \( \beta \) in a computer model for given values of leg length \( L = \sqrt{x(0)^2 + y(0)^2} \) and landing velocity \( v(0) = \sqrt{u^2 + v^2} \) if the model must be periodic. After all the leg angle of attack and the leg angle of take-off must be equal for a periodic motion. Under the given circumstances the following condition can be used to distinguish between aerial and contact phase: when \( v = L \sin \alpha \), there is ground contact and the leg can be considered as a linear spring with stiffness \( C \).

Let us now derive the equations of motion for bouncing gaits. The spring force \( F \) during contact phase is according to Hooke’s law of elasticity given by \( F = C(L - r) \), where \( r = \sqrt{x^2 + y^2} \) is the length of the spring and \( C \) is the stiffness of the spring. This spring force must be decomposed into horizontal and vertical components in order to derive the equations of motion: \( F_{x} = F \cos \phi, F_{y} = F \sin \phi \). Thus: \( F_{x} = F_{x} x/r, F_{y} = F_{y} y/r \). After moderate algebraic manipulation we obtain the following initial value problem from Newton’s second law of motion:

\[
\begin{align*}
x' &= -\omega^2 x + \frac{g\lambda x}{\sqrt{x^2 + y^2}}, \\
y' &= -g - \omega^2 y + \frac{g\lambda y}{\sqrt{x^2 + y^2}}, \\
x(0) &= -L \cos \alpha, \quad x'(0) = u, \quad y(0) = L \sin \alpha,
\end{align*}
\]
\(y'(0) = -v\). Here we have introduced the parameters \(\omega_x^2 = C/m\), \(\omega_y^2 = g/L\), and \(\lambda = \omega_x^2/\omega_y^2\). The main difference between vertical and forward hopping is that the leg length, parameterised by \(\lambda\), comes seriously into play in the modelling of the body motion during ground contact. The main application of the analytical methods like the one discussed in this section is that it allows investigating the influence of gait parameters on the body motion and researching dependencies between the various parameters in the mathematical model.

We are almost ready for constructing a computer model of periodic forward hopping like a kangaroo. But first we must realise that the body centre follows under the given assumptions a parabolic curve during aerial phase that starts in \((L \cos \alpha, L \sin \alpha)\) with a horizontal speed \(u\) and a vertical take-off speed \(v\). After all, the initial value problem for the motion during aerial phase is:

\[
x'' = 0, \quad x'(t_0) = u, \quad x(t_0) = L \cos \alpha, \quad y'' = -g, \quad y'(t_0) = v, \quad y(t_0) = L \sin \alpha.
\]

These equations of motion of the spring-mass model of forward hopping can easily be implemented in the graphical modelling tool of Coach. Only a solution for moving the coordinate system from one stance point to the other must be found. The fact that Coach is designed as a hybrid system that combines a classical system dynamics approach with event-based modelling for processes that change abruptly helps solve the implementation problem of a moving frame. As triggering condition for the landing of a hopping person we can use the Boolean expression \(y \leq y_{\text{num}}\) and \(y' \leq 0\) and \(x + x_{\text{num}} > 0\), where the initial conditions \(x_{\text{num}}\) and \(y_{\text{num}}\) are given by \(x_{\text{num}} = -L \cos \alpha\) and \(y_{\text{num}} = L \sin \alpha\). The event is defined behind the upper-left icon in the below graphical model. The idea behind the event handling is that one starts with a coordinate system of which the origin coincides with the first stance point. In the variable \(x_{\text{num}}\) we store the current value of the \(x\)-coordinate of the origin of the moving coordinate frame. Each time when the event of touch down occurs, the variable \(x_{\text{num}}\) is refreshed with the \(x\)-coordinate of the new stance point via the assignment \(x_{\text{num}} = x + L \cos \alpha\).

In order to evaluate the suitability of the spring-mass model for human hopping like a kangaroo we compare model results with experimental results obtained by motion analysis. To this end we recorded the motion of a hopping girl on a motorised treadmill going at a speed of 3 km/h on video and constructed the height-time graph via automated point tracking of a hip joint marker. We use this measurement as a background graph in the modelling activity to find by trial and improvement appropriate parameter values. It is quite tricky to set the values such that the spring-mass model runs periodically for a long time: the leg angle of attack must match with other parameters in order that the take-off velocity equals the landing velocity. But the reward is great: Figure 5 illustrates a perfect match. The upper-right \(y-x\) diagram illustrates the periodicity of the simulated motion. The computer model can be used to study common forms of energy such as the kinetic energy \(E_{\text{kin}} = \frac{1}{2}mv^2\), the gravitational energy \(E_g = mgv\) and the spring potential energy \(E_s = \frac{1}{2}C(L - r)^2\). The sum \(E_{\text{tot}}\) of these

\[E_{\text{tot}} = E_{\text{kin}} + E_g + E_s\]
three energies is constant, as shown in Figure 5. The parameter values found for the hopping girl weighing 53 kg were: \( C = 28 \text{ kN/m}, \ u = 0.84 \text{ m/s}, \ \nu = 1.95 \text{ m/s}, \ L = 0.91 \text{ m}, \ \alpha = 86.0003 \). Other quantities can then be computed: \( \omega_s = 23.0 \text{ Hz}, \ \omega_p = 10.8 \text{ Hz}, \ t_c = 0.40 \text{ s}, \ t_r = 0.2 \text{ s} \). What the results indicate is that leg stiffness and spring frequency \( \omega \) are greater in hopping forward like a kangaroo than in hopping upward is greater in this case. Yet this had little or no effect on aerial and contact time. All this is not so very strange considering the low speed of the treadmill. Apparently, only leg stiffness must be increased to maintain a short contact phase.

**Conclusion**

Finally, let us return to the two questions raised in the introduction. How simple or complex must such a mathematical model be to describe reality to a reasonable extent? It is striking that a relative simple spring-mass model describes bouncing gait patterns so well. Bullimore & Burn (2007) confirmed that the model allows prediction of several gait characteristics such as contact time, vertical momentum, and stride length. But they also noticed that it often overestimates the horizontal ground reaction force, the flight time and the change of mechanical energy of the body centre. Geyer (2005) successfully adapted the spring-mass model to walking and running gait patterns. The actual power of the mathematical models is that they help researcher investigate various aspects of bouncing gaits, such as step frequencies, forces, stability of gait patterns, costs of energy, etc., and compare gait patterns.

The second question was whether students at secondary and undergraduate level with modest knowledge of mathematics and physics can actually do such investigations. Our experience is that students with a keen interest and good ability in mathematics and physics can master the modelling process. Less gifted students are still expected to be able to grasp the one-dimensional inverted spring-mass model, which was after all used in a nationwide examination. More importantly, all students can do with modest means biomechanical research in much the same way as ‘real’ scientists and they can practise herein mathematical knowledge and skills such as graph comprehension, numerical differentiation and integration, data processing and analysis, regression, etc. They can develop the critical attitude that is necessary for successful modelling of natural phenomena. For this it is very important that the students can compare the results of computer models with real data, preferably collected in an earlier measurement activity. Confrontation of a model with reality turns graphical modelling not only into a fun way of learning, but it also makes it exciting, challenging, and concrete work. It is joyful when experiment, model, and theory are in good agreement, as in this paper.

**References**


Exploring the giant circle on the high bar with ICT tools

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Daan Knobbe and Nic Nijdam were secondary school students with artistic gymnastics as a hobby. They jointly investigated the mechanics of the gymnastics swing around the high bar. They not only did this to meet the curriculum requirement of a carrying out a large research project, but also to satisfy their curiosity regarding the sports science subject. With a high-speed camera they recorded the motion of a backward giant circle on the high bar. Hereafter they analysed their data both quantitatively and qualitatively. We present the results of the students’ research work about the backward giant circles on the high bar, which resembled the practice of sports scientists. We discuss the authenticity of this project, the role of ICT, and potential further exploration at student level. In our opinion, the presented work is a nice illustration of authentic experiences of secondary school students in doing sports science that they will never forget.

Introduction

In the Dutch secondary education system, students must carry out at the end of their school career a research or design project with a study load of 80 hours to demonstrate their ability to apply acquired competencies while pursuing a research question or design goal in some depth. Students are encouraged to choose the topic themselves and they are to some extent free in setting up their work. Normally they work in small teams over a long period parallel to the regular lessons and they deliver a report and a presentation about their results. Ideally, the students do not only see it as a compulsory subject but also enjoy the stimulating aspects of doing their own research or design. Challenging and authentic projects, which are representative for actual research and design work done by practitioners, and projects on subjects to which students can personally relate seem effective in this respect. This is at least the experience of the first author in earlier work with student research projects in the context of human movement (e.g., Heck & Ellermeijer, 2009; Heck & Holleman, 2003; Heck & Van Dongen, 2008). In these projects, the focus was on providing students with opportunities to experience how science is enacted, i.e., with authentic inquiry, and in particular on providing students with ICT tools that allow them to act as ‘real’ scientists (Heck, 2009).

In this paper we report on a project of two secondary school students who investigated the backward giant circle on the high bar. We present the results of their research work, which resembled the practice of sports scientists, and we discuss the authenticity of this project and the role of ICT. This means that we go into questions like “What did the students actually do in their project and what could they have done more or better?”, “How did ICT contribute to the realization and the success of the sports science project of the secondary school students?” and “To what extent did the students’ work resemble scientific practice?”.

The backward giant circle on a high bar

![Figure 1. Graphics sequence of the traditional technique of a backward giant circle, in which the gymnast circles backwards (anti-clockwise direction) from a handstand on the high bar through 360° (after Hiley & Yealdon, 2001) and (Tsukiyama, Murata & Fukunaga, 2004).]
The high bar is one of the six pieces of apparatus used in Men’s Artistic Gymnastics competitions. A high bar routine consists of a number of circling skills, flight elements, and a dismount. The backward giant circle is used to link the circling techniques and to provide the required flight and rotation for flight elements and the dismount. It is an artistic gymnastics element in which the gymnast passes through a handstand position above the bar and fully rotates around it, without releasing the bar at any time, without bending arms and knees, and with only smooth changes in hip and shoulder joint angles that have little influence on the aesthetic execution (cf., www.fedintgym.com). This at least is the description of a traditional backward giant circle in which the gymnast extends his body close to the highest point of the circle, maintains it in the downward phase and flexes the shoulder and hip joints during the early upward phase (See Figure 1).

The students teamed up to investigate the mechanics of the backward giant circle on the high bar, triggered by their own hobby and interests. They were given printed copies of relevant literature (e.g., Hiley & Yeadon, 2001) to put them on the way. Their physics teacher and gym teacher advised them during their research. The students formulated the following main purpose of their study: “We investigate the influence of the shoulder and hip joint angles on the angular velocity in a backward giant circle on the high bar and we investigate how a gymnast can optimize these and other factors in his performance in order to achieve the highest possible angular velocity and still perform well in the eyes of the gymnastics jury.”

**Video analysis of the backward giant circle on the high bar**

In this section we present the results of the students’ research work about the backward giant circle on a high bar. We discuss in depth their experimental design, data analysis, and use of ICT. In order to get a better impression of the students’ performance and to assess their results, we have also redone the video analysis of one giant circle in a more advanced way.

**Experimental design**

For the purpose of exploring the influence of the shoulder and hip joint angles on the angular velocity in a backward giant circle with both quantitative and qualitative methods, the student researchers designed an experiment in which they could collect position, angle, velocity and time data for a backward giant circle on a high bar through video measurement and in which they could use video tools for analysing video clips of various types of swing motions. In the previous sentence we deliberately wrote, “designed an experiment”, because a lot of thinking and preparation went into the set-up of the experiment.

First of all, the students arranged the location of their experiment, namely, the practice room of their gymnastics club where the high bar apparatus could be used on a quiet moment during daytime. A performance of several subsequent giant circles was required for a good analysis of a backward giant circle in which a gymnast just swings about the bar without the goal of increasing or decreasing the angular velocity at the highest point above the bar. In order to be able to make several full swings after another, the students used a training tool for the apparatus that reduced the friction when the gymnast circled about the bar, made it easier to do a full swing, and simplified the control over experimental conditions.

The next thing to watch in the experimental set-up was the type and positioning of the camera(s) with which the movements were recorded. The student researchers used a point-and-shoot high-speed camera operating at a frame rate of 120 fps to get enough data for a quantitative video analysis and to get at the same time a good resolution quality of the video clip. Ideally, the camera for recording the giant circle would have been positioned perpendicular to the plane of motion, at a distance and height that reduce perspective distortions, and with the camera oriented such that the x- and y-axis are aligned horizontally and vertically, respectively. In reality, these experimental conditions are often difficult to arrange. The student researchers did their best and luckily the video analysis tool of the Coach environment (Heck, Kędzierska & Ellermeijer, 2009) provides a tool to correct perspective distortion afterwards.

Before recording body movements, a researcher must at least have an idea of what (s)he will do with the video clips. The questions that the student researchers asked themselves already give a clue of
the type of body segments model of a gymnast performing a backward giant circle that they had in mind. The rigid 3-segments model is shown in Figure 2 (adapted from the students’ report). To make it easier to measure positions of wrist, elbow, shoulder, hip, knee and ankle, the students attached markers to the gymnast’s skin and shorts over the right body side joints. The markers over elbow and knee joint were attached only to verify whether the arms and legs were kept straight during the giant circle, but were not used in the video measurement and in the biomechanical analysis of the motion. Body orientation was defined as the angle between the vertical and a line from the bar to the body centre of gravity. The body orientation angle, shoulder angle, and hip angle were estimated on the basis of displacement data using the convention illustrated in Figure 2.

The students used the video tool of the Coach software environment (www.cma-science.nl) to measure the distance of the body centre of gravity and to compute its velocity during the giant circle. They used a fixed position for the body centre of gravity, slightly above the hip joint as shown in Figure 2. To calibrate distance, they used the known gymnast’s height and the height of the high bar measured from the bar at rest to the upper face of the landing mat.

The student determined the joint angle–rotation angle graphs in the following way. They utilized the open source video analysis software Kinovea (www.kinovea.org) to collect data about shoulder and hip joint angles for certain predetermined body rotation angles per giant circle. The students decided to do measurements only at body rotation angles 0°, 45°, 90°, 135°, 180°, 225°, 270° and 315° for the 6th up to and including the 10th giant circle in a sequence of 14 consecutive giant swings on the high bar. The picture on the right-hand side of Figure 1 gives an impression of a subject’s body configuration at these rotation angles.

Using an Excel spreadsheet, the students averaged the measurements for the 5 consecutive giant circles in order to deal with small changes between giant circles. In this way, they obtained a data plot of 8 points, in which the shoulder and hip joint angle were plotted against the body rotation angle. In order to get line graphs, they utilized the demo version of the Igor scientific graphing and data analysis program (www.wavemetrics.com) for curve fitting. They used a sinusoidal regression type. Figure 3 shows the graphs obtained by the students.

The students also used the measured shoulder joint angles at the given eight rotation angles per giant circle and the known body segment dimensions to compute the distance of the hip joint to bar at the particular rotation angles. This was a nice application of the cosine rule learned at school to biomechanical problems in a real context.

The students qualitatively analysed their graphs in much the same way as we present in the next subsection. In addition they analysed whether a gymnast can return to handstand when his shoulder joint angle gets small during the movement. They concluded that only a free hip swing to handstand on the high bar is possible, but this is not considered a giant circle.

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The motion analysis of the student researchers continued with reflection about other factors that influence the performance of a backward giant circle on the high bar but are not under less control of the individual gymnast. Using qualitative arguments, based on fundamental biomechanical principles about angular motion such as moment of inertia, angular velocity, and angular impulse, and based on formulas for these quantities in simple cases, the students came to the conclusion that gymnasts can swing faster about the high bar when they are less heavy and have a shorter distance between their body centre of gravity and the bar.

The behaviour of the student researchers in their video analysis of the backward giant circle on the high bar resembled the attitude of scientists in that they tried to explain the effects of various factors on the gymnast’s performance by scientific reasoning and also by quantitative analysis, if possible. Their data collection, processing and analysis were based on the same methods that sports scientists use in practice and references to research literature were made. The students also reflected on their research design and the obtained results, looked for alternative methods and explanations of phenomena, and made suggestions for further investigations in their detailed research report. This is what professionals do in practice, too. In the choice of ICT, the student researchers had the same attitude as most scientists: they just used whatever tools were available to them and seemed useful for their research aims.

**Video analysis based on a computed body centre of gravity**

In reality, the body centre of gravity is not always located at the same body point: due to changes in body configuration, the location changes and may even be outside the body (as shown in Figure 4). In a more advanced video analysis one would compute the location of the body centre of gravity during the giant circle from the recorded position of the markers and from anthropometrical data for human body segments. It is more work, but the procedure is standard (e.g., Robertson et al., 2004) and the benefit is an improved quantitative video analysis of the motion. We used the same five-segment model of the gymnast (two upper and low extremities, plus the head and torso as one component) as Townend (1993) and utilized anthropometrical data as presented in Robertson et al., 2004) to predict the location of the body centre of gravity during the giant circle.

![Figure 4. Screen shot of a Coach video analysis activity about one backward giant circle.](image)

Figure 4 is a screen shot of the video analysis of one giant circle (actually the 6th in the uninterrupted sequence of 14 giant circles performed by the gymnast) based on the described methodology. The upper-left window in the above screen shot show a still from the recorded video clip in which the gymnast has bent his shoulder and hip joints in the upswing phase of the giant circle. The points overlaying the video clip are the measured positions of the shoulder, hip and ankle joints and the computed position of the body centre of gravity with respect to the moving reference frame that has the right hand of the gymnast as its origin. Since the video clip in which spatial and time data are collected and mathematical representations like graphs and tables are synchronized in Coach,
pointing at a graph automatically shows the corresponding video frame, and selecting a particular frame highlights the corresponding points in diagrams, when scanning mode is on. This makes scrubbing, i.e., manually advancing or reversing a clip, an effective means to identify and mark interesting events in the video clip and to relate them with graphical features. This feature is much used in motion analysis. In the diagrams of Figure 4 are shown sinusoidal regression curves for the data, the formulas of which have been used in computer modelling of the motion to be discussed in the next section. Below we describe some diagrams in detail.

Going in clockwise direction through the above screen shot, the first diagram consists of the distance–time graph of the body centre of gravity and a curve obtained by approximating the data with a sum of sine functions. In this particular case, a sum of two sinusoids (plus a constant) gives a remarkably good approximation. The distance–time graph of the body centre of gravity can be understood in the following way. Initially, when the gymnast is in a handstand above the bar, his body is not fully extended and the distance of the body centre of gravity is not maximal. During the first phase of the downswing, this quantity actually increases to a peak value when the body is close to horizontal orientation. Hereafter the distance of the body centre of gravity to the bar decreases to a (local) minimum when the gymnast passes under the bar. His hip joint is maximally hyper-extended at this time. In the first phase of the upswing the gymnast closes his hip joint angle. First his body configuration becomes more extended, with the effect that the distance of the body centre of gravity to the bar increases to a local maximum when his body is close to full extension. But as the upswing phase continues, the gymnast’s shoulder and hip joint angles close more, with the effect that the distance of the body centre of gravity to the bar decreases to a minimum when the gymnast is already in the second phase of the upswing and is going to pass the bar soon in handstand. The minimum distance of the body centre of gravity coincides more or less with the minimum of the shoulder joint angle. The gymnast in the last part of the upswing extends the shoulder joint again so that the distance of the body centre of gravity to the bar increases.

The rotation angle–time graph and the angular velocity–time graph are displayed together in the second diagram, in which the left vertical axis is used for the measured rotation angle and the right vertical axis is used for the angular velocity. The time profile of the angular velocity, which was determined via an advanced numerical method for computing smooth derivatives, can be understood in the following way. Angular velocity increases in the downswing as the gymnast falls from the handstand position above the bar and reaches a maximum value shortly before the gymnast passes below the bar when the hip joint is maximally hyper-extended. The angular phase velocity decreases when the gymnast starts moving to a more extended body configuration and progresses to the upswing. If the gymnast were a rigid body swinging about the bar, then his angular would decrease monotonously in the upswing. The angular velocity–time graph reveals that this assumption is not true: the gymnast opens and closes his shoulder and hip joint angles. The changes in body configuration during the upswing result in a second small peak in the graph due to closing of the shoulder and hip joint angles. The local maximum coincides with maximal flexion at the hip joint.

The hip joint angle–time graph and the shoulder joint angle–time graph show clearly when the gymnast flexes and extends his body, and they reveal that the gymnast did not perform optimally: Apparently the gymnast did not deliver the required hip joint torque to avoid opening of the hip angle in the downswing and his shoulder joint was hyper-extended at the beginning of the upswing, which is also considered a weakness in the gymnast’s performance.

The relationships between shoulder joint angle, hip joint angle and distance of the body centre of gravity, and certainly the special body configurations during the backward giant circle become more visible when these quantities are plotted against the rotation angle. This has been done in the lower-left diagram of Figure 4. These graphs are in good agreement with the graphs found by the students (Figure 3) and graphs found in the research literature (e.g., Tsuchiya et al, 2004; Sevrez et al., 2009).

**Modelling and simulating the giant circle**

By use of basic principles of physics and simple models of the gymnast-apparatus system one can already analyse the backward giant circle. For example, Townend (1993) was able to estimate the magnitude of the reaction force experienced by the arms and shoulders of the gymnast via a simple body model: he found a reaction force of about four times the gymnast’s body weight. It is clear that...
a gymnast has to be strong, as does the apparatus. Taking away simplifying assumptions, such as rigidity of the gymnast, no friction and inelasticity of the bar, complicates the dynamics of the motion enormously and makes it difficult to construct a computer program that simulates the gymnast’s motion so that there is good agreement between empirical data and modelling results. Such advanced models would be beyond the abilities of secondary school students (and probably also beyond the level of physics undergraduates). But students could explore simpler models under supervision of a knowledgeable physics teacher.

In this paper we consider a model of the gymnast as a one-segment body consisting of a slim, uniform, circular cylinder of variable length \( r \) and mass \( m \) attached to a rigid axis of rotation and subject only to gravity. Note that this model is merely a mathematical model, which has little resemblance with reality, but nevertheless serves the purpose of analysing the swing motion and may be a source of inspiration for studying more complicated models. The equation of motion is written as 

\[
(I\omega) = mg r \sin\theta, \text{ where } \theta \text{ is the angle of rotation, } \omega \text{ is the angular velocity, and } I \text{ is the moment of inertia given by } I = \frac{1}{2}mr^2.
\]

Using the rules of differentiation and after further algebraic manipulation, we get the following system of differential equations 

\[
\dot{\theta} = \omega, \quad \dot{\omega} = \frac{g \sin\theta - \frac{1}{2}r\omega}{\frac{1}{2}r}.
\]

When input for initial values and data for \( r \) are available, or when \( r \) is modelled by a given mathematical function, this system of differential equations can be solved numerically. Considering the movement as a periodic motion, the function \( r \) can be described as a sum of two sinusoids (this already gives a reasonable approximation). We have done this with the graphical system dynamics-based modelling tool of the Coach learning environment (Heck et al., 2009) and compared the model results with the data obtained from a video analysis of the recorded motion of the backward giant swing performed by one of the student researchers. The results are shown in Figure 5. Background graphs came from the video analysis described in the previous section.

Figure 5. A graphical model implementing the system of differential equations driven by the sinusoidal model of the distance \( r \). The diagrams on the right-hand side show the calculated graphs of the rotation angle and angular velocity, respectively, together with graphs of experimental data.

The phase of the backward giant circle where the model results actually differ from the empirical data is the upswinging phase, when the gymnast changes his configuration by strongly bending his shoulder and hip joints. Obviously our model of the gymnast as a circular cylinder fails under these circumstances. But overall, we consider the quality of this rather simple model as remarkably good.

Once a computer model has been accepted as useful one can use it to see what happens when the gymnast changes the timing of his movements. In this way, one can search for a more optimal performance of a swing technique and learn about possible improvements in techniques. In more advanced computer models, researchers use multiple-segment models of the human body, include elasticity of the bar and friction, and they use joint angle histories or models of applied torques as driving quantities for computer simulations. The interested reader is referred to the biomechanics research literature (e.g., Arampatzis & Brüggeman, 1998; Yeadon & Hiley, 2000; Sevrez et al., 2009).
Conclusion

The educational issue in the presented students’ inquiry work is the ICT-supported interaction between the world of experiments, in which empirical data are obtained, and the world of theories, in which ideas are scientifically developed and further explored. The acquisition and analysis of empirical data is based on methods from mathematics, science and technology. The fact that students must apply their mathematical and scientific knowledge and their experiential knowledge in a meaningful way in a concrete context leads at the same time to consolidation and deepening of this knowledge. The example in this paper shows that affordable technology (both hardware and software) can bring students into contact with the field of biomechanics. In particular, ICT can contribute to the realisation of authentic inquiry and can raise its level by allowing students to (1) gather information about the subject and be in contact with experts (at least in the form of literature and by supervision of knowledgeable teachers); (2) collect real-time data of good quality; (3) process, analyse and visualise data; to do computations that are otherwise impossible; (4) work in much the same way as practitioners do; and (5) report results in a professional way. Besides, students can develop, practise and demonstrate research abilities in such inquiry activities. We consider the student-driven experimental design, the underlying thinking processes, the effective use of ICT, and the improvement of students’ mathematical and scientific literacy as more important in the students’ work than the obtained results. All the same, it is joyful when experiments, theory, and experiential knowledge are in agreement, as was the case in the presented research project of understanding the biomechanics of the backward giant circle on the high bar.

References


Adopting the gamed-based learning software Racing Academy in engineering education: results from the University of Portsmouth and the University of Bath

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¹ University of Portsmouth, UK, ² University of Bath, UK

Following the implementation of the game-based learning software Racing Academy at the University of Bath, the software was adopted for the teaching of year-one Mechanical Engineering students at the University of Portsmouth. ‘Racing Academy’ employs principles of engineering dynamics to simulate and display, in real time, a car drag race in which students’ modify their car from a menu of components. The aim was to complete a drag race in the minimum time, display the time histories of velocity and acceleration and interpret the results from an engineering perspective. The display of velocity and acceleration in real time was intended to make intuitive connections between physical observations and operations of integration and differentiation. Pre and post questionnaires at Bath and Portsmouth measured students’ experience of gaming and motivation towards Racing Academy and studying engineering and a comparison between these two cohorts is presented in this paper.

1. Introduction

Engineering dynamics, taught as a common core of first year Mechanical Engineering degrees in the UK, has been perceived by many students as a demanding subject. The conventional lab sessions for this unit only address a narrow range of topics which students found ‘unexciting’ and ‘disconnected’ to the lecture content. Recent changes to the technical support available within departments mean that changing these conventional lab exercises or adding lab sessions maybe unrealistic.

Requirements for engineering courses are outlined in the UK SPEC (Engineering Council UK, 2010). To satisfy curriculum alignment defined by QAA (2006), elements of the current curriculum for engineering dynamics may require an update.

Recent case studies based on an engineering dynamics module that used computer games to support students’ learning highlighted the effectiveness of visualizing physical phenomenon using computer graphics and a social interface of learning (Joiner et al., 2007). At the University of Bath, the investigators employed a gaming software ‘Racing Academy’, which used principles of engineering dynamics to simulate and display on a computer, in real time, a car drag race. Students could then modify their car from a set menu of components (Darling et al., 2008).

Working closely with the authors from Bath, Racing Academy was introduced into the Engineering Dynamics syllabus at Portsmouth, using a work sheet to lead the students through the games-based learning process and push them into thinking about the science, rather than just the game playing. An online forum answered queries about running the game as well as theories behind the game. The exercise was assessed through a technical lab report.

This paper evaluates and compares the success of game based learning at Portsmouth and Bath University with particular reference to the primary aims of:

i. supporting students’ learning in engineering dynamics

ii. enhancing motivation of the subject.

2. Racing Academy

The detailed learning objectives and specifications of Racing Academy are outlined by Darling et al. (2008) and Joiner et al. (2011).

The learning objectives of the Racing Academy exercise were:

• to understand the fundamental principles of dynamics

• to develop judgement in system design and modelling

• to use computer gaming software in an engineering context

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to work independently and as a team in a competitive environment.

Racing Academy uses a powerful physics engine to embody the principles of engineering dynamics to simulate and display in real time a drag race (Figure 1). In the race, students were able to change vehicle parameters by selecting i) engines, ii) tyres and iii) gearbox ratios from a set menu. In this way students could optimise their vehicle performance and get an intuitive understanding of the system dynamics that influence behaviour.

![Opponent](image)

**Figure 1  Racing Academy Screen During Race**

In order to illustrate the engineering dynamics taking place, the user can produce a number of different graphical outputs, including acceleration against time and speed against time. This way the user is encouraged to understand the engineering dynamics and use an iterative design process to optimise their vehicle.

### 3. Implementing Racing Academy into the Syllabus

This section specifies how Racing Academy was used at Portsmouth. The format was similar to that described by Joiner et al. (2011) from Bath so as to compare data from the two institutions.

#### 3.1 Participants

At Portsmouth 118 students answered the pre questionnaire of computer gaming (110 males and 8 females, mean age 18.5 years, and standard deviation SD = 0.9), while 82 students (76 males and 6 females) answered the post questionnaire. The students were from the first year undergraduate course of Mechanical Engineering in the Department of Mechanical and Design Engineering at the University of Portsmouth.

At Bath, 158 students (143 males and 15 females) participated in the study, with an average age of 18.5 years (SD = 0.9). They were from a first year undergraduate course in the Department of Mechanical Engineering at the University of Bath.

#### 3.2 Procedure

While the structure of the Racing Academy exercise undertaken at the two institutions was similar, there were variations due to logistics and resources available. Both institutions used the same pre and post questionnaires (see Section 3.3), the same pre and post tests, and the same lab sheet to help students to complete a lab report. The programmes at the two institutions, in general, started with an introductory lecture, a pre questionnaire, then followed by different forms of tutorial sessions, and finally after submission of the lab report a post questionnaire.

#### 3.3 Materials

The students answered a pre questionnaire before playing the game, and a post questionnaire after playing the game. The two questionnaires were the same, measuring experience with digital games and motivation towards RA and studying engineering. The post questionnaire included questions regarding how much Racing Academy was used, and how motivating the students found playing RA.
Details of questionnaires and analysis were provided by Joiner et al. (2011).

The pre questionnaire measured students’ experience of playing digital games. In both the pre and post questionnaires, three different aspects of motivation towards engineering were assessed. These were enjoyment of engineering, perceived competence in engineering and how important engineering was to them personally. Students answered the questions using a five point Likert scale ranging from 1 strongly disagree to 5 extremely agree.

In the post questionnaire we assessed how motivating the students found playing RA by measuring how much they enjoyed playing RA, how good they were at playing RA, how much effort they put into playing RA, and how valuable playing RA was.

4. Results at Portsmouth and Bath

4.1 Gaming behaviours and preferences

4.1.1 Age started playing

<table>
<thead>
<tr>
<th>Age group</th>
<th>Number of students Port</th>
<th>Number of students Bath</th>
<th>Percentage of students Port</th>
<th>Percentage of students Bath</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;5</td>
<td>7</td>
<td>1</td>
<td>6.1%</td>
<td>0.6%</td>
</tr>
<tr>
<td>5-7</td>
<td>34</td>
<td>6</td>
<td>29.8%</td>
<td>3.8%</td>
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<tr>
<td>8-10</td>
<td>43</td>
<td>43</td>
<td>37.7%</td>
<td>27.2%</td>
</tr>
<tr>
<td>11-13</td>
<td>14</td>
<td>58</td>
<td>12.3%</td>
<td>36.7%</td>
</tr>
<tr>
<td>14-16</td>
<td>9</td>
<td>40</td>
<td>7.9%</td>
<td>25.3%</td>
</tr>
<tr>
<td>&lt;16</td>
<td>4</td>
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</tr>
<tr>
<td>Null</td>
<td>3</td>
<td>3</td>
<td>2.6%</td>
<td>1.9%</td>
</tr>
</tbody>
</table>

Table 1 Age students started playing video games

Students were asked when they first began to play video games in order to measure their playing history. Approximately 36% of students started playing video games under the age of 7 and less than 4% of students had not started playing videogames until after they were 16. The majority of students from Portsmouth started between the ages of 5 and 10, with the most common age group being between 8 and 10. Students from Bath appeared to start gaming at an older age, the most common category being between 11-13. Table 1 shows a breakdown of the different age ranges.

4.1.2 Frequency and breadth measures

Most students play games once to several times a week, but only play racing games less than once a week if at all. Table 2 shows a complete breakdown of how often students play video games in general. There are slight differences between the Bath and Portsmouth cohorts although these may be attributed to the surveys being undertaken three years apart. For instance, Portsmouth students who completed the survey most recently were more likely to play games on their mobile phones, using a technology that has only recently become widely available.

<table>
<thead>
<tr>
<th>Number of students</th>
<th>Video games</th>
<th>Racing games</th>
<th>Play on computer</th>
<th>Play on mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port Bath</td>
<td>Port Bath</td>
<td>Port Bath</td>
<td>Port Bath</td>
</tr>
<tr>
<td>Never</td>
<td>11 0</td>
<td>32 1</td>
<td>31 0</td>
<td>35 0</td>
</tr>
<tr>
<td>Less than once a week</td>
<td>22 28</td>
<td>55 64</td>
<td>37 131</td>
<td>35 131</td>
</tr>
<tr>
<td>Once a week</td>
<td>17 64</td>
<td>15 70</td>
<td>18 15</td>
<td>17 19</td>
</tr>
</tbody>
</table>
4.1.3 Breadth measures

In order to further examine the breadth of video game play, students were asked the types of games they played. The results are summarised in Table 3. The most popular category amongst the Portsmouth cohort appeared to be Action games (90/118), followed by Sports (67/118) and then Puzzle games (61/118). Role play game (RPG) scored the lowest (39/118). These results were more or less mirrored in the Bath cohort.

<table>
<thead>
<tr>
<th>Types of games</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port</td>
</tr>
<tr>
<td>Action</td>
<td>90</td>
</tr>
<tr>
<td>Adventure</td>
<td>48</td>
</tr>
<tr>
<td>Fighting</td>
<td>50</td>
</tr>
<tr>
<td>Puzzle</td>
<td>61</td>
</tr>
<tr>
<td>RPG</td>
<td>39</td>
</tr>
<tr>
<td>Simulation games</td>
<td>47</td>
</tr>
<tr>
<td>Sports</td>
<td>67</td>
</tr>
<tr>
<td>Strategy</td>
<td>58</td>
</tr>
</tbody>
</table>

Table 3 Type of Games Played (Portsmouth, Bath)

4.1.4 Depth measure

The questionnaire took a measure of how long students spend playing games, to establish the depth of game playing. During the week, a gaming session seemed to last approximately 1 hour (SD=1.02).

4.2 Engineering identity

Both the pre and post questionnaires contained a measure of how important engineering is in a student’s life. This measure had 3 subscales.

i. How much they enjoyed studying engineering.

ii. How important engineering was to the students’ own self identity.

iii. How competent they were at engineering.

The mean measures of enjoyment, identity and perceived competence from the pre and post questionnaires are reported in Table 4.

The scores for Portsmouth students are all above three before they played RA and remained unchanged after they have played RA. Thus playing RA had neither a positive nor a negative impact on how important engineering was in a student’s life. The only significant difference in the Bath cohort was that RA appeared to have significant influence on how important engineering was to the student’s own self identity. This may be attributed to the timing of the RA activity at Bath that took place during their second week at University during a period when the students were beginning to engage with engineering for the first time.

<table>
<thead>
<tr>
<th></th>
<th>Pre</th>
<th></th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port</td>
<td></td>
<td>Bath</td>
</tr>
<tr>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Enjoyment</td>
<td>4.2</td>
<td>0.4</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Edited by Marie Joubert, Alison Clark-Wilson and Michael McCabe
4.3 Frequency of playing RA

The post questionnaire asked students about how frequently they used RA. Over half of the Portsmouth students (54%) played RA at least once per week, while over 86% of the Bath students played the game to the same extent. Table 5 shows a summary.

<table>
<thead>
<tr>
<th>Number of students play RA</th>
<th>Percentage of students play RA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port</td>
<td>Bath</td>
</tr>
<tr>
<td>Never</td>
<td>3</td>
</tr>
<tr>
<td>Less than once a week</td>
<td>33</td>
</tr>
<tr>
<td>Once a week</td>
<td>37</td>
</tr>
<tr>
<td>Several times a week</td>
<td>5</td>
</tr>
<tr>
<td>Daily</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5 Frequency of playing RA (Portsmouth, Bath)

4.4 Motivation of playing RA

The post questionnaire contained measures of how motivating the students found Racing Academy. There were four subscales.

i. How enjoyable they found Racing Academy.

ii. How competent they felt playing Racing Academy.

iii. How much effort they put into playing Racing Academy.

iv. How valuable Racing Academy was in their study.

<table>
<thead>
<tr>
<th></th>
<th>Port</th>
<th>Bath</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Enjoyment</td>
<td>3.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Competent</td>
<td>3.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Effort</td>
<td>3.8</td>
<td>0.6</td>
</tr>
<tr>
<td>Value</td>
<td>3.6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 6 Motivation of playing RA (Portsmouth, Bath)

Table 6 shows the impact Racing Academy had on student’s motivation. The students from Portsmouth scored 3.6 on the measure of enjoyment suggesting they enjoyed playing RA. They felt competent, and thought that it was worth putting the effort into the game and that it was a valuable experience. The Bath students, who had a more academic background than their counterparts at Portsmouth, were generally less keen on the RA experience.
4.5 Perceived success of playing RA

Table 7 shows a breakdown of how successful the students’ thought Racing Academy was at supporting their learning. The majority of students thought the implementation of Racing Academy was either a little bit successful or quite successful. Interestingly, a few students from Bath thought that Racing Academy was not successful at supporting their learning.

<table>
<thead>
<tr>
<th>How successful</th>
<th>Number of students play RA</th>
<th>Percentage of students play RA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Port</td>
<td>Bath</td>
</tr>
<tr>
<td>Not at all</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>A little bit</td>
<td>17</td>
<td>71</td>
</tr>
<tr>
<td>Quite</td>
<td>50</td>
<td>66</td>
</tr>
<tr>
<td>Very</td>
<td>13</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7 Perceived Success (Portsmouth, Bath)

5. Conclusions

The Racing Academy project at Portsmouth clearly met and extended requirements specified by the Engineering Council UK (2010) and the QAA (2006) in terms of engineering knowledge and teaching and assessment methods.

Pre and post questionnaires that assessed the motivation and perceived success of Racing Academy showed a positive response from both the Portsmouth and Bath students. Informal feedback from students and the lab report results indicated that the comprehensive supporting materials provided a ‘safe’ learning environment. The quality of the lab reports from both the Bath and Portsmouth cohorts indicated that the learning outcomes had been met by the majority of students. Future development of online gaming communities could further improve the engagement of large groups of students although simulation software should not completely replace practical laboratory activities.

References

An intervention addressing inequity of access to ICT for pre-service mathematics teachers

Rosalyn Hyde and Julie-Ann Edwards
University of Southampton, UK

This study describes an intervention to address inequities in secondary mathematics pre-service teachers’ access to models of effective use of information and communication technologies (ICT) in classrooms and the opportunities afforded to these pre-service teachers to use the skills and knowledge they possess in developing pedagogical strategies for the effective use of ICT in the classroom. It explores the relationship between the skills necessary to use digital technologies and pedagogical subject-specific knowledge. We report on a key aspect of a wider study on a series of interventions to connect pre-service teachers’ ICT attitudes, confidence and skills with their pedagogical readiness to use ICT effectively in their teaching.

Introduction

The Standards for Qualified Teacher Status in England require pre-service teachers to demonstrate an understanding and use of information and communication technologies (ICT) in the classroom. In secondary mathematics departments in our university-school partnership, in which one-year postgraduate pre-service teachers undertake practice placements, we are aware of a range of provision of experience for our pre-service teachers as they work towards attaining these statutory standards. This anecdotal evidence is supported by Cuckle, Clarke & Jenkins (2000) who claim that some pre-service teachers possess personal IT skills on entry to a Postgraduate Certificate of Education (PGCE) course which they find difficult to transfer to the classroom. Their findings indicate that pre-service teachers used their IT skills, instead, to support their role as a teacher in planning for teaching, recording assessment and in undertaking PGCE course requirements. They argue that part of the reason for this is “poor access to hardware and software in schools” (p 18).

The experience of some of our pre-service teachers does not necessarily involve a lack of availability of hardware in schools, rather a lack of access to this. Supportive political resolve and significant financial input into schools in relation to ICT provision, in England, means that a lack of availability of digital technologies in schools is no longer a problem (Enochsson & Rizza, 2009). However, evidence from Kennewell, Tanner, Jones and Beauchamp (2008) and others indicates that the potential of this investment has not been realised. In an Australian study, Gill and Delgarno (2008) suggest that this is a more global situation which needs addressing.

The purpose of this study is two-fold. Firstly, we aimed to address the inequity of pre-service teachers’ access to pedagogical practice which supports effective use of ICT for teaching and learning mathematics and, secondly, we aimed to actively reconstruct their positive attitudes, skills and self-confidence in the use of ICT and connect these to pedagogical decision-making.

Research background

There are two arguments about an approach to using ICT in classrooms (Miller & Glover, 2010). These are, firstly, that teachers become proficient in the use of ICT and then recognise and incorporate this into their classroom pedagogy and, secondly, that pedagogical realisations by teachers necessitate a new means of use of the available ICT. Findings from Minaidi and Hplanas (2005) suggest that there are difficulties in addressing effective ICT use in classrooms by training teachers in a skill-based way to use ICT. Their evidence suggests that such training induces a further demand for skill-based training, while these participants equally recognised the pedagogical importance of the use of ICT in educational settings.

In contrast, Mercer, Hennessey and Warwick (2010) describe classroom situations in which the pedagogical perspectives are of prime importance for ICT use, rather than the technological capacities of both the user and the hardware, in this case the interactive whiteboard. With a focus on dialogic teaching as a pedagogical aim, the hardware (and implicit software) became a means to a particular pedagogical end. The authors argue that a minimal level of technological competence is required to
use ICT effectively and that teachers’ propensity towards its use for teaching and learning has a much greater influence on the effectiveness of their use of ICT.

Within the secondary mathematics PGCE course at the University of Southampton, we acknowledge and work with both of these approaches differentially. We recognise pre-service teachers’ lack of pedagogical knowledge early in the course and understand that they come to the course with significantly diverse technological skills, levels of confidence, and beliefs about the use of ICT. Using Mishra and Koehler’s (2006) framework, representing the complex interaction between a pre-service teacher’s technological expertise, their subject knowledge and their understanding of subject-specific pedagogy, we address “the question of what teachers need to know in order to appropriately incorporate technology into their teaching” (Mishra & Koehler, 2006, p. 1018). The ways in which we combine each of these three elements (Holmes, 2009) is personalised for our pre-service teachers, as much as is possible, and depends upon the pre-service teacher’s underlying competence, confidence and beliefs in relation to both mathematics and ICT.

**The study setting**

The one-year secondary mathematics PGCE course at the University of Southampton is a large course recruiting up to 50 pre-service teachers each year. Course participants usually represent three groups: those recently completing a first degree, those returning to the workplace after a career break, and those making a career change much later in their working lives. Up to half of these pre-service teachers have previously undertaken our pre-PGCE course focusing on subject knowledge enhancement (SKE), designed for those with insufficient degree-level mathematics for direct entry to the PGCE. All pre-service students are expected to begin the PGCE course with basic skills in word processing, email and internet use. Many are highly digitally literate and some of the career changers have had previous employment experience in computing or information technology fields.

Our teaching philosophy is based on developing an integrated and personalised approach to pre-service teachers’ learning of effective use of digital technologies in the mathematics classroom. This approach acknowledges the diversity of school placement experiences, identified by Cuckle et al (2000), and provides opportunities for developing personal technological skills. It also places all their learning in the context of developing sound pedagogy both for mathematics learning and for the use of ICT in enabling that learning. Approaches we use to developing both skills and pedagogy are wide-ranging and include: 1) modelling by university tutors and school-based mentors, 2) group and individual tasks to evaluate existing resources and to develop their own, 3) peer support, and 4) skills-based and more pedagogically-oriented sessions early in the course. Pre-service teachers’ progress towards effectively using ICT to support students’ learning is guided and informed by an on-going audit focusing on personal technological confidence, classroom competence with digital technologies, and a developing personal pedagogy for their use.

**Data collection and analysis**

The data collection instruments for the study reported here are: 1) semi-structured interviews with two focus groups of secondary mathematics pre-service teachers in June, 2010, as they completed their PGCE course, 2) the usual initial audit of personal technological competence and confidence in using ICT with a new cohort of 43 secondary mathematics pre-service teachers, in September, 2010, 3) ICT conference evaluations, on a 5-point Likert scale, in January, 2011, and 4) the final audit of personal technological competence and confidence, in June, 2011.

One of the focus-group questions was designed to explore what opportunities and experiences could be usefully provided in order to help pre-service teachers more rapidly develop their learning to teach effectively with ICT. Part of the discussion in this section of the focus group interviews was about being inspired by the possibilities for using ICT and extending their horizons for the future. In one of the focus group interviews, a pre-service teacher described some of the university skills-based sessions as making some of the software more “…accessible to me …” and that she “didn’t actually use them in the classroom, but I feel like I could use them”. Another describes how she found it hard, earlier in the course, to see how ICT fitted into teaching but that she “…thought it might have been useful to have maybe another session for people to go in to later on …. I just wasn’t sure where we
would actually use it”. This same theme was explored by another pre-service teacher in the second focus group who said that the university skills-based sessions were good but that “I felt that more of a chance to use them and more practical examples of where they could be used in a lesson would have been good.” Another suggested that he would have liked to have a teacher demonstrate some innovative possibilities with commonly available software in order to inspire him, showing his level of expectation of the demonstration by saying, “…if you’re really, really hot, this is what you can do”.

The initial audit results informed the choices we made about specific forms of input on the course for the academic year 2010-11. Taught skill-based sessions were offered early on in the course focusing on using spreadsheets, LOGO, Autograph and Geometers’ Sketchpad. This audit also indicated pre-service teachers’ levels of personal confidence with a range of content-free software packages, small mathematics-specific software, and with use of hardware and peripherals. Such attitudinal information is considered by Teo (2008) to be important and necessary in order to support pre-service teachers to improve their pedagogical use of ICT. Scores (from 1-4) were allocated to pre-service teachers’ reported levels of confidence and then totalled for individual items in the audits and for individual pre-service teachers. Unlike evidence from Markauskaite (2006), gender was not a factor separating those self-reporting high or low levels of personal confidence in the aspects of ICT use described earlier.

![Figure 1: Overall personal competence score](image)

Twenty of those on the PGCE course had previously studied with us on the SKE course, one on a similar course elsewhere and 22 had been accepted for direct entry onto the PGCE course. Figure 1 shows the ‘overall personal competence scores’ of the pre-service teachers. Of the 22 reporting the highest overall levels of personal confidence with ICT tools, 18 had participated in the SKE course.

Whilst use of ICT is not a specific learning objective or course aim for this SKE course, the audit results clearly suggest that these participants started their PGCE with a significantly better perception of their personal ICT skills across a range of relevant software and hardware than those who did not attend the SKE course. Such results should, however, be interpreted with some caution. Pre-service teachers were asked about their level of personal confidence but it may be that those from the SKE course reported on a partial level of confidence with classroom use of digital technologies gained from their SKE course through short school-based placements and the tutors’ modelling and teaching strategies underpinning the course. Conversely, some of the direct-entry PGCE course members may
have been reporting on their perceived lack of experience, or anxiety about, the use of digital technologies in the classroom.

**Intervention design**

These audit results, and the suggestions made by the pre-service teachers during the focus groups, for 1) a later opportunity to contextualise the learning from the university skill-based sessions and 2) demonstrations by ‘expert’ teachers of innovative uses of ICT for the classroom, formed the basis for our design of the first intervention. This was a one-day ICT conference for our pre-service teachers timed for the half-way point of the course, prior to their final long school-based teaching placement. One of the aims of this intervention was to provide a similar opportunity for all the pre-service teachers to experience modelling of good pedagogical practice in relation to both mathematics and ICT, thus reducing the levels of inequity we identified in their school-based experiences. The conference comprised of a range of workshops to both support those who were less confident in their ICT skills and to inspire the more competent with some of the possibilities. Five local teachers, with acknowledged expertise in using ICT to teach mathematics, offered three parallel workshops for approximately 14 participants. Another mathematics teacher delivered a plenary lecture to the whole cohort on “Getting the best out of ICT”.

The pre-service teachers were offered the opportunity to choose one workshop and then we allocated them to two further workshops using the information drawn from their ICT audits. Each pre-service teacher therefore attended three workshops and all the supporting files were placed on our Virtual Learning Environment. The foci of the workshops were: 1) **Exploring the paths of projectiles using ICT**: Led by a teacher who teaches both mathematics and ICT, the classroom activity was designed to inspire confident users of ICT and the learning was focused on the motion of projectiles. The activity combined the use of video, specialist software (“Tracker”) and graph plotting or dynamic geometry software; 2) **Using Google Sketchup to teach volume and surface area**: Again for confident ICT users, this session explored the use of a piece of freeware familiar to many school-age students for learning and teaching mathematics. It focused particularly on creating and manipulating nets and solids; 3) **Using the SMARTboard tools to teach mathematics**: Aimed at the less confident users of ICT, the intention was to provide the opportunity to learn further skills in using the interactive whiteboard tools in ways that supported learning mathematics specifically; 4) **An introduction to using Geogebra**: Geometer’s Sketchpad is commonly used by our partnership secondary schools so we had offered our pre-service teachers the opportunity to attend a skill-based session on this software early in the course. This Geogebra session was intended to develop this area further, providing basic instruction on using the software and then the opportunity to explore a range of aspects of mathematics suitable for 14-18 year olds through a series of worksheets appropriate for the classroom. The initial ICT audits identified dynamic geometry software as an area of weakness in terms of personal competence, with 74% of our pre-service teachers reporting that they had never used this software; 5) **Using Internet resources in lessons**: Aimed at less confident users of ICT, the session provided the opportunity to explore a broad range of pre-selected resources suitable for classroom use to give some starting points for teaching. All the resources selected were those actively used and valued by the teacher leading the session.

**Discussion**

Conference evaluations, on a 5-point Likert scale, show that all but one of the 43 pre-service teachers rated the day as good or very good overall, in the top two categories. The individual sessions were also all rated similarly highly, with Geogebra having the highest positive feedback. The dissatisfactions expressed were the need for 1) the opportunity to attend all five sessions and 2) a much longer time working in each of the sessions. Evaluations also clearly indicated that many of the pre-service teachers were already, half-way through the course, focusing on the ways ICT could enhance their students’ mathematics learning and looking for ways they could utilise ICT in the classroom. However, at this point in their learning journey, this is not often expressed in pedagogical terms. Phrases from the evaluations such as “… Interesting sessions but unable to see direct link to teaching sessions in schools” and “complicated resource to use in class” (about Google Sketchup) indicate that there is recognition that they should be making such pedagogical connections. Such responses reflect
the findings of Gao, Choy, Wong and Wu’s (2009) study on implementing a technology-based pedagogy for student-centred learning. They acknowledge the need for increased levels of guidance, modelling and collaborations between pre-service teachers and teachers to improve pre-service teachers’ understanding and use of a technology-based pedagogy.

The analysis of final audits of confidence in specific areas of ICT use show that pre-service teachers’ confidence was significantly improved between September, 2010 and June, 2011, as indicated by data handling software and dynamic geometry software analyses, in figures 2 and 3.

Positive responses from the conference evaluations, such as “I enjoyed it and learnt a lot and got a lot of useful resources from it” and the many requests for other resources to use in classrooms, demonstrate that these pre-service teachers are beginning to make connections between their technological skills, their mathematics-specific subject knowledge and the pedagogy which connects these. They also show a shift towards positive attitudes to the use of ICT for teaching and learning mathematics. This one-day conference mirrors, on a much smaller scale, the five-day intervention described by Campbell and Kent (2010) which had similarly high levels of positive feedback from pre-service teachers and recognition of the importance of modelling effective use of ICT by ‘expert’ teachers.

As Steketee (2006) argues, pre-service teachers can have a crucial role in developing effective use of ICT in schools. The intervention reported here offers one element of a wider range of interventions
which seek to do this. Working with ‘expert’ mathematics teachers in a conference-style forum provides all pre-service teachers with a window into effective pedagogical practice for mathematics teaching using ICT, thus reducing the randomness of access to such practice in school-based placements for each of them.

References


Automated assessment and feedback on MATLAB assignments

Alan Irving¹ and Adam Crawford²

¹University of Liverpool, UK, ²Loughborough University, UK

We describe an automated system for the assessment and provision of feedback on computational mathematics assignments using MATLAB. The assessment code checks whether the submitted function runs and, if it does, tests the requested output against a correct reference code, using a variety of inputs. A version of this is made available to students to assist with development prior to submission. The results, anonymised total marks and individual feedback files are made available via the virtual learning environment and web. Marking details and summaries are provided for the tutor to assist with assembling generic feedback for the whole class. We have also developed code which compares the active portions of all submitted code to check for collusion. It assigns a numerical correlation coefficient to similar pairs of files and collects these into groups which exhibit common coding and so warrant further investigation.

1. Background

Automated and semi-automated assessment of programming language assignments has long been in use within the computer science community – see for example the review by Ala-Mutka (2005). In most systems, the focus has been on assessing the correctness and structure of the programmes and the tools employed are specific to the language being taught. When the number of submissions to be checked and evaluated is measured in hundreds rather than tens it is not practical to do this manually. Simply running the code to see if it does what it is supposed to takes a lot of time. Creating feedback in time to be of use to students moving on to later assignments is impossible. A quick scan of a piece of code to see whether the student looks as though they might have the right idea is insufficient. Both tutor and student need to know whether it works as advertised. Solutions involving farming out assessment to large numbers of teaching assistants is problematic, not just because of the manpower involved but because it becomes hard to ensure uniformity of treatment. For many assignments, particularly in mathematics and the sciences, the task being assessed can be quite well-defined and lends itself well to automatic and hence objective treatment. Here, perhaps more so than in the computer science context, the functionality of the programme and the results themselves are usually more significant than the programming style being employed. The system described here was initially developed from 1998 at Loughborough University (Tolley, Huntley & Crawford, 2008) and was designed to make practical the assessment of, and provision of feedback on, large numbers of MATLAB files constructed by engineering students. The current project funded by HE-STEM is to develop further this system and facilitate its use by third parties in other institutions and disciplines. At Liverpool we are using the latest code to assess monthly submissions by 250 students who are being taught numerical analysis methods relevant to engineering applications. The code being assessed, as well as the assessment code itself, is written in MATLAB. Implementation in other high level languages is possible but has not yet been fully addressed.

MATLAB is a fully comprehensive programming language with a particularly intuitive grammar and structure and is quickly accessible to anyone with a mathematical or science and engineering background. At its most basic level, it can be used as a simple calculator. Since it has many built-in functions, students are quickly able to build up more complicated calculations than would be feasible using a calculator or spreadsheet. By editing and saving files containing the corresponding commands, students quickly enter the world of programming. Because MATLAB’s syntax is quite flexible and forgiving compared with many traditional computer languages, the reward of meaningful results is more rapidly gained.

2. Assessment and feedback strategy

The system can be used to assess a wide range of learning outcomes ranging from elementary MATLAB coding skills to success in carrying out complex numerical calculations using MATLAB in any area of mathematics, science and engineering. However, it is important that an appropriate level of MATLAB skill has already been demonstrated before students are asked to submit work under this system. The meaning of ‘appropriate’ will become apparent in what follows. The basic system relies
on the student constructing, from scratch or from a supplied template, a function MATLAB file (FMF) for short with specified functionality:

```matlab
[out_1  out_2  ...  out_n] = fname(in_1, in_2,....,in_m)
```
% fname.m calculates these n output arguments
% from these m input arguments
% specification of the type and number of input and output arguments
% is critical
% ....... student code
% end fname.m

Thus, for students at an early stage, they must have already appreciated the significance and appropriate use of both MATLAB script files and function MATLAB files (FMF). The number and type of input/output variables should also be kept minimal and simple. For more sophisticated calculations and assignments, it is a good idea to include a range of outputs including some relatively straightforward ones to enable partial credit to be given. From an assessment point of view, the function should be thought of as a black box. There is input and there is output. The student task is to create the black box which takes whatever input is fed into it and returns the corresponding output for the problem in hand. The assessment code probes the black box by varying the input and checking that the corresponding output is as expected.

The feedback consists of five types:

1. **Syntax errors** – these are where MATLAB declares that the code does not obey the basic rules. Fortunately MATLAB is usually able to pinpoint where the code has gone wrong and why. These errors are trapped by MATLAB in the standard way and are returned to the student feedback file. Unfortunately, this means the student gets little more feedback. There is no easy way to give additional hypothetical feedback on what might have happened had they not made the first error encountered by MATLAB. This is the first hard lesson in programming.

2. **Run-time errors** – these occur when a disallowed operation (e.g. diving by zero) occurs and, again, these are trapped and returned to the student without further comment.

3. **Marks for correct answers** - the code returns marks and comments for correctly getting a specified numerical output, or otherwise as the case may be.

4. **Marks for nearly correct answers** - the code can return marks and comments for getting numerical outputs nearly correct (i.e. within some tolerance set by the tutor). One can also test for, and comment on, predictable common errors.

5. **Marks for checks** - the code returns marks for good checking/handling of the input parameters. This can be used as a way of rewarding the more sophisticated student who has coped rather too easily with the other requirements.

From the above, it is clear that the marking scheme should have sufficient structure so the student can (a) get partial credit where appropriate and (b) get enough useful information to make sense of any model solution provided along with the feedback.

Implementation of the above is aided by provision of a test script allowing students to check how things are going. This is a version of the marking script which is produced by simply selecting a flag and ‘packaging’ the marking code into a binary (.p rather than .m) version so obscuring its details. This provides comments without revealing detailed marks. One can also help things along by providing a driver script with suitable input values already provided to get the initial stages of FMF development off to a good start. When showing students how to get the best out of the system, one can advise them to identify exactly which output arguments are required and, for example, what dimensions they are expected to have. When coding their FMF they can then start by assigning ‘place-holder’ values to them pending a complete calculation as their code development proceeds. In this way, they can more successfully develop and test the required FMF.

3. **Code functionality and structure**

The main assessment script file, MarkAss.m, sequentially processes all student M-files (FMFs) StudentID.m found in some marking directory and, for each one, constructs a simple feedback file StudentID.txt which contains simple comments to accompany the marks and feedback described in the previous section. In simple terms, MarkAss.m checks to see if each StudentID.m runs and, if it does, tests the requested output against a correct reference code, CorrectSoln.m, using a variety of inputs. For each application/assignment, some tutor effort is therefore required to design the
relevant checks and an associated marking scheme, as described in Section 4 below. We have made some effort (but more is certainly possible) to organise MarkAss.m so that the code changes required are localised and well-flagged. When editing MarkAss1.m for one assignment to produce MarkAss2.m for the next, experience shows that a couple of hours’ work is enough to design, implement and test the new code, assuming one is starting from a tested piece of reference code CorrectSoln.m. The overall structure of MarkAss.m is shown in Fig 1.

MarkAss.m also produces the separate web page html files for students, to be linked to the VLE (virtual learning environment), and for tutors. The tutor sees a list, student by student, of the feedback provided and the marks allocated according to the scheme (see Fig. 2). The submitted FMF code is accessed by a web link. The full compilation of student feedback, including MATLAB errors is useful in identifying common problems. The student, for their part, sees a list of all marks, identified by student ID rather than name, and a link to the corresponding feedback file. The header page also links in a general feedback page where a tutor has provided, manually, some additional comments and advice on the outcome of the assessment.
4. Setting up an assignment

In a typical implementation, the requirements for the assignment are posted on the VLE. The students download any relevant notes, complete the work and submit, by a deadline, their FMF via the VLE. The tutor then downloads (en bloc) all student FMFs, unzips, renames and marks them using MarkAss.m. If all is well, marking takes about a minute or two on a PC. In practice, perhaps 10 of the several hundred files have been misnamed or have some serious deficiency (such as a recursive call) which requires manual intervention and a re-run. Since the marking is very quick, this is not usually a problem. The general procedure in setting up an assignment is as follows:

1. Check one has a correct solution FMF CorrectSoln.m. which conforms to the assignment notes provided for the students.
2. Copy/rename the previous version of the marking script MarkAss.m.
3. Locate the flagged portions of the code: %SET begin/end which will require editing.
4. Edit the Marking Scheme Summary for the new assignment. This and the FMF specification determine what further tests need editing.
5. Systematically work down the marking script updating the code to conform to the new FMF specification and marking scheme.
6. Test the marking script using both the solution FMF and various other versions of it where deliberate errors are introduced, both minor and major. Look at the feedback files produced and modify accordingly.
7. Check that the student assignment notes are still consistent with any changes to the requirements that have been flushed out by the above tests.

If the assignment, or something like it, has been used previously one can use the experience gained to anticipate common errors and incorporate helpful feedback at an appropriate point.

5. Collusion detection

There exists a variety of on-line systems for detection of code similarities resulting from plagiarism or collusion (see for example the Information & Computer Sciences resource pages of the Higher
Education Academy: www.ice.heacademy.ac.uk). For convenience, we have provided our own MATLAB script, collision.m, as part of the automated assessment toolkit. Like many other systems it uses a string comparison algorithm to detect pairs, and indeed groups, of code files with a high degree of similarity. A numerical correlation value is used to identify and sift these ready for further investigation by an academic. The manual investigation prior to taking any formal action is simplified by the production of a report which accompanies the actual code files involved. One can use this tool to check similarities with related model assignment solutions retained from previous years. It can be run conveniently within the same assessment environment without resubmission to an external service. We have not found plagiarism to be a problem since assignments have tended to be quite specific. Collusion is a more likely problem when students have the time and ability to exchange apparently successful solutions.

6. Student and staff experience

Staff
Those who would otherwise be submerged in hand-marking hundreds of scripts obviously appreciate the labour-saving aspect. As a rough guide, the setup time for an assignment is whatever time it takes to design the assignment and create a module solution (which would have to be done in any case) plus roughly 2 hours to adapt/edit the marking script. For the assessment, one might spend 1 hour fixing files incorrectly named or with some terminal deficiency requiring manual intervention. The marking takes about 1 minute per hundred files. Detection of collusion can take up to one hour running time on a PC depending on the degree of rigour required. A further 2 hours might well be spent reviewing common errors and creating general feedback to link in with the individual feedback. Of course the real frustration is the tutor’s inability to assess the problem-solving or coding style being adopted. With small or modest cohorts of students one might adopt a combined approach of using the automated assessment to take care of the more objective aspects of the assessment and feedback, coupled with some form of real-time inspection of student code perhaps on a selective basis.

Experience suggests some other considerations to bear in mind:

• It may be a good idea to give students some form of ‘driving test’ before exposing them to automated assessment. This could be a simple multiple choice test of their basic knowledge of MATLAB syntax and use of M-files and functions, followed by a simple submission test using the format to be employed later in the assessed work.

• For the system to work, one needs to underpin it with a good active-learning programme for the student. Passive, content-focussed lecture preparation is unlikely to be sufficient.

• One needs to think carefully about the marking scheme while a large fraction still have difficulty in writing syntax error-free code.

• Provision of the student test version of the marking script MarkAss.m can distract students from more systematic coding and debugging.

• There is a decision to be made on whether the work is expected to be carried out, perhaps collaboratively, over a period of time or in a more exam-like and time-limited situation. There is no reason why this system cannot be used to assess projects carried out individually or as group work.

Students
Student feedback was obtained via at least three routes. Online questionnaires were conducted by the School of Engineering (as part of their own regular module survey in first semester) and by the Department of Mathematical Sciences in second semester. We also ran a ‘blog’ on our VLE to try and extract suggestions for improvements during the year. Much of the feedback focussed on the teaching and learning aspects of the module as opposed to the assessment and has provided many useful pointers for future improvements.

Students appreciate the immediacy of the feedback and the objectivity of the marking. Of course, they would like more sophisticated individual feedback than is possible with an automated system. They are less interested in being told that something doesn’t work, or is wrong, than in knowing why it is wrong and what they have to do to fix it. The Liverpool implementation via individual time-limited class tests is very demanding on students. These tests expose the individual’s skill level in a rather ruthless manner. Students would like a softer form of assessment particularly at the early stages of learning. Also, students understandably want more discrimination between those who can write more sophisticated code but with minor errors and those who can hardly code at all. As noted above, the automated system can only provide this to a limited extent.
5. Conclusions

Automated assessment of MATLAB-based assignments is indeed possible. MATLAB provides a very suitable environment for teaching and assessing computer programming and problem solving. Automated assessment and feedback has some very positive features:

• it can deliver objective and timely feedback;
• it can be used in an almost interactive mode so allowing students to improve work prior to summative assessment;
• it can make practical the otherwise impossible task of assessing very large numbers of scripts;
• it encourages student engagement if used sensitively;
• with some care in design, it can be applied to the assessment of a useful range of learning outcomes;
• the detection, and hence prevention, of collusion is relatively straightforward.

There are however a number of limitations – some obvious and some less so:

• it is difficult to assess and provide feedback on the 'softer' skills of programming or problem solving unless coupled with human intervention of some kind;
• considerable care is required in the design of marking schemes and the preparation of students for successful participation in the process;
• even with intelligently designed mark schemes, the mark distributions tend to be bimodal with limited discrimination between low achievers.

Overall, the experience has been sufficiently positive that the system will be exploited again in future academic years. We expect further improvements will be implemented as more and diverse applications are encountered.

References

Home technologies: how do they shape beyond-school mathematical problem solving activity?

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In this paper we analyze and discuss the mathematical activity of a participant in a web-based problem solving competition – the Sub14. Our purpose is to contribute to the knowledge on the learning processes that occur outside the classroom, constantly and necessarily entangling technologically rich environments. The theoretical framework supporting our analysis draws on the concept of humans-with-media (Borboded & Villareal, 2005) and the notion of mediational artefact (Wertsch, 1991), which are vital for understanding the unbreakable entity resulting from the interactions between subject, object and action. We illustrate how the computer is a mediating artefact in this participant’s submitted solutions.

The context – an online mathematical problem solving competition

Our study looks at specific aspects of a web-based mathematical problem solving competition, named Sub14 (http://fc.tec.ualg.pt/mathematica/Sestrelas/subs/sub14.html), which addresses 13 to 14 year old students from the south of Portugal. The Sub14 runs annually since the academic year 2006/2007, and is organised by the Mathematics Department of the Faculty of Sciences and Technology of the University of Algarve.

The Qualifying part of the competition consists of ten problems, each one posted every two weeks. The organizing committee uses the website for publishing every new problem, providing updated information, and it allows participants to send their answers using a simplified text editor in which they can attach a file containing any work they wish. The participants may solve the problems working alone or in small teams and using their preferred methods and reasoning. They must send their solution and a complete explanation through the website mailing device or using their personal e-mail account. The organizing committee assesses every answer and replies to each participant with some constructive feedback about the given answer. The Final consists of a one-day tournament where the finalists solve five problems, individually, with paper and pencil, also having to explain their reasoning.

An earlier study (Jacinto, 2008) aimed at describing and understanding the participants’ perspectives regarding their mathematical activity and the role of the technological tools they used in the competition. The main conclusions were that: (i) the participants faced the usage of Internet quite naturally, (ii) revealing a rather sophisticated technological fluency in the presentation of their answers to the problems posed, and (iii) such fluency seemed to be developed mainly outside the mathematics classroom.

Up to the present, participants keep sending their work in several digital formats, using many different technological resources such as text editors, spreadsheets, presentation editors, editing tools and image processing software, scanning software, digital camera or mobile phone, dynamic geometry software, online repositories, or even programming languages. Therefore, we use the concept of “home technologies” to refer to all kind of digital tools that are constantly available to the young participants in their everyday live.

However, many questions emerged since the earlier study and a broader investigation is taking place. The aim of the present study is to establish how 13 to 14 years old youngsters – surrounded by home technology and revealing a clear predisposition to use several resources like the Internet, the computer or the smartphone – extend their learning of mathematics beyond the classroom. In particular, we hope to understand the mediating role that technology plays in solving mathematical problems as part of an online problem solving competition.
Theoretical framework

In this paper we are reporting on part of an ongoing larger study and therefore we refer to particular theoretical aspects of the overall framework. Nevertheless, there are four focuses in the theoretical approach, which covers (i) mathematical problem solving as a (ii) human activity (iii) mediated by technological tools (iv) beyond the classroom.

Human activity mediated by technological tools

A number of theorists describe the human mind as an ecological system, in which the individuals interact with the cultural tools existing in such system, in order to produce thoughts and actions (Shaffer & Clinton, 2006). According to Hickman (1991), Dewey also supported the idea that knowledge does not occur only in the brain. Instead, it is a form of activity in the surrounding world, which comprises both the whole body and the cultural tools available.

The Theory of Virtual Culture (Donald, 1991; Clark, 2003) describes a cognitive ecology that considers thoughts as arising from the interaction between humans and technology, and suggests that the computational resources are creating new forms of cognitive activity. Levy (1990) discussed these new forms of activity, stating that cyberspace “supports intellectual technologies that amplify, externalize and modify the number of human cognitive functions: memory (databases, hyperdocuments, digital files of all kinds), imagination (simulations), perception (digital receivers, virtual reality), reasoning (artificial intelligence, modelling complex phenomena)” (p. 187). The author also stresses that these intellectual technologies sustain new forms of access to information, and promote the emergence of “new styles of reasoning and knowledge” (Lévy, 1990, p. 188).

Borba & Villarreal (2005), influenced by Levy (1990) and others, agree that the technological tools do not replace or simply complement the human being in their mathematical learning activities. The authors argued that the processes mediated by technology lead to a reorganization of the human mind. Knowledge does not result from the individual alone, since it is an outcome of a symbiosis between humans and technology – an entity that they called humans-with-media: “we believe that knowledge is produced together with a given media or technology of intelligence. [...] we adopt a theoretical perspective that supports the notion that knowledge is produced by a collective composed of humans-with-media, or humans-with-technology, and not, as other theories suggest, by individuals alone, or collectives composed only by humans” (Borba & Villareal, 2005, p. 23). In fact, human thought – which used to be logical, linear and narrative – progressively became an hypertextual thinking and nowadays it is being expressed through other forms of language that not only involve orality or writing, but also image, video or instant messaging.

More recently, Villarreal & Borba (2010) discussed that (i) cognition is not a single work but rather it has a collective nature, and that (ii) cognition includes tools, artefacts and means by which knowledge is produced. Within this theoretical concept, the separation between humans and media makes no sense, since the media are not mere accessories, but an essential and constitutive part of the concept. To these authors media are extremely relevant, so that the use of different media leads to the production of different types of knowledge.

Several authors (Tapscott, 1998; Prensky, 2001; Carreira & Jacinto, 2010), identifying such symbiosis, propose that young people today – digital natives – have special characteristics and unique modes of action which are directly related to the technology they use in a daily basis. These singularities influence the way youngsters think, access, absorb and interpret information, communicate and, consequently, how they learn (Oblinger & Oblinger, 2005).

Wertsch (1991) proposes a similar unit of analysis through an analogous syntactical shape: person(s)-acting-with-mediational-means, claiming that “from this perspective, any tendency to focus exclusively on the action, the person(s), or the mediational means in isolation is misleading” (p. 119). Moreover, the author places an emphasis on the meaning of mediational means related to action. There is no magical power in any artefact or technological tool; instead, its power depends on how such artefacts become part of action. Thus, mediational means are inherently related to action and impossible to dissociate from it.
Another crucial idea about mediational means within the analysis of students activity on Sub14 concerns the type of discourses that one may describe as informal/formal approaches (linking to the notions of horizontal and vertical mathematization) or else to non-instructional/instructional discourses (in terms of Wertsch’s outside and inside school semantic content). For instance, the use of photos or a video taken with a digital camera or a computer program to illustrate and communicate the problem solving process would hardly be seen as conventional and institutional in the typical classroom practice of solving mathematical problems. In many cases, such devices served and defined an exploratory strategy related to students’ concrete knowledge, to manipulation of objects, to experimentation and creation that are akin to actions and tools “outside” of a classroom discourse.

In this sense, we adopt the notion of humans-with-media that Borba & Villarreal (2005) propose and discuss, to describe a symbiotic relationship between the individual and the technology seen as a tool that both changes the person and is changed by the person in the course of mathematical learning experiences. Accordingly, we suggest that (i) the problem solving activity of the participants is mediated by home technologies and (ii) digital tools become mediating artefacts within students’ activity.

Methodology

Our results refer to an exploratory study that is part of a broader investigation, still in its developmental stage. According to an interpretive perspective, the research design follows a concurrent mixed design (Teddlie & Tashakkori, 2006), that embraces both quantitative and qualitative collecting and analyzing techniques.

Nevertheless, the data presented and analysed in this paper stems from the qualitative strand of the design, involving an extensive process of documents’ collection (the outcomes of the participants in the 2010 edition of the competition) and contemplating the analysis of a set of problems solved by the young participants that we identified as potential case studies. After gathering the files containing their work, we selected the ones that best illustrate a strong mediation of home technologies, regarding the reasoning approach and the communication process, i.e., referring to the mediating role of technology in the participants’ problem solving activity.

Here, we select one participant and her answers to two of the problems posed, and the text that she sent explaining her reasoning. For the data analysis, we used an interpretative perspective (Patton, 1990) and an inductive process (Merriam, 1988).

Home technologies mediating problem-solving activity

Leonor was attending the 7th grade and had been participating in the online competition since her 5th grade. She always stood out for her commitment, for the simplicity in her reasoning, for the clarity in her arguments, but also for the creativity and the technological fluency she showed in her solutions.

She often resorted to a Word attachment, containing an explanation of her strategy, layering images to illustrate her reasoning, with detailed descriptions that allow us to understand fully and in depth, the course of her thinking and her global understanding of the problem, as revealed in her solution to Problem # 7 of the competition.

<table>
<thead>
<tr>
<th>Problem # 7 – Group photo</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the last day of school, Patricia’s class took a group photo. The photographer placed all the students in three rows. Patricia was in the first row, four girls stood by her right and two boys sat at her left. In the second row, there were as many boys as girls, and there was one girl less than in the first row. In the third row, the number of girls was twice the number of boys. Peter stood at one edge of the third row while Andrew stood at the other edge of the same row. The class had 24 students. How many boys and girls were there in the class?</td>
</tr>
</tbody>
</table>

Figure 1. Statement of problem # 7 from the competition
Leonor starts her activity on this problem by presenting her interpretation of the first sentences. The text document she sent has a combination of writing and drawing, which she used to create her strategy and express her reasoning. Leonor draws three lines to illustrate the three rows where boys and girls would sit for the picture. She represents the boys by dark circles and the girls by light circles. Most significant is the fact that she places the boys on the left of Patricia, and progressively matches the other conditions of the problem, from row 1, to row 2, and to row 3 (Figure 2). So she gets the answer to the problem: 9 boys and 15 girls.

![Figure 2. Leonor’s sequential drawings matching the conditions of problem # 7](image)

Among the several problems proposed, problem #10 became a surpassing challenge since many participants asked for help from the organising committee, and most of the answers contained errors or did not present the correct solution.

**Problem #10 – Cat and mouse**

A hungry cat surprises a mouse. Immediately, the mouse starts running and the cat follows in pursuit. When the mouse starts its escape, it has a lead of 88 mouse’s little steps on the attacker. It turns out that 2 steps of the cat are equivalent, in distance, to 12 little steps of the mouse. Moreover, while the mouse takes 10 little steps, the cat takes 3 steps.

How many steps must the cat take to catch the mouse?

![Figure 3. Statement of problem # 10 from the competition](image)

In the file she sent, Leonor also starts by presenting her analysis of the problem conditions. Only then, she engages in “building a scheme” (Figure 4). The assemblage of such scheme starts with the depiction of the 88 little steps of the mouse, that separate it from the cat (initial conditions), coloured in dark blue small dots: “the first 88 dark blue dots stand for the 88 little steps advantage that the mouse had”.

![Figure 4. Leonor’s drawing matching the conditions of problem # 10](image)

The combination between *description* and *iconic arrangement* offers the reader a view of a progression, most likely the same as Leonor pictured during her reasoning process. The “hunt” begins. The scheme starts to take shape as Leonor interprets the conditions of the problem and simultaneously uses objects and colours to display a dynamic process.

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The spiral line with the attached small dots together with the divisions containing sets of six dots suggest a representation of a timeline, which also conveys the relationship between the length of the cat’s step and the length of the mouse’s step. In fact, each set of six dots correspond to the length of the hunter’s step which is equal to the length of the prey’s six small steps. Initially, colouring the dots was not part of Leonor’s strategy. However, later on, it proved to be very useful, as she stated — “to simplify the scheme, I organized the following steps of the two animals with the same colours considering the rule: as the mouse takes 10 small steps, the cat takes 3 big steps”.

It seems that Leonor uses simple and familiar digital options, such as formatting iconic forms, to organize her strategy and to assess the results of her problem solving iterations. The ten small orange dots represent the first ten little steps of the mouse as it moves away from the starting position; while the larger orange circles represent the three steps of the cat, during the same time period. She continuously represents the movement of both animals using the same colour, and creates groups of six little steps of the mouse (which corresponds to a step of the cat). The scene ends when the cat catches the mouse, which means that the last set of ten little steps of the mouse meets the last set of three steps of the cat, both in the same colour. This is how she found the solution: “by counting the steps the cat took to catch the mouse, I can conclude that the cat took 33 steps to catch the mouse.”

One of the interesting aspects of this problem solving activity lies in the quasi-dynamic nature of the picture, in the sense that it suggests the evolution of a system composed by two objects moving with time. By changing the colours of icons, which virtually represent the two animals, Leonor digitally emulates the progress of hunter and prey, describing the change in their relative positions and the decreasing distance between the two. This is extremely important regarding the mathematical activity involved in her strategy. Many students show difficulties in similar problems that demand the transformation of a dynamic phenomenon into mathematical conditions that “freeze” the situation. Leonor overcomes this obstacle by revealing competence in translating her thoughts into iconic and quasi-dynamic representations. This is a widely recognized competence of individuals who naturally interact with the modelling, simulating and imaging that characterize the digital contemporary culture.

**Discussion and conclusions**

The solutions presented by the large number of participants of the Sub14 (around 600) allows us to recognize the diversity of learning experiences that are connected both to their school environment and to other social contexts where they take part in.

The use of digital tools is consistent with the rules of the competition (namely the use of Internet and e-mail as communication tools) but furthermore gives students the chance to draw upon other significant experiences they have gone through in different social settings, revealing the diversity of their knowledge and contexts of learning.

In fact, each particular problem not only exhibits several ways of thinking and mathematical reasoning but also highlights a strong variety in the forms of communicating and explaining a solving strategy, which seems to be stressed by the wide presence of several digital tools in the daily life of the participants. Nevertheless, we wish to assert that the two issues – particular forms of reasoning and certain modes of representing and communicating the process – are inseparable.

A closer look at the solutions presented point out the new forms of activity to which Levy (1990), Borba & Villarreal (2005) and others refer. Each solution not only exhibits several ways of mathematical thinking and reasoning but also highlights a strong variety in the forms of communicating and explaining a solving strategy. These so-called “digital natives” use other forms of language, other forms of communication, characterized by their intensive use of home technologies. The quick and easy access to any technological tool, and the emerging social changes, allow youngsters to develop a large number of skills that increase their ability and sophistication in seeking knowledge in contexts that go beyond the school environment.

These preliminary results point out that there is a range of young people participating in the Sub14 that have a natural predisposition for using technological artefacts as mediators of their problem solving activity. Unlike most competitors, Leonor does not use digital tools exclusively to express herself or to display her results. As also observed in some other participants (Jacinto, 2008; Carreira &
Jacinto, 2010), Leonor exploits digital tools and their flexibility to build up her own reasoning and elaborate on a strategy. The computer is more than a means to present a solution in a neat form. The tool becomes an integral part of the problem solving process and, in a certain sense, the participant uses technology as a “native language” to think with, act with and communicate. For young people like this participant, it is trivial to be connected to the Internet, so it is natural to learn, share and communicate through images, photographs, videos, icons or hypertext. We suggest, therefore, that Leonor is a person-acting-with-mediation-artefacts (Wertsch, 1991) as an indivisible unit, and we argue that the computer was a significant mediating artefact of her problem solving activity and her answer a computer-mediated-solution.

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References


“For show” or efficient use of ICT in mathematics teaching?

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The aim of this paper is to address the important topic of ‘good examples’ in the context of educational environment that enables the use of technology in teaching mathematics. The principles for selection of suitable enriching examples in teaching are formulated and illustrated by examples. One complex example, the “Bullet and target”, is presented as an activity that successfully combines all the discussed principles and whose potential both in mathematics and in the use of ICT in various educational environments and for various ages and skills of students is enormous.

Introduction

Use of computers in mathematics education on primary and secondary school levels (e.g. Goldenberg, 2000; Ruthven, 2007) and in pre-service teacher training (e.g. Turker, Sağlam, Umay, 2010; García-Campos, Rojano, 2008) is a very up-to-date topic. Computers have become a tool of motivation and foster comprehensible interdisciplinary links between mathematics and other subjects. However, use of computers in teaching asks for new approaches to exposition and to mathematical content. This might be one of the reasons why recent studies in mathematics education show that, despite many national and international events targeting integration of ICT into mathematics classrooms, this integration in schools remains underdeveloped. The rate of this integration increases markedly slowly when compared to the speed of evolution of the technology.

Even decades after the introduction of ICT into classrooms there are still unanswered questions about the long-term and short-term impact of technology on students’ learning, and questions on how it affects simple and complex learning tasks. (Cox, Marshall, 2007)

One of the causes for this state is the huge diversity of ICT resources, which often leaves teachers unsure of which to use, as well as when and how to use them. Another important retarder of successful use of ICT in teaching is lack of information on the potential, advantages and dangers of inclusion of activities using ICT into teaching (see e.g. Aktumen, Kacar, 2008). Despite the fact that ICT has a huge potential in teaching (for an example see e.g. Jančařík, Novotná, 2011), examples from practice show that in many cases, “for show” examples contribute very little to development of mathematical knowledge and may be even contra productive.

Role of ICT in Education

Implementation of ICT into mathematics teaching and learning should never be purposeless. When speaking about the role of ICT in science and mathematics education, it is necessary first to identify the objectives of that education and then to disaggregate various forms of ICT in order to discuss the potential relevance or otherwise of each (McFarlane, Sakellariou, 2002).

The aim of this paper is to propose the rules of efficient implementation of ICT into teaching and of selection of appropriate ICT tools. As Osborne and Hennessy (2003) state, implementation of computers should be an ‘added value’ to learning activities. The fundamental question raised in this context is: What could the added value brought by the use of ICT be?

Undoubtedly, added value may lay in the area of students’ motivation (e.g. bringing in elements of competitiveness, using graphics or group work). If added value is connected directly to the subject content, computer may serve in the following two activities:

- For processing of large amounts of data whose processing would be too time consuming. Use of computers should not replace students’ cognitive processes; they should develop, not substitute thinking.
- For finding numerical or algebraic solutions of problems (often from a real-life context) whose solution is not achievable by students themselves.

In both cases it is necessary to take into account the relationship of problems to students’ prior knowledge and to offer a suitable space for discussion and discovery (Osborne, Hennessy, 2003). The
discussion is more efficient if the solved problem is connected to a real situation, e.g. through processing of real data or simulation of a real situation (Good, Berger, 1998, Lukac, Engel, 2008).

In the following text we will demonstrate the following principles on particular problems:

- The use of computers cannot be autotelic but must be linked to a specific educational content.
- The computing power must be used effectively; the results should be presented in a comprehensible way.
- The results from the computer should be further interpreted; they should provide space for follow-up discoveries.

Subsequently we will introduce our own problem posed in accord to these principles.

**Example 1 – Connection to specific educational content**

The following problem was used several times in research in the Czech Republic: The number \( N \) (e.g. 3080) is the product of two consecutive natural numbers. Find these numbers (Hošpesová, 2002).

On every occasion when students from various types and levels of schools were presented with the problems and offered the use of ICT, they tried to model the situation by replication of the process of formation of the number – multiplication of the selected numbers (Fig. 1). Nobody used a quadratic equation or square roots.

![Fig. 1](image-url)

This problem is a convenient tool for demonstration of the importance of a square root when looking for an approximate solution and for the introduction of quadratic equations. It is also a good example to demonstrate the value of considering an inverse function when trying to discover the numerical solution to a problem.

However, we also came across the same situation when the teacher modified the problem and substituted a sum for the product. Students then used the same solving procedure as in case of multiplication. In this case we find that use of ICT is strongly counterproductive, despite the fact that from the perspective of general pedagogy the solving processes were similar in both cases. The problem, with a sum, can be easily solved by using the arithmetic mean, with which students are familiar. Here, the contribution to mathematical thinking is minimal or even none, so students do not learn any new knowledge.

**Example 2 – Use of computing power**

Use of ICT at lower and upper secondary school offers new possibilities if the computing power of ICT is used for graphical data processing. Computers are able to draw graphs of functions quickly and precisely and in such a way that they illustrate to students the graph of a function, even in...
dependence on parameters. The teacher and his/her students can discuss the influence of different parameters on the behaviour of the function directly from empirically obtained data.

This use of ICT is demonstrated on the graph of the quadratic function $f(x) = ax^2 + bx + c$. It is easy to explain the meaning of the parameters $a$ and $c$. The influence of the parameter $b$ is much more complex. When using ICT we can, in real time, draw the graph of $f$ depending on $b$. By mere observation of the graph behaviour, students can discover e.g. the fact that when changing parameter $b$, the vertex of the parabola will move on another parabola (see Fig. 2); this discovery can be then verified by calculation.

![Fig. 2](image)

In school practice, the teacher might use an unsuitable tool for drawing graphs and individual parts may be drawn in various scales (Fig. 3). Due to the simultaneous change both of the parameter and the scale, the graph of the function neither changes nor moves. Therefore the students fail to see the difference between the functions and the impact of ICT is counterproductive and results in misunderstanding.

![Fig. 3](image)

**Example 3 – Space for discoveries**

The computational strength might be also used for collection of a large amount of data that can then be used by students for observation and formulation of hypotheses. The following problem illustrates this: Decide which of the numbers 1 to 50 can be written as a sum of several (at least two) consecutive natural numbers.

A student’s solution uses the construction of the corresponding sums of all pairs, triples, ... (Fig. 4).
functions. Motivation to this task is the solution of the classical problem of the bullet-target type. The problem creating a suitable environment in which students can use ICT for discovering properties of bullet and the speed and trajectory of the target.

The problem does not represent a single task; it is a set of mutually related tasks. The difficulty of the tasks increases. To begin with, students are assigned problems that can be solved using mathematics taught at secondary school. However, this is certainly no limit to the potential of this problem – activities carried out in this problem can be naturally stretched to university mathematics, as will be demonstrated in this paper.

The main activity consists in the choice of the trajectory of target and the following calculation of the bullet trace approximation. The main aim of the activity for students is to learn about graphs of various (especially trigonometric) functions in an indirect way (e.g. they want the target to zigzag or run around).

The problem was prepared in three software versions: spreadsheet (MS Excel), CAS (TI Nspire) and dynamic geometry software (GeoGebra). This allows the teachers to select the environment of which their students have prior knowledge.

The activity in its basic design has a preparatory stage. In this phase, students use their prior knowledge from mathematics and physics for solving problems using a direct bullet trajectory. The activity requires the following mathematical knowledge: linear functions, Theorem of Pythagoras, circumference, length of a circular arch. Use of ICT is not required at this stage. In case of necessity to solve a quadratic equation, it is possible to use a computer/calculator or spreadsheets. Students work with concrete numbers.

The introduction to the main type of activities are video recordings of real situations in which “bullet” and “target” change their paths – air battle from the 2nd World War, touchdown from American football etc. The activity is introduced by the following questions posed by the teacher: What happens with the trajectories if the target changes the direction? How should the bullet behave, under which conditions can it hit the target? Where should the bullet direct?

Subsequently, the students are assigned more complicated tasks that cannot be solved outright. These problems demand that the bullet should continuously be oriented on the target and the solution is constructed numerically. In given intervals, the bullet redirects on the target and modifies its path. The solution is modeled with the help of various tools – MS Excel (Fig. 5), Geogebra (Fig. 6),

|   | A | B | C | D | E | F | G | H | J | K | L | M | N | O | P | Q | R | S | T | U | V | X | Y | Z |
| 1 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 3 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 4 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 5 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 7 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 8 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 9 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

Fig. 4

The obtained data do not only enable the students to answer the original question but also provide plenty of space for their own discoveries. They may e.g. discuss the question of what all numbers obtained as a sum of two, three, four etc. consecutive natural numbers look like and then say what all numbers that cannot be written as a sum of several consecutive numbers look like. The answer to this question may be found with thanks to the sufficient amount of data offered by computer, and by their analysis.

Bullet and target

Bullet and target is an example of a complex problem meeting all the above mentioned principles for use of ICT. The activity was developed by the authors in the Socrates Comenius project EdUmatics as a problem creating a suitable environment in which students can use ICT for discovering properties of functions. Motivation to this task is the solution of the classical problem of the bullet-target type. The problem is assigned to secondary school students in different modifications. The students’ task is to simulate an optimum path of the bullet under a set of given conditions – the initial point, speed of the bullet and the speed and trajectory of the target.

The introduction to the main type of activities are video recordings of real situations in which “bullet” and “target” change their paths – air battle from the 2nd World War, touchdown from American football etc. The activity is introduced by the following questions posed by the teacher: What happens with the trajectories if the target changes the direction? How should the bullet behave, under which conditions can it hit the target? Where should the bullet direct?

Subsequently, the students are assigned more complicated tasks that cannot be solved outright. These problems demand that the bullet should continuously be oriented on the target and the solution is constructed numerically. In given intervals, the bullet redirects on the target and modifies its path. The solution is modeled with the help of various tools – MS Excel (Fig. 5), Geogebra (Fig. 6),
TI Nspire. The students’ task is not only to model the bullet path, but also to model the requested trajectories of the target with the help of known mathematical formulas.

Extension 1: What about the situation when the target moves on a sine curve?

In more complicated cases, the paper-and-pencil calculations of the curve become more and more complex. The necessity to use ICT arises naturally from the development of the activity.

Students first derive the formula for one step. They assign the target a position and the program calculates the position of the bullet after this step. Using the formula, students themselves create a model of the situation in the selected program. The data are stored in a table with 4 columns (two for the position of the bullet, two for the target). In the beginning, both positions are entered manually; later students program the change of the position of the target and the program calculates the position of the bullet. Students propose various paths for the target and represent them by a function (e.g. a piecewise linear function, absolute value, trigonometric functions with various parameters); the changes in the path of the bullet are observed. The changes of parameters leading to e.g. the graph of the target “higher”, “denser” etc. are sought.

The activity provides a lot of space for discussion and modification of the assignment according to students’ proposals. There are various modifications and extensions of this activity. Some examples of such questions follow:

- The bullet moves at a constant speed.
  - What is the speed of the target?
  - Is it also constant?
  - Is it possible to calculate it?
  - When is the speed lowest/highest?
  - What are the changes in acceleration when the speed changes?

Students may observe/analyze the differences between the cases of constant speed and varying speed, the influence of the slope of the function, the relationship between the tangent and instantaneous speed etc. The meaning of the point of inflexion as the transition between speeding-up and slowing-down becomes evident.

**Experience from piloting**

The problem was piloted in the Czech Republic with students of the 9th grade in compulsory mathematics lessons and in France with students of the 10th grade in the optional module Methods and Scientific Practices. The teachers chose different approaches depending on the participating students’ experience with ICT. In the Czech Republic, students worked with MS Excel where they entered all points of the trajectory from the table into a graph. In France, students modelled the trajectory using macros in GeoGebra.

In every lesson a new aspect of the potential of the activity for mathematics education is disclosed. These aspects touch upon various areas in a diversity of functional relationships, addressing a lot of grade 9 and 10 mathematics and preparing students’ further development.
Concluding remark

Use of ICT in teaching and learning mathematics does not necessarily mean it’s improvement. It is still up to the teacher to contemplate on what the use of ICT brings and how to involve ICT in the educational process. Assigning complex problems using ICT, as we demonstrate in the problem presented above, offers students a new way to approach mathematical problems. Students learn to model real situations, find approximate numerical solutions and gain experience with input data. Simultaneously, problems are both linked with the current subject matter – in this case functions and also have the role of preparation for calculus and integral calculus.

References


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Implementing the Dynamic Geometry Approach in Classrooms

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Abstract: The reported study compares effects of the dynamic geometry approach with standard instruction that does not make use of computer tools. The basic hypothesis of the study is that the DG approach results in better geometry learning for most students. The study tests that hypothesis by assessing student learning in classrooms randomly assigned to treatment and control groups. Student learning is assessed by a geometry test, a conjecturing-proving test, and a measure of student beliefs about the nature of geometry and mathematics in general. Data for answering the research questions of the study are analyzed mainly by appropriate HLM methods. The analysis on the geometry test data is discussed in detail. The experimental group significantly outperformed the control group in geometry performance.

Steven Rasmussen (the founder and CEO of Key Curriculum Press) and Nicholas Jackiw (the developer of the Geometer’s Sketchpad) first invented the term “Dynamic Geometry”, which denotes computer-based interactive geometry in which students and teachers explore mathematical ideas through active experimentation such as dragging, measuring, observing, conjecturing, conjecture testing, and reasoning. We are interested in thoroughly investigate the efficacy of an approach to high school geometry that utilizes Dynamic Geometry (DG) software and supporting instructional materials to supplement ordinary instructional practices, which we refer to as the DG approach.

A four-year research project has been funded by the US National Science Foundation to conduct repeated randomized control trials of the DG approach. The primary goal of the research project is to investigate the efficacy of the DG approach on students’ geometry learning over the course of a full school year. Together with other team members, we have just completed our project year 2 – a year in which we conducted the first implementation (the first of two consecutive implementations) of the DG treatment and related data collection and initial data analysis.

Rationale

Along with the widespread use of DG software, many related research studies (e.g., Choi-Koh, 1999; Gerretson, 2004; Vincent, 2005; Hollebrands, 2007; Baccaglini-Frank and Mariotti, 2010) have been conducted. According to these research studies, if DG software is used effectively, it can make significant difference to students’ learning; when used as a cognitive tool, it can facilitate students’ exploration and investigation activities, promote their conjecturing, verifying, explaining, and logical reasoning spirits and abilities, and enhance their conceptual understanding of important geometric ideas. This project will build upon these studies. However, almost all of the studies were either exploratory phenomenological studies that involved a small number of participants, or comparative studies that were conducted during a relatively short period of time (ranging from a week to less than one semester).

The exploratory qualitative studies are important since they can reveal students’ actual, detailed learning processes. However, there is a need for education research design using modern statistical methodologies “if the quality of education research is to meet the requirements that government policies and societal expectations are placing upon it” (American Statistical Association, 2007, p.44). If one wants to find the convincing efficacy of DG, quantitative comparative studies are necessary (Schneider et al, 2007). To address the potential threats to internal (and external) validity, true experimental designs with randomized assignment to treatment and comparison conditions should be used whenever and wherever possible. To find significant development of students’ geometric thought such as having moved from one mathematical thinking level up to the next level, a relatively long term of instruction (such as a full school year) is very much needed.

Theoretical Foundations

The constructivist perspective suggests that knowledge cannot be passively transmitted from one individual to another but is actively constructed by the learners themselves (Steffe & Cobb,
1988). The traditional approach to geometry instruction is teacher centered and based on definitions, theorems, and proofs, with little attention to whether students understand teacher’s lecture. In contrast, the DG approach to geometry instruction is based on students’ experimentation, observation, data recording, conjecturing, and proving. As Olive (1998) indicates, “Such an approach would give students the opportunity to engage in mathematics as mathematicians, not merely as passive recipients of someone else’s mathematics knowledge. From a constructivist point of view, this is the only way children can learn mathematics” (p.399).

van Hiele (1986) postulated that students progressed through a sequence of five discrete thought levels in geometric reasoning. The five levels are: 1. Recognition, 2. Analysis, 3. Order, 4. Deduction, and 5. Rigor. According to the van Hiele theory, the main reason the traditional geometry curriculum fails is that it is presented at a higher level than those of the students (de Villiers, 1999). The DG learning environment is a suitable environment in which students can explore geometry at their geometric thinking levels. Teachers can prepare activities that match students’ current van Hiele levels so that students can continue their explorations with little help from their teachers and make the transition to the next higher level.

Research Design

The research study follows a mixed-methods, multi-site randomized cluster design, with teachers as the unit of randomization. 76 geometry teachers from high schools in five Central Texas school districts participated in the project initially. Half of them were randomly assigned to comprise the experimental group in our study, while the other half of the teachers were assigned to comprise the control group. For schools where the selected teachers teach more than one class, only one class per teacher was randomly selected to participate in the study. Therefore each teacher is represented in the study with measurements from only one classroom of students, and the classroom and teacher unit of analysis will overlap, yielding the design where the students are nested within teachers/classrooms, which are nested within schools.

In this project, the DG software used is mainly the Geometer’s Sketchpad (GSP). The 38 teachers randomly assigned to the experimental group have participated in a GSP workshop, of which the main part was conducted in the summer of project year 1. Six half-day Saturday sessions were conducted in the 2010-2011 school year (project year 2) as the follow-up of the summer institute. The control group is a “business-as-usual” group. The teachers in this group teach as before. They also participated in a workshop, in which the same mathematical content taught in the GSP workshop was introduced to them, in a non-GSP environment. The amount of instructional time spent on this regular workshop was the same as that for the GSP workshop.

Measures and Data Collection

The instruments used for students’ knowledge and skills tests were a geometry pretest and a geometry posttest. The geometry pretest is Entering Geometry Test (ENT), which was first used by Usiskin (1982) and his research team at University of Chicago. The reason for using ENT for the pretest was that this test had been used by numerous studies on students’ geometry learning over the past 28 years, and had been considered as a good and easy-to-administer multiple-choice geometry test to assess students’ geometric background before entering a full-year high school geometry course.

The geometry posttest is End of Geometry-Course Test (EGT), which consists of the selected items from released questions from California Standards Test: Geometry (CSTG). The reason of using selected items from CSTG for the posttest was: All questions on the California Standards Tests were evaluated by committees of content experts, including teachers and administrators, to ensure their appropriateness for measuring the California academic content standards in Geometry (California Department of Education, 2009).

Student-level measures also included a Conjecturing-Proving Test and a student belief questionnaire. Both of them were developed by the project team. The Conjecturing-Proving Test was used as a pilot test at the end of 2010-2011 school year, and the data will be analyzed mainly for establishing the validity and reliability of the measure. The student belief questionnaire was developed to measure student beliefs about the nature of geometry and their beliefs about the
nature of mathematics in general, and used as both the pretest and the posttest. It was adapted from the mathematics version of the Views about Sciences Survey (Halloun, 1996).

To determine how to capture the critical features of the DG approach, we designed measures of fidelity of implementation - both a DG implementation questionnaire and a classroom observation instrument. The DG implementation questionnaire was adapted from a teacher questionnaire developed by the University of Chicago researchers (Dr. Jeanne Century and her colleagues) in an NSF funded project. We made significant changes and additions to address the extent to which the DG approach was implemented by the experimental group teachers. An equivalent but different version of the questionnaire was administered with the control group teachers to examine the degree to which they faithfully implemented the business-as-usual approach. The classroom observation instrument – the Geometry Teaching Observation Protocol (GTOP) was developed by adapting the Reformed Teaching Observation Protocol (Piburn et al, 2000) based on the critical features of the DG approach. GTOP was used for both groups of teachers. The scores provided data to compare the teachers’ teaching styles and strategies.

To probe more deeply into the teachers’ and students’ thinking processes, and to gather evidence about the range and variability of students’ development of the most important abilities that the DG approach fosters, this study used in-depth interviews of selected students and teachers to collect qualitative data. Interview protocols were designed and used for the interviews.

This article concentrates on the student geometry post-test (EGT) to address our primary research question: How do students in the experimental condition perform in comparison with students in the control condition on geometry performance? When developing EGT, the project team worked carefully in selecting the items from CSTG that were closely aligned with Texas geometry standards and the geometry curricula of the participating school districts. 30 items were chosen and pilot-tested in the non-project classes at a participating high school. Based on the pilot-test results and the feedback of the master teachers (who are high school geometry curriculum and instruction experts working for the project), five items were removed. The final version for EGT has 25 multiple-choice items. Based upon the pilot results, the instrument has high reliability (α = .875). Factor analysis provided strong evidence that EGT corresponded to uni-dimensional scale.

Data Analysis

We have conducted some initial data analysis (the analysis on the geometry pretest and posttest data and the psychometric analysis on the project developed instruments). More thorough analysis of the collected data is still going and will be conducted during project year 3. An IRT analysis on the geometry posttest data and the outcome analysis on the geometry pretest and posttest data are reported below.

Upon scoring the 25 items for each student in the sample, the resulting right/wrong data was calibrated under the Item Response Theory (IRT) framework. IRT allows for the development of item indices independent of the examinee group, examinee ‘abilities’ independent of the item parameters, among other features such as individual ability-specific measures of precision (Hambleton, Swaminathan & Rogers, 1991).

Two-level hierarchical linear modeling (HLM) was employed to model the impact of the use of the Dynamic Geometry approach on student achievement while taking into account the nested structure of the data (i.e. students nested within teachers’ classrooms). The student posttest (EGT) data were first analyzed without pretest (ENT) scores included as a covariate. The rationale for excluding the ENT scores was that teachers were randomly assigned to each of the treatment groups, so pretest control was not required to determine accurate estimates of the treatment effect. In fact, a separate analysis of the pretest, not presented here, showed no significant difference (p = .724) between the two treatment groups. To see if we could find consistent results, however, we also analyzed the EGT data with ENT scores included as a covariate.

Results

IRT analysis results

The data were calibrated using a two parameter logistic (2PL) model and a three parameter logistic (3PL) model. A two-parameter model estimates two item parameters – difficulty and
discrimination. A more difficult item requires more of the student latent trait to attain a correct answer (in this case, the latent trait is Geometry ability). Conversely, an easier item requires less of the latent trait to have a higher probability of answering correctly. A highly discriminating item is able to differentiate between students with different levels of ability, whereas an item with low discriminating is less able to differentiate between students with lower levels of ability and higher levels of ability. The additional parameter in the 3PL model estimates a guessing parameter, representing the probability that examinees with very low ability answering an item correctly (presumably due to guessing correctly). Although residual analysis suggested that the 2-PL model may be a better fit, the 3PL item parameter estimates support the inclusion of a guessing parameter. The guessing parameters were generally low (which means low ability students do not have high probabilities of getting items correct by guessing) but were far enough from zero that they should not be ignored (as would be required in the 2PL model).

Thus, item parameter estimates (and student estimates of ability) were adopted from the 3PL calibration. The item parameter estimates for all 25 items on the DG geometry posttest were generated. As an example, item 6 (see Figure 1) on the assessment is a particularly well-performing item. The difficulty estimate was moderate; the discrimination estimate was relatively high; while the guessing parameter was relatively low. These estimates suggest that approximately half of the students will probably get this item correct, that the item will do a good job of discriminating between low and high ability students, and low ability students have a low probability of guessing correctly. As another example, the guessing parameter estimate for item 1 was noticeably high (.46). Closer examination of the response frequencies across the answer choices showed that nearly 94% of students selected option A, the correct answer, or option C. Since response options B and D were chosen so infrequently, item 1 in essence had only two viable response options, which may explain why the guessing parameter estimate was so close to .5. Thus, this item should be considered for further review.

![6.)](image)

Figure 1. Item 6 of the geometry posttest (EGT)

In summary, the IRT analysis generated examinee 'abilities' and item parameters. The item parameters allowed us to determine how well each of the items on the post-test performed. Aside from a few items (such as item 1) in particular, collectively the items included on the posttest provided a range of performance that holistically represented a well-functioning instrument. The adherence of the data to the three-parameter logistic IRT model, which has known psychometric properties, provided evidence for the assessment's construct validity. One of the important properties of this model is that the assessment measures a single construct, which in this case is dynamic geometry. No major violations of uni-dimensionality were noted with this DG assessment using a multidimensional scaling procedure.

**HLM analysis results**

The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking Pre-AP Geometry). Since the classroom expectations and quality of the students in each of these levels is very different, the factor Class Level
was included in each HLM model (with or without pretest as a covariate). Additionally, the years of classroom experience of the teachers in the sample varied a lot, ranging from 0 years all the way up to 35 years. Given the emphasis of technology in the study, entering the study the possible effect of teaching experience was unclear. A more experienced teacher may have greater command of the classroom but be less able to implement the technology. For this reason, the covariate Years Exp (number of years of classroom experience) was included in the models.

During the project year 1 professional development workshop, the participating teachers completed a demographic survey that included information about years of teaching experience, the level of the class chosen and gender. From our initial teacher sample (76 participants), six teachers didn’t compete project year 2 mainly due to either family/health or job displacement reasons. Additional six teachers submitted incomplete posttest data or failed to submit the data. Therefore, 64 teachers submitted complete posttest data for analysis in the study. Among them, 33 were in the experimental group (DG group), and 31 were in the control group.

For both HLM models described above, full factorial designs were explored and insignificant interactions were discarded. The final models are discussed below.

**HLM results without pretest as a covariate:** Model 1, shown in Table 1, examines the differences in student outcomes between the DG group and the control group while accounting for years of teaching experience and level of the class. The DG group significantly outperformed the control group (p = .000, ES = .3327). As expected, level of the class was highly significant as well (p = .000). Due to the coding of variables, the intercept reflects the Middle School Pre-AP group performance. Examining the coefficients in Table 1 and the mean values in Table 2, we see that the Middle School group substantially outperformed the high school Pre-AP students, who in turn outperformed the students in Regular Geometry. In particular, controlling for experience of the teacher and treatment group, the Pre-AP students scored 21.15 points lower than the Middle School students, while the Regular students scored a full 35.1 points lower than the Middle School group.

Though not indicated in Table 1, the difference between the Pre-AP and Regular students is statistically significant (p = .000). Interestingly, the effect of years of experience differed by level of the class as well. Experience had a positive effect on the two higher performing groups, but had a negative effect on the achievement of the students in Regular Geometry classes. However, the effect of experience in the middle school group was not significant and the size of the coefficients in all groups is somewhat small. For the Regular group an increase of 10 years of experience corresponded to a drop of 3.2 points on the EGT, while for the Pre-AP group a similar change in experience was associated with a 4.8 point increase. Compare this to 7.4 point increase of the DG effect. Note the main effect for Years Exp is not shown in Table 1. A model including this effect was considered, but it was insignificant.

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>Standard Error</th>
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<th>Approx d.f.</th>
<th>p-value</th>
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<tr>
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<td>DG Effect</td>
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<td>-5.587</td>
<td>88</td>
<td>.000</td>
</tr>
<tr>
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<td>-3.221</td>
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<td>.002</td>
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<td>Level*Years Exp</td>
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<td></td>
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<td>91</td>
<td>.740</td>
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Table 1. HLM Results without Pretest as a Covariate

Table 2 shows the summary statistics for each level of class separately. The DG group significantly outperformed the control group in each level, but the effect size was largest for the students in the Regular Geometry classes.
<table>
<thead>
<tr>
<th>Level of Class</th>
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<th>Control</th>
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<tr>
<td></td>
<td>n</td>
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<td>Middle School</td>
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<td>88.25</td>
</tr>
</tbody>
</table>

Table 2. Summary Statistics for EGT by Treatment and Level

**HLM results with pretest as a Covariate:** Model 2 (not shown here) examines the effect of the DG intervention when taking into account entering geometry test (ENT) as well as Class Level and Years Exp. As with Model 1, the results for Model 2 indicate the DG effect was strongly significant (p=.002), and the effect size was largest for the students in the Regular Geometry classes.

**Discussion**

The HLM data analysis model showed that the Dynamic Geometry group significantly outperformed the control group in geometry achievement. This project used random assignment to form the treatment and control groups. The teachers in the control group also attended a professional development workshop. The amount of instructional time spent on this regular workshop was the same as that for the GSP workshop offered to the DG group teachers. The purpose of holding this workshop was to address a confounding variable. With this comparable amount of professional development, if differences appear on the project’s measures between the treatment and control groups, we are able to rule out the possibility that the professional development activities can account for them rather than the DG learning environment. This true control group, in addition to random assignment, provides strong evidence to support the finding that the DG approach did make a difference – it did cause the improved geometry achievement observed in the study. In the first efficacy study on the DG approach at a moderately large scale in the nation, this finding is a noteworthy contribution to the field of mathematics education.

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Working in the 21st Century: Moving teacher professional development online

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2010 began a time of change for senior secondary education in New Zealand as schools and teachers began progressively introducing the revised New Zealand Curriculum with senior classes and preparing for associated changes in assessment for the National Certificate in Educational Achievement (NCEA), the national school qualification. The Ministry of Education (MOE), in association with the New Zealand Qualifications Authority (NZQA), has been reviewing all curriculum-related NCEA achievement standards so they are aligned to The New Zealand Curriculum. To help teachers with these changes, the MOE is providing best practise assessment resources for each new NCEA Level 1 standard. (NCEA Level 1 has both internal and external assessment components, and is generally introduced at Year 11). The New Zealand Association of Mathematics Teachers (NZAMT) supports teachers by providing secure, quality assessment resources. This paper explores the way in which NZAMT used a combination of synchronous and asynchronous online platforms to develop teachers’ competency in using the new standards.

Introduction

The revised New Zealand Curriculum (NZC) for primary and secondary schools was launched in November 2007 and became mandatory for all students from year 1 to year 10 from the beginning of 2010. The revised curriculum was developed through an extensive consultation process which included a broad cross section of stakeholders, including business representatives, employers and leading entrepreneurs. This process provided the curriculum with an agreed set of competencies to enable students to participate effectively as adults in a 21st century global economy and society in its broadest sense.

In schools themselves, the curriculum provides a framework within which teachers and principals can select the curriculum content that best suits their students. This approach ensures that what schools teach has an external, outward looking, competency-based focus encouraging a more challenging rigorous approach.

The revised curriculum means a corresponding realignment of the achievement standards that form the basis for assessment for the National Certificate of Educational Achievement (NCEA), the national school qualification.

This alignment will ensure that the key outcomes to be assessed for NCEA are the same outcomes on which the teaching and learning programmes are based. Previous experience in changing senior secondary school assessment practices has shown how important it is to ensure that the implementation of the changes is aligned with the capacity of the system, and the people in that system, to deliver. Accordingly, the implementation of the standards aligned with The New Zealand Curriculum is being phased in over three years from 2011.

This phased implementation allows time for assessment resources and exemplars to be developed, and it also acknowledges the scale and nature of the change in subject areas. It also allows the process to be managed in a way that will ensure the on going quality of the NCEA as our national secondary school qualification.

The Ministry of Education, in partnership with NZQA and the educational publishing company Learning Media Ltd, is developing assessment resources based on the aligned standards. These resources are being trialled in schools, and publication of the resources will include student exemplars to help teachers and schools clarify the grade distinctions for the standards.

Mathematics and statistics is one of the learning areas where there have been significant curriculum changes and thus changes in standards as they are aligned to the new curriculum. The curriculum changes mean teachers must develop their pedagogical content knowledge and make changes to their classroom practice – something which takes time and professional development support.
Face to Face professional development:

In 2010, to support teachers as they prepared for to use the new achievement standards for the first time in 2011, the Ministry of Education, through contracted professional development providers, offered a series of workshops for the teachers of mathematics and statistics. Centrally generated material was delivered on a regional basis, with all schools able to send heads of departments or faculty to a series of face-to-face workshops. The participants attended four workshops to assist them develop their practice with the realigned mathematics and statistics standards at NCEA Level 1. The professional development emphasised the issues and implications around new standards with the consequential changes needed in teaching and learning programmes.

A significant change in the mathematics and statistics standards, and one that was a key focus of the workshops, is the notion that grade distinctions must be based on qualitative differences in achievement rather than students being required to acquire and retain more subject-specific knowledge. To express the qualitative differences in achievement, the mathematics and statistics standards writers used a framework that was loosely based on the SOLO thinking taxonomy (Biggs & Collis, 1982). This approach is different from that used until 2011 when achievement was quantified, at each level, by more and often harder or different content. Thus the draft assessment activities developed by the MOE to exemplify this change were critical for teachers. Unsurprisingly, it was only when they saw these activities that many teachers began to understand the impact of the change and consequential implications.

Each of the four professional development workshops focused on a particular standard. Exploring the curriculum underlying the standard, its connections to the SOLO taxonomy, and examining the levels of thinking in student work needed to meet the standard. In the period between workshops, which could be up to six weeks, participants were expected to experiment with one of the draft assessment resources with their own students and to run follow-up sessions for teachers from their department who had not attended the workshops. Teachers who attended all the workshops indicated that they had enhanced their knowledge of the curriculum, standards and the SOLO framework. Additionally, many reported that they would appreciate further sessions in 2011, all of which indicated that the sessions were adding value.

In 2010, the New Zealand Association of Mathematics Teachers (NZAMT) writing group had begun working online and as a consequence of feedback from these, it was decided that three sessions, focusing on the SOLO framework, be offered to the wider mathematics community. These were developed and facilitated by one of the contracted professional development providers late in 2010 (using a synchronous platform called dimdim) with a secure forum space for discussion in the periods between the presentations. The forum was located on the MOE-funded website NZMaths. There was considerable interest and, despite problems with the synchronous platform and very little use of the forum, the feedback was extremely positive, and it was the firm wish of those who participated for such sessions to continue.

The mathematics and statistics community goes online

Historically NZAMT has supported teachers by providing them with secure assessment resources. These resources were usually developed during a week-long residential workshop held in January of each year. Previously the focus was on activities to support the assessment of what were called unit standards. However, the alignment of standards to the revised curriculum meant many of these unit standards would no longer be available for schools to use in the future. Hence NZAMT decided a much better way to support schools in 2011 would be to produce activities that could be used in the assessment of the new achievement standards.

The challenge for the writing group in January 2011 would be that there were not enough teachers with the detailed understanding of the new achievement standards and how they fitted with the curriculum objectives to write quality resources. One solution to this problem would have been to bring a group of teachers together during 2010 to provide them with the necessary professional development. However, the costs of bringing a face to face group together in 2010 were both prohibitive and the time constraints unrealistic.
Thus it was decided that the solution to providing such professional development, in an efficient, timely and cost effective way, would be to provide an online environment where teachers could work together. The structure would be based on the successful face-to-face professional development that advisors had delivered during 2010 (discussed above), with four facilitated synchronous one hour workshops, held two weeks apart, focussing on a particular achievement standard. There would be a facilitated online space where artefacts could be lodged and where discussions could continue between the live sessions. For this to be successful, it would be essential that the outcomes from the project be clear and tightly defined. These outcomes were as follows:

- two assessment tasks would be produced to a ‘ready-to-use state’ for use by teachers
- an online community of teachers from different schools and locations would be developed; these teachers would be confident in their dealings with each other and the new achievement standards which can be viewed at http://www.nzqa.govt.nz/ncea/assessment/search.do?query=mathematics&view=achievements&level=01
- participants would develop competence in the use of the web conferencing software
- participants would have a shared understanding of the SOLO taxonomy on which the achievement standards are based
- participants would have a shared understanding of the holistic nature of the marking required for the internally assessed tasks.

The initial design was extremely informal. Volunteer participants were called for; knowledge of and the ability to work online or total understanding of the new standards was not a prerequisite; the only criterion was the willingness to participate in the process of generating resources. This design presupposed that the volunteers would be motivated to engage in the material and, in fact, this has mostly proven to be the case – I believe this is not only because teachers see an intrinsic value in writing tasks but also because task-writing that supports the curriculum will be a vital ingredient for success in their own classrooms in 2011.

An initial survey of the teachers involved produced an interesting profile of skills and competencies, both with the online environment and with the newly developed standards and assessment materials. For example whereas most participants were familiar with email and google docs, over 80% had little or no experience with web conferencing, video conferencing or learning management systems such as moodle.

Research tells us that learners usually don’t interact online unless they see a good reason to engage. With any learning, but particularly for teacher professional development, the focus should be on doing. Based on their results from a national sample of teachers in the United States Garet et al (2001) summed up the importance of engagement as follows:

Our results provide support for previous speculation about the importance of collective participation and the coherence of professional development activities. Activities that are linked to teachers’ other experiences, aligned with other reform efforts, and encouraging of professional communication among teachers appear to support change in teaching practice, even after the effects of enhanced knowledge and skills are taken into account.

In order to produce the best final output this engagement may take a number of forms; including negotiating, critiquing, debating, arguing or agreeing. In the case of the New Zealand mathematics and statistics teachers, this involvement enabled the group to be resilient enough to overcome a number of the inevitable technology issues.

During the first round of resource writing there were up to 16 teachers (this varied from session to session), 2 teacher advisors and myself involved. As mentioned earlier, it was envisaged that the group would participate in four synchronous one-hour meetings held two weeks apart (in a synchronous meeting room with recording capability, ellluminate). Between meetings there was ‘homework’ which involved further developments of tasks, revisions, and forum discussions using email and posting in a wikispace (asynchronous). The wikispace was a late addition and was used as it was the only suitable platform available as a repository for materials with a place for discussions. Although the wikispace design may use familiar technology, facilitating this process needs time and
attention to detail for it to be a success. Although online technology can be a great enabler, unresolved issues or problems can quickly have the opposite effect. Consequently, and in hindsight, the background effort and time required to make the wikispace happen successfully was underestimated.

For similar reasons, time was spent familiarising and establishing usability of the synchronous platform during all the online workshops. The main focus of the first workshop was a facilitated discussion on the SOLO taxonomy and the design of the new achievement standards, as well as establishing how the group would proceed with the writing of the resources. A wikispace was created containing all the documents and ideas for viewing, downloading and discussion. The next two workshops involved separating the group into two smaller groups, with one advisor assigned to each to lead the development of the assessment tasks. These advisors took the lead role and worked with the teachers as they refined the tasks, the marking schedules and the annotation of student work that provided exemplars of grade boundaries. Although this structure was not part of the original design, it freed me up to facilitate the virtual room, fix any technical problems and to give guidance both with tasks and with any official information when needed. As a consequence, by the third workshop, the groups were firmly established and needed little guidance from me. Before the fourth workshop, the NZAMT organising group revisited what had happened and began to set out a more planned road map for the next round of assessment writing.

Conclusion

Jennifer Lock (2006) speaks of the importance of taking a new view of professional development in an online environment:

The realization of online learning communities to facilitate teacher professional development is a matter of carefully and deliberatively designing dynamic learning environments that foster a learning culture. This requires a pedagogical framework that nurtures the establishment of relationships, intimacy, and trust, where people engage in shared learning experiences mediated through technology. Designing an online learning environment that fosters the development of a learning community is not about adding technology on to current professional development practices. Rather, it is about designing, building, and supporting a structure and a process that are purposeful and fluid in nature and in meeting the personal ongoing professional development needs of teachers.

The NZAMT approach was initially informal and drew on the successful face-to-face professional development that advisors had delivered during 2010. As the sessions progressed, it developed its own momentum, and building on this experience enables the development of an enhanced structure for future online professional development.

These enhancements would include:

- selecting the correct online environment - this is critical to the success of the professional development as is careful preparation in advance of teacher participation
- providing clearly reasoned, articulated and documented outcomes which set the parameters for the professional development
- developing specific resources for use in the real classroom environments which provides a focus that is motivating for teachers
- providing a first session on the SOLO taxonomy directly focused on the standard being used in the development of material, thereby cutting down on facilitator discussion and input at later stages of the task writing
- improving the design of the wiki or asynchronous working space.

Additionally my preference would be to move the group to a learning management system LMS (such as moodle) which would enable a more efficient organisational structure, with better reporting and logs of usage. At this stage this might be a step too far for some of the teachers involved but will be worth investigating in the future. Either way, nested pages, better navigation and a “What’s New” section on the front of the wiki will all be part of the build for future online work.
This experience has shown that an online environment is an extremely cost- and time-efficient way of enabling scarce resources to be targeted effectively while at the same time mitigating issues of time, availability and the problems of geographic separation of participants. Experiencing the process itself provides teachers with both a window on, and a learning experience for, other forms of learning in the future.

As a result of the online writing groups work in 2010, six teachers plus myself were able to attend the face-to-face residential session in January of 2011 and, while working with six others and bringing them up to speed, we managed to develop 13 resources for teachers.

This successful outcome has meant that NZAMT has plans to continue working with teachers in 2011, using the model developed in 2010. The current proposal being discussed includes groups who write activities online for the next level of achievement standards to be registered (NCEA Level 2 standards) and other presentations which will include the SOLO taxonomy and how to use the new resources developed by NZAMT.

The building of an online community developed a learning community that lasted beyond the life of the project. There is no doubt that by their involvement in the development of assessment tasks for use in the classroom, the teachers involved in this online writing group have engaged with the underlying taxonomy and the new standards in a richer way than they expected and they are looking forward to future online professional development.

Perhaps the final word should go to the teachers involved:

“I found the experience of the NZAMT writing group to be amazing!! While it is always good to get together with colleagues and discuss issues, the most amazing thing to me was the technology. ... I borrowed a microphone / ear piece from school and suddenly I could be in contact with Mathematics teaching professionals around the country. The personalities of people involved came through with jokes as well as serious issues - by being able to speak, type, put a hand up, give a tick or cross, smiley face etc., communication was great. It was MUCH better than the teleconference thing where you get distracted because you can see others (or, worse, myself !) and completely lose the train of thought.

I started using a wiki, but (my problem really) am still not confident with this even though I added files, etc. There seemed to be too many threads of discussion – and some of that was my fault because I wasn’t sure how to add to a thread and seemed to start up my own. ... It was great to be able to take the material and work on it at a time that suited me – if I felt enthusiastic immediately after the discussion I could do that. Or I could go and have a break and come back to it that night or a week later.” – Gwenda

“The mechanics of the use of the software/virtual room was straightforward, and required little time to become familiar with after a brief initial instruction. The sessions were managed in such a way as to allow people to hear what others had to contribute and the facilitator could move things along as needed. The whole process was extremely valuable as it allowed capable people with an interest in the given topic throughout New Zealand to [have] input. It was particularly valuable when material had been distributed, people had worked through, and then the session allowed feedback based on this material. The term one work on the new AS has gone really well for us. I was a bit apprehensive, but the AS have been good and it has been a case of adapting to change. The work I did with the writing was invaluable.” – Lewis

“The use of illuminate in our online writing group was fantastic and very easy to use. It was nice to be able to talk to a whole group without the room becoming “cluttered”. The white board where ideas were pasted and participants could write comments was fantastic for this purpose. Regular meetings made the topic (writing assessments) “do-able” as we would discuss all points including problems then leave for two weeks to prepare our answers. In this way we got around the problem areas in a neat, efficient manner. Two weekly meetings meant it did not become too time consuming and meant I could participate fully yet still cope with my normal (excessive) teaching work. On line meetings meant I did not have the problems of travelling to a venue and spending maybe all day there as would have normally been the case if I had wanted to participate in such an activity.

The fact that our team consisted of teachers from all over NZ meant we could establish a working friendship with other teachers we might never, otherwise, have met. It also meant that the team, included teachers from all parts of NZ and all types of schools in NZ.

I had never used either Illuminate of Wiki Space before so for me it was very educational and useful. I now use Wiki space for my class at school as a comment space and it works well.” – Denis
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Teachers’ designs with the use of digital tools as a means of redefining their relationship with the mathematics curriculum

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The present paper reports a study concerning the analysis of nineteen activity plans (we call them ‘scenarios’) developed by mathematics teacher educators-in-training for the pedagogical use of digital tools. The development of these scenarios took place during their training program and was designed as an activity for increasing reflection, for expressing creative pedagogical ideas and for an active engagement in the design of curricula enriched with the use of technology. Our analysis showed that the trainee teacher educators deconstructed and reconstructed respective parts of the formal curriculum regarding the mathematical concepts they chose to embody in their scenarios.

Theoretical framework

This paper focuses on the relationship between teachers and curriculum, approached through the study of the teaching designs – in the form of scenarios – developed by mathematics teacher educators-in-training. The study took place in the context of an in-service educational program at the University of Athens. The program was part of a large scale nationwide initiative of the Ministry of Education concerning teachers’ familiarization with the use of digital technologies for teaching and learning in their respective subjects. As for mathematics, the aim of the program was to provide the participants who were selected experienced mathematics teachers with methods, knowledge and experience in in-service teacher education and to educate them in the pedagogical uses of expressive digital media and communication technologies for the teaching and learning of mathematics. The Educational Technology Lab (ETL) had the responsibility to develop curricula for this program and the completion of training four educator groups in the University of Athens.

The Greek educational system is characterized by a central national curriculum in which goals, concepts and activities are explicitly described. Despite existing efforts to implement an interdisciplinary curriculum, the teaching of extended material continues to be linear. The concepts are sometimes introduced with activities. However, there are few provided representations and, still, with limited connections inside and outside mathematics. Even though there are abundant concepts, the conceptual fields (Vergnaud, 1996) finally constructed are particularly fragmented and rigid. The teacher has the role of the technical implementer of this curriculum, as she/he strictly follows the textbook, along with the guidelines of the special book addressed to the teacher. The use of technology, in spite of the fact that in some cases is proposed, it is rarely put into practice and when that happens takes the form of a simple demonstration. The integration of technology in the teaching and learning process implies new kinds of learning activity, which focuses on the generation of meanings, problem solving, students’ mathematizations, use of representations, creation of socio-cultural norms in the classroom (Kynigos et al, 2009). These aspects of the teaching and learning process, require the development of new curricula and change of the teacher’s role, not only as a facilitator to this kind of learning, but as a decision maker with an active role in designing innovative curricula (Budin, 1991) and this was one of the program’s purposes.

The teacher – curriculum relationship has been studied less than other fields in the domain of teacher education (Remillard, 2005). When curriculum is referred to, we have to distinguish between the intended curriculum (that which resides in state frameworks, guides, textbooks, and in teachers’ minds as they plan what they will do) and what it appears to be, which is curriculum as enacted by teachers in the classroom and curriculum as experienced by students (Gehrke et al, 1992). This distinction recognizes the significant role of the teacher as mediator on what is planned and what really takes place in the classroom. Very important factors that influence teachers’ mediation are their knowledge, beliefs and experiences. We consider that this mediation phase is of great importance, as it is not the practice itself, but it is directly interlinked with practice. According to Carlgren (1999) teachers should not be considered as reflective practitioners (in the sense of Schön) only for actions internal to the classroom, but for external ones as well. As such, she proposes the development of local curricula by the teachers, through reflective processes incorporating knowledge, beliefs and classroom experiences, which are transmitted to their colleagues as a kind of
The computational environment

The scenarios analyzed here involve the use of “Turtleworlds” a programmable Turtle Geometry medium designed to integrate formal mathematical notation with dynamic manipulation of variable values (Kynigos et al, 1997). In Turtleworlds, the elements of a geometrical construction can be expressed in a Logo procedure. After a variable procedure is defined and executed with a specific value for each variable, clicking the mouse on the turtle trace activates the “variation tool”, which provides a slider for each variable (see at the bottom of Figure 1). Dragging a slider has the effect of...
the figure dynamically changing DGS-style, as the value of the variable changes sequentially. The novel character of dynamic manipulation in Turtle Geometry is that what is manipulated is not the figure itself as in DGS, but the value of a variable of a procedure. The variation tool provides an alternative way to control changes that affect both the graphics and the symbolic expression through which it has been defined, combining these two kinds of representations which appear rather static in most other geometrical construction computational settings. Thus, employment of the variation tool provides opportunities for the students to take a reflective view on the geometrical construction process by confirming or dismissing conjectures derived from the use of the tool in conjunction with the other representations (e.g. by adding/erasing commands or variables, by expressing or modifying magnitudes with variables etc.).

**Method and research questions**

The research presented in this paper is part of a wider one, concerning the trainees’ choices in teaching design in different educational contexts. It is an intervention research, aiming to study the features of human artifacts produced in a specific educational context designed by us (Cobb et al., 2003). Our aim was firstly to gain insight into the nature of the teaching design of technology enhanced mathematics learning developed by the trainees in the form of scenarios based on the use of specially designed computational tools. Secondly, we were interested in investigating if and how these tools shaped the mathematical content of the respective scenarios. In particular, we were interested in identifying: a) The main characteristics of the teaching design (concepts involved, situations/contexts of tasks, connections within or outside mathematics, use of representations) and their relation to the characteristics of the formal curriculum, b) The way the trainees used the available tools as teachers to orchestrate learning situations that promote conceptual understanding and generation of mathematical meanings. Our emphasis was on the ways in which they integrated the dynamic manipulation feature of Turtleworlds in their design of tasks and if and how this integration favored the possibility of introducing new components in the formal curriculum. To analyse the scenarios we organized their content into tables. Every line of the table corresponded to a scenario and every column to the specific characteristics according to the scenario structure we described earlier. This process led to the categorization of the scenarios according to their topics. For each topic we traced the sequence of the chosen concepts through diagrams in order to clarify the connections between them. Afterwards, the teaching sequence was studied in each scenario and new tables related to the use of technology were created. Some of the parameters that we studied were the software tools that the teachers used for designing their microworlds and the potential contribution of these tools to the students’ meaningful engagement with the underlying mathematical concepts. In particular, the analysis of the anticipated students’ use of dynamic manipulation lead to the definition and refinement of new codes characterizing a number of qualitatively different dragging categories that are presented in the second part of the analysis.

**Results**

**A. Domains, topics and mathematical concepts**

Geometry almost monopolized teachers’ interest, since eighteen out of nineteen scenarios had Geometry as the starting point of their activities and only one had Trigonometry. Two kinds of connections were sought: a) Connections inside mathematics: The majority of scenarios (eight) were developed on a specific topic of Planar Geometry, with no further connections. Six of the scenarios connected Geometry with Trigonometry, three of them Geometry with Algebra and one Planar Geometry with Stereometry. b) Connections outside mathematics: There were no connections with other disciplines. We also examined the different situations and contexts upon which the teachers based their scenarios: four scenarios involved Arts as a context, eight scenarios were developed under specific situations of every-day life (jewels, wheel of the amusement park, houses, kites, building plots, sailing-boats and stairs) and seven scenarios were developed into the mathematics world. The nineteen scenarios were developed around six topics: regular polygons (6), triangles (6), quadrilateralers (4), slope of a straight line (1), similarity (1) and areas (1). A notable finding stemming from the examination of the 19 scenarios is that the trainees followed different paths for approaching the same topic not only regarding the targeted concepts but also the technological tools used. The
same mathematical concept could be included in different topics through the exploitation of its different properties. For example, the equilateral triangle appeared: a) in the topic of regular polygons, both as the polygon with the minimum number of sides and as the structural element for the construction of a regular hexagon, b) in the topic of triangles as a special case, c) in the topic of areas as a unit for covering plane shapes and d) in the domain of Algebra as a unit for Pascal’s triangle.

B. Phases of anticipated learning activities and students’ tool use

Two representative scenarios from the topic of regular polygons are analyzed in the next paragraph. Our aim is to indicate the variety of the teaching designs for the same topic involving also distinct differences in the exploitation of the available tools. In the Greek formal curriculum, this topic is approached through a formal definition of regular polygons which is followed by the construction of a regular polygon with n sides (i.e. through the division of the cycle’s periphery into n equal arcs) and the proof of the correctness of this construction. The description of the teaching sequence included in the scenarios S12 and S13 that we chose to present here highlights in each case the teaching process as anticipated by the designer according to his/her own choice of activities and tool use.

S12) «Let’s create regular polygons»: 1st phase: A) Process - activities: Three half-baked microworlds are given to the students, providing open jagged lines of 3, 4, 6 equal sides respectively (fig. 1: jagged line of 6 sides). The students work in groups and try to modify the code so that the lines close and create an equilateral triangle, a square and a regular hexagon respectively. B) The role of the tools: Students are expected to correspond the sliders of the variation tool to the variables of the code, by dragging them and observing the graphical outcome. The students are expected to recognize that the slider determining the line’s closure corresponds to the turtle’s turn. After moving the side’s slider on a certain value, the students are expected to experiment by dragging the turn’s slider to different values until they discover that the regular polygon is created when the turtle’s turn becomes 120°, 90°, 60° respectively. The students are expected to relate the turn to the polygon’s exterior angle and conjecture its relationship with the number of sides. Then they can modify the code by taking off the variable that corresponds to the turn and replace it by the functional relation 360/n. These changes can be further tested by executing the code and observing the graphical outcome.

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to line :n :b :c repeat :n [fd :b rt :c] end
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2nd phase: A) Process - activities: The students have to modify a given half-baked microworld, called “line” which constructs an open jagged line of n equal sides with length :b. B) The role of the tools: Students correspond the sliders of the variation tool to the variables of the code and ascertain that the slider which does not influence the closure of the line is the one of length, which is moved on a certain value. By dragging the slider (:c) of the exterior angle, they find out that for every value of n the line closes (the regular polygon whose exterior angle equals :c). By dragging the slider (:n) of number of sides, they find out the value of the angle which closes the line (the exterior angle of a regular polygon of n sides). They conjecture that the exterior angle which creates a regular polygon of :n sides, equals 360/n. They take off the variable :c from the code, replacing it by 360/n. Their conjecture can be verified by executing the code and taking the graphical feedback.

3rd phase: The scenario is extended to the circle, and then, to the circumscribed circle of a given regular polygon (the description cannot be given here due to the space limits).

S13) «The wheel of the amusement park»: 1st phase: A) Process – activities: The students have to modify a given half-baked microworld, called “scheme”, which constructs an open jagged line of three equal sides, length’s :a, in order to construct an equilateral triangle. B) The role of the tools: The students correspond the sliders to the variables and ascertain that the one which does not influence the line’s closure is the slider of length :a. They move it in a certain value. By dragging the sliders of angles :b and :c, they find out when the line closes. Then the students put forward the conjecture «each triangle with equal sides, has also equal angles». They are expected to recognize that the slider :d controls the turtle’s orientation for the construction of the equilateral triangles in succession. They modify the code, by eliminating the variables to one
and use the command repeat 3 [fd :a rt 120] for the construction of the equilateral triangle. The new code is called “triangle”.

**2nd phase:** A) Process - activities: A class discussion is scheduled to be conducted related to the way a regular hexagon derives from an equilateral triangle and the students are expected to create a microworld of 6 successive equilateral triangles with common vertex. B) The role of the tools: In order for the students to create the code “hexagon”, they initially correspond sliders to variables. The slider of the side’s length is moved to a constant value. Then, they drag the slider :b which ‘turns’ the equilateral triangle. If, for example, :b=10°, then six equilateral triangles will be created (each one of them derives from its previous by being rotated 10°).

Here the concept of rotational symmetry is embodied. In the instances shown besides six equilateral triangles are positioned in the plane after their rotation of 60°, 120° (two by two coincide) and 180° (three by three coincide) respectively.

**3rd phase:** A) Process - activities: For the construction of “the wheel of the amusement park”, the students have to create an equilateral triangle of length :a which have to be repeated for k times after being rotated by b degrees each time. The code “wheel” is expected to be created by the students through modifications of the code “hexagon” (i.e. insertion of a variable :k for the number of repetitions of the equilateral triangle). B) The role of the tools: Students are expected to experiment with the slider :k in conjunction with the turtle’s turn :b and replace the turn by 360/:k. An instance of :a=72 and :k=19 is shown in the figure besides. In this phase, the students generalize their understandings and connect them to a specific situation of everyday life.

Through the analysis of the activities’ phases it can be noted that:

I) The concept of regular hexagon is approached in two different ways: a) Differentially in S12 through the intrinsic turtle’s movement (every position and turn of the turtle is defined according to its previous one) in a situation oriented within the mathematics world. In this case the focus of the designer is on the polygonal line rather than the polygonal surface. This approach favors the extension of the scenario to the concept of circle. b) Using a simulation of a real-world object, the designer of S13 attributes emphasis on the polygonal surface and this approach favors the connection of the scenario with the topic of Areas. In S13 the regular hexagon derives from transformations of an equilateral triangle and particularly through rotational symmetry of 6th order. Two other scenarios for the construction of regular hexagons used the concept of rotational symmetry in a different way than the S13. According to the formal Greek mathematics curriculum only few teaching hours are devoted to the concepts of central and axial symmetry during the first grade of the secondary level. The concept of rotational symmetry is not taught at any grade of secondary education despite the fact that central symmetry is a specific case of rotational symmetry (i.e. rotation of 180°). We suggest that the use of the concept of rotational symmetry in the design of these particular scenarios was favored by the potential of the available tools. Specifically, the use of the ‘repeat’ command in conjunction with the VT facilitates the generalization of the concept of rotational symmetry of different orders and seems to have challenged teachers to adopt an alternative didactic approach by-passing the formal curriculum.

II) As far as the role of the tools is concerned, the vast majority of the activities revolved around the use of the variation tool. From the body of the nineteen scenarios we distinguished four expected dynamic manipulation schemes (DMS) taking into account findings of previous researches focusing on the use of the variation tool (Psycharis & Kynigos, 2009). These schemes are present in the anticipated students’ use of the variation tool in the scenarios S12 and S13 mentioned before. Next, we describe briefly these schemes with particular references to the corresponding stages indicating the evolution of the anticipated learning process.

**Reconnaissance DMS:** The students are expected to begin with random manipulation of sliders and finally to correspond sliders to particular magnitudes of the geometrical construction and recognize the interdependence between the variables used.
Correlation – Orientation DMS: Two levels can be described: After distinguishing which sliders influence the construction of the geometrical figure and which do not the students are expected to experiment with the varying magnitudes so as to complete the construction of the figure. If there is only one slider which influences the construction of the figure, the students determine the value or the values for which the figure is completed. If there are two or more sliders that influence the construction of the figure, the students move them in order to find the ‘combination’ of particular values which leads to the completion of the figure. In this level dynamic manipulation is oriented towards the completion of the figure through the use of specific techniques involving the change of the initial and end value of specific variables, the change of the variation step and the identification of the marginal values. The students are expected to develop conjectures for possible relations between variables and modify the code.

Verification DMS: After having corrected the code, the students are expected to verify the correctness of their conjectures by dragging the sliders for enlarging and shrinking the geometrical figure. According to the provided feedback the students confirm or dismiss their own conjectures.

Design DMS: In order to compose a new figure from the initial one, the students can use the variation tool to control the number of iterations. This kind of dragging signals students’ ability to use a particular geometrical figure as a unit for the creation of more complicated ones.

Conclusion
Our concern was to shed light on the nature of the teaching design based on the use of digital tools and on how these tools might shape the mathematical content in comparison to the formal mathematical curriculum. According to our findings, the development of scenarios based on the use of specially designed digital tools can create contexts for teachers to build meaningful situations for their students. Our analysis reveals that these scenarios might constitute curriculum units which take into consideration different aspects of mathematical concepts and introduce new components in the formal curriculum and create new widening conceptual fields out of the limitations of the formal curriculum. In that sense, designing scenarios based on the use of digital tools can be a professional activity which redefines the relationship between teachers and curriculum.

References
The role of interactive assistance in discovering geometrical theorems at secondary school

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This presentation submits a proposal to amend the traditional learning with interactive materials. The issues covered relate to secondary school level geometry. The material was compiled using GeoGebra software. It can be used on personal computers as well as in a classroom while working with an interactive whiteboard. The concept implies working in direct contact with the group as well as using the e-learning platform. It is heavily based on the concept of blended learning. The usefulness and effectiveness of the developed method were verified during the research experiment conducted on two groups of secondary school students. The article describes the materials, concepts of the study, findings and conclusions resulting from the research.

Introduction

In the twenty-first century, new technologies are present in every area of social life. This also applies to education. Media and multimedia create a new educational environment. It is attractive to young people and competitive to traditional measures. Currently, the hybrid learning is used increasingly. „Blended Learning is learning that is facilitated by the effective combination of different modes of delivery, models of teaching and styles of learning, and founded on transparent communication amongst all parties involved with a course”[2]. Blended learning implies the consolidation of the traditional teaching with the concept of e-learning. It does not favour any of the concepts as the more important or superior. They are, however, closely linked. They interpenetrate and complement each other. Together, they provide a coherent curriculum design. By using these methods together, we can take advantage of their strengths and, at the same time, avoid their drawbacks if possible. E-learning enables you to work with means of electronic transmission, software, interactive drawings and visualizations as well as the Internet.

Blended learning – description of the concept of teaching

As far as the teaching concepts are concerned, e-learning allows you to approach mathematics in a functional manner. The A.Z. Krygowska’s concept describes solving problems by performing actions consciously [3]. The student is able to name and prioritise them. The problem solving is performed actively – either through trial and error or by using ready-made patterns. E-learning course allows student to pass through each stage of the problem solving step by step. The computer allows the student to stop at any place in order to better understand the topic. The student may also return to the selected text and analyse the issue again. The teacher, in turn, is able to consciously organise the problematic situations. They lead students from concrete actions through imaginative activities to mathematical abstractions. The use of interactive drawings and visualisations lets you visualise a new concept. It also provides the possibility of quick analysis of numerous cases, including special cases. It also makes making observations, formulating and verifying hypotheses as well as searching for formal proofs easier.

Advantages of blended learning are also evident when we take into account the phenomenon of social facilitation [1]. It is based on the fact that new material is easier to learn without the presence of observers. There is no tension then. The student does not work under pressure. The student works at his/her own pace, under preferable conditions. It is worthy of paying attention to, while planning the activities. The student’s first encounter with some problems can be performed via e-learning platform. This may facilitate dealing with a new task.

Creating materials for the student is not an easy task. It should be a well considered project, factually and didactically correct. It must also contain a carefully selected exercises and examples. It is essential that they are coherent and form an integral whole. After designing, developing and providing the materials to students, they have to be evaluated. Their accuracy, relevance, effectiveness, the role
played in the learning process are all subject to evaluation. This results in the analysis of materials and drawing proper conclusions. It is then possible to modify and adapt the presented content, so that it can be used by the next group of students. This schema is shown on figure 1. A research experiment has been conducted in order to evaluate the effectiveness of the described concept.

![Figure 1](image)

**Research Methodology**

The material presented below describes the concept of working with the youth during math classes with the support of IT resources. This is a preliminary, exploratory study. It was conducted on two groups of students – attending first and second grade of secondary school. In both cases, the participating students declared their interest in mathematics, computer science, and science subjects by selecting a course of studies in the secondary school. In addition, they had to be successful students at the preceding level of education in order to be enrolled to the selected group. Therefore, theoretically, these are students who do not have major difficulties in learning mathematics or solving typical problems. Both groups were given the opportunity to expand their interests through additional math classes with the use of IT tools. One of the classes took place in the school computer lab; the students also worked with the interactive whiteboard. The e-learning platform was also made available to them. We can therefore conclude that teaching was based on the concept of blended learning.

The subject matter of these materials focuses on plane geometry issues. On one hand, they repeat, systemise and organise the knowledge that the students should possess when they start secondary education. On the other hand, a large part of them have been supplemented and extended. Therefore, these are partly new materials; the students are confronted with some issues and concepts for the first time. In addition, some presented issues are not subject to the requirements of the school curriculum. These are extra-curricular materials for students, designed to develop their mathematical reasoning and interest in the subject. The discussed issues include both definitions and relations between geometric objects and properties of figures in the plane. Specific topics can be divided into three main scopes:

- basic geometric figures and concepts - the concept of collinearity of points and properties of this relation have been discussed. The definitions of the following figures have been recalled: line segment, straight line, half-line, polygonal chain, circle, disk. Materials concerning the convex set, non-convex set, plane figure as well as the shapes’ boundaries, interiors and exteriors were included as an additional issue.

- relative position of the circles, relative position of the straight line and circle – issues relating to incircles, circumscribed circles and rings thus created, have been added to the aforementioned subjects.

- properties of angles, angles in a circle –. this part recalls the definitions of the angle in a plane and the angles in a circle. New issues concerned the exploration of measurement relationships between
these angles and examination of relationships of angles created by tangents to circles or bisectors of selected angles.

Working on the issues was based on specially designed worksheets, provided to students via the e-learning platform. Furthermore, the materials were accompanied by interactive applets developed with the support of GeoGebra software and also located on the platform. The platform was the main form of communication outside of school. It provided quick contact, place for discussion and exchange of views in the forum; it also made submitting one’s results easier. The applets provided to students played a number of roles. They helped with the presentation and visualisation of new concepts. The analysis of other interactive drawings was designed to help students to make appropriate observations, make a statement on a perceived property as well as to help students find the formal substantiation of the perceived fact or find a suitable counterexample to refute it.

Providing students with materials via Internet allows them to repeatedly return to the issues of the students’ particular interest. The student can work on an issue in the most favourable conditions and in the place of his/her choice.

These worksheets have been designed in such a way that the initial part of the issues was discussed during one of the school classes. The work was based on the analysis of the guiding text accompanied by the appropriate guidance provided by the teacher. Other tasks presented to students were solved independently, in a time period specified by the teacher. During this period, the students could consult each other and the teacher via Internet. The students sent the completed worksheets for evaluation. While working independently, the students were able to use the GeoGebra files provided by the teacher, or, when needed, create them themselves. The whole cycle of classes was summarised by the students solving a list of problems – a form of test verifying the knowledge and skills gained. At this stage, the students were not allowed to use the computers.

Observations, results, conclusions

Observations made and analysis of the materials gathered during the study confirm the assumptions. In the analysis of the results obtained, a significant role of the IT resources in the teaching process can be observed. In most cases, interactive materials actually gave students a broader view of the discussed issue. They also enabled a more detailed analysis of the presented problems. In many cases, GeoGebra software served as a tool to quickly create drawings (fig. 2).

Pupils used files which they received to make illustration of the task. This allowed them to investigation of all possible cases of the reciprocal position of circles fulfilling given in the task conditions.

Computer program made it easier for the students to document their work and make presentations of their observations and conclusions. Several students used the software to examine and illustrate the special cases (fig. 3).
The discovery of the relationship among three contiguous circles in the determinate position was pupils task. It was quite difficult, but investigation of the special case – two overlay circles – make further work easier.

The files prepared by the teacher also facilitated the analysis and searching for dependencies. The ability to make measurements using the software as a tool, allowed the students to spot the measurement relationships they were looking for. Making these sort of observations with traditional means would be much more difficult and time-consuming. Virtually all students were able to write a formal substantiation of basic facts that they observed. It is worth pointing out that even the students who were not able to write a proper substantiation could describe and explain it verbally. Several of them carried out the formal proofs of more complex properties of plane figures - extracurricular material (fig. 4). The description on figure: Written down by the pupil reason, figure with made measurements.
Virtually all students performed typical tasks which did not require the computer with no problems. The few errors that appeared in the solutions were mostly minor errors of calculation. These observations suggest that software can play a positive role in solving various types of problems. There are some risks however, that are worth taking note of. First of all, it should be noted that there was a large group of students who, while working with the provided file, have not modified the initial objects. They made all observations using the original drawings – default positions of figures, and thus, have not used in full the interactive features provided by the software (fig. 5).

The position of circles on the figure is identical like in the prepared by the teacher file. Not propeling figures the pupil makes measurements in program and formulates some of conclusions related to dependence among them. He treats them as static objects.

This may be due to the habits acquired in their daily work at school. These students have not fully recognised the concept of interactive drawings. In fact, they examined one example only – as if they were working with a drawing on a piece of paper. It is important that students take full advantage of the opportunities provided by the software. The example provided shows that they have to learn it, abandon old habits. Another important aspect is the accuracy of measurements made using the software. The values displayed are often approximate. This may result in arriving at the wrong conclusions. The issue of collinearity of points may be a good example. The analysed drawing seems to show collinear points, the displayed lengths of the line segments, however, refute this fact (fig. 6).

![Figure 5](image1)

![Figure 6](image2)
However, this does not necessarily disqualify the use of GeoGebra software or justify questioning of its usefulness. It is the teacher’s task to prepare students to work with the software, to prepare them for these kind of situations and explain the reasons behind the occurring discrepancies. In many cases, you can also bypass the troublesome cases by better preparing the file for the students or by adding the appropriate comments to it.

Zad. 4
Punkty A, B, C są niezależne, a punkt X należy do odcinka AB. Sprawdź, że: $2|CX| > |AC| + |BC| - |AB|

Po wykonaniu konstrukcji w GeoGebra, okazało się że, założenie $2|CX| > |AC| + |BC| - |AB|$ jest prawdziwe.

![Diagram](image)

Figure 7

The most important conclusion after conducted observations relates to realizing and writing down formal proofs. Pupils working with interactive files very often were able to notice feature, property of considered geometrical objects and dependence among them. However writing down the formal reasoning was problem in the majority. Part of the pupils did not feel the need of making such note, and it’s very danger situation. In such situation the teacher ought to pay pupils attention, remind them, that computer visualization does not make up the mathematical proof and can never replace it.

The above observations are only general findings and preliminary conclusions. They are the results of the preliminary stage of research. The research is currently conducted and will be continued. This will confirm or verify the obtained findings and conclusions.

References:

Combining theoretical frameworks to investigate the potential of computer environments offering integrated geometrical and algebraic representations

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In this paper we aim to address the problem of fragmentation of theoretical frameworks within the field of mathematics education with technology while exploring the potential of computer environments offering integrated geometrical and algebraic representations. We follow a ‘cross analysis’ method to analyse an experiment taking place in the Greek context under a constructionist theoretical perspective through the lens of the theory of didactical situations and of an epistemological model of activity with functions from the French research context. The analysis indicates that the aforementioned method enhances our efficiency to capture the potential of computer environments linking geometrical and algebraic representations.

Introduction

Several computer environments now offer integrated geometrical and algebraic representations and functionalities. Researchers and innovators stress their potential to support especially the teaching and learning of functions at various levels. However, it seems difficult to really appreciate this potential, since every author usually bases his/her findings on a vision provided by his/her specific framework and because of “the fragmented character of the theoretical frames which have been developed in order to approach learning and teaching processes in such environments” (Artigue et al. 2009, p. 5). Our aim in this paper is to combat fragmentation, trying to connect visions based on different theoretical perspectives. We have chosen the metaphor of networking theoretical frameworks and the idea of combining and coordinating frameworks “for the sake of a practical problem” (Prediger et al., 2008 p.172). We also chose to analyse concrete teaching experiments taking place in real classroom settings. Consequently, we adopt “cross analysis” as a method, according to which researchers analyse jointly an experiment taking place in a given national and didactic context under different theoretical frames and research approaches.

Each of us used a software environment that has been developed under specific theoretical frameworks in specific research and national contexts. The first one (called Turtleworlds) is a piece of geometrical construction software which combines symbolic notation, through a programming language (Logo), with dynamic manipulation of variable values (Kynigos, 2004). The design and the research on the use of Turtleworlds is inspired by constructionism (Harel & Papert, 1991). The second software (called Casyopée) offers a dynamic geometry window connected to a symbolic environment specifically designed to help students to work on functions. Casyopée’s design and experimentations occurred in a French context shaped by didactical theoretical frameworks and epistemological considerations. We focus here on a framework preeminent in the French context—the theory of didactical situations (TDS, Brousseau, 1998), and on an epistemological model of activity with functions built to make sense of the potential of computational environments with interconnected algebraic and geometrical representations, especially of Casyopée (Lagrange & Artigue, 2009).

We separately conducted research about the potential of computer environments offering integrated geometrical and algebraic representations, each using his own software in his specific context and with his specific frameworks. We are interested in exploring deeper this potential and the question we aim to tackle is: what new insight about this potential might be gained from analysing an experiment carried out by researcher A involving his own environment within his specific framework, when combining this framework with researcher B’s framework? We consider here a teaching experiment designed and implemented with Turtleworlds in the Greek context. As a way to combine A and B’s frameworks we cross a constructionist analysis of this experiment with analyses carried out by way of TDS and of the epistemological model mentioned above as distinctive features of the French research context. We introduce briefly situated abstraction as the main idea of constructionism for mathematical learning since it was the framework that informed the design of Turtleworlds and the implementation of the teaching experiment and then we report on the teaching
experiment. In the cross-analysis we first highlight the elements of an analysis from the constructionist perspective before we demonstrate how key notions of TDS and of the epistemological model shed light upon these elements.

Constructionism and situated abstraction

Constructionism is a theory of learning that incorporates and builds upon constructivism’s connotation of learning as “building knowledge structures” through progressive internalization of actions, in a context where learners are consciously engaged in constructing (or de/re-constructing) something on the computer. Constructionism attributes special emphasis on students’ activity when using mathematics to construct a model on the computer: the notion of construction refers both to the ‘external’ product of this activity as well as to the theories constructed in students’ minds (Papert, 1980). Aiming to elaborate a theoretical account of how mathematical meanings can be situated—in terms their genesis, means of expression and use - and yet abstract in that they extend beyond immediate concerns to more formal conceptions of mathematical knowledge, Noss and Hoyles (1996) introduced the notion of situated abstraction to describe how learners construct mathematical ideas by drawing on the linguistic and conceptual resources available for expressing them in a particular computational setting. A basic tenet of situated abstraction is that the computational tools, in turn, shape the ways the ideas are expressed and thus the respective computational environment can be considered as a system through which mathematics can be expressed.

Turtleworlds

Turtleworlds is a microworld designed to integrate formal mathematical notation with dynamic manipulation of variable values (Kynigos, 2004). In Turtleworlds, the elements of a geometrical construction can be expressed in a Logo procedure. After a procedure depending on variables is defined and executed with a specific value for each variable, clicking the mouse on the turtle trace activates the “variation tool”, which provides a slider for each variable (see at the bottom of Figure 1). The dragging of a slider results in a continual reshaping of the figure according to the corresponding variable value. Thus, the user is able to use Logo formalism (a) for describing geometrical figures algebraically by using variables and/or relations to represent segments and/or angles and (b) for dynamically manipulating the geometrical objects embedded in the construction of these figures for controlling their shape according to geometrical properties. The software thus behaves like a hybrid between programming tools (e.g. Logo-like microworlds) and expressive tools (e.g. Dynamic Geometry Systems) proposing formalism as a means of representing mathematical ideas in a geometrical construction context. The user is able to write, run and edit Logo procedures to ‘drive’ the construction of geometrical figures by the turtle providing also a way to construct relationships that render the construction visible.

The experiment

The experiment with Turtleworlds took place in a secondary school in Athens with two classes of 26 pupils aged 13 years old and two mathematics teachers. All teaching sessions were video-recorded by a team of two researchers who acted as participant observers in the classroom. The constructionist theoretical framework that underlies the study suggests designing a task that engages students in producing a meaningful outcome, and simultaneously in appreciating the utility of the respective mathematical ideas, the why and how these ideas are useful (Ainley et al., 2006). The task – called Dynamic Alphabet – engaged each class in constructing enlarging-shrinking models of all the capital letters (i.e. of variable sizes) with one variable corresponding to the height of the respective letter. Moving the slider of the variation tool in this case would result in the visualisation of the letter as an enlarging-shrinking geometrical figure. In formal mathematical terms this means that each letter procedure had to contain only one variable, so all of its varying lengths would be expressed with appropriate multiplicative functional relationships. We stress that these relationships were not initially explicit to the students, the aim being that they experience visually-based cognitive conflict, particularly when using additive strategies.
Early in their work most of the students constructed a model of their letter – which we refer to as the original pattern – sometimes without using any variables. In subsequent phases of their exploration, students experimented with the use of variables for all of its segments, to change it proportionally, until they built their final model with one variable. We focus on a pair of students - Christina and Alexia- exploring the construction of an enlarging-shrinking model of the letter N that they completed in the following successive six phases during four classroom sessions:

Phase 1: Construction of the original patterns of two models of N: N (35°) (vertical segment=200, slanted segment=240, internal angle=35°) and N (45°) (vertical segment=100, slanted segment=145, internal angle=45°).

Phase 2: N (35°) construction with two variables for the vertical segments and the slanted segment respectively. Recognition of the interdependence of variables. Exploration of the construction of similar N (35°) models of different sizes.

Phase 3: N (35°) construction with one variable and specification of an additive functional relation between the vertical segment and the slanted segment. Experimentation with changes to the constant value of the additive functional relation used to represent the slanted segment.

Phase 4: N (45°) construction with one variable and specification of a multiplicative functional relation between the vertical segment and the slanted segment (not appropriate function operator). Experimentation with changes to the constant value of the function operator used to represent the slanted segment.

Phase 5: N (45°) construction with one variable and appropriate multiplicative functional relation between the vertical segment and the slanted segment.

Phase 6: Exploration of the construction of different models of N (25°, 30°, 35°, 45°) and specification of appropriate multiplicative functional relations between the vertical and the slanted segment.

During phase 3, in order to challenge these students to consider the co-variation of the two variables (:r and :t) for constructing similar models of N (35°) in different sizes, the researcher asked them “how many times one segment is the other”. The students translated the relation of the two values as “200 plus forward 40” and substituted variable :t with the functional expression (:r+40). Dragging the only slider :r, students realised that the figure was obviously distorted for most of the values of variable :r. However, they observed that when :r took values ‘near 200’ -which was the value of the vertical lengths in the original pattern- figure distortion seemed to be minimised. Thus, they tried to identify a sub-domain of the functional relation to control figure distortion and validate the effectiveness of their method for enlarging-shrinking the geometrical figure. According to the graphical outcome they concluded that the figure was less distorted when variable :r took values between 195 to 205. In the stream of their subsequent exploration the students experimented with new additive functional relations (i.e. :r+45 and :r+50 respectively) trying again to identify new sub-domains of these relations for controlling the graphical distortion on the figure caused by different values of the only variable :r. In this vein, they drew a line at the letter base so as to precisely evaluate the accuracy of their method. Again, testing dragging on the variation tool confirmed that the use of an additive algebraic expression constituted an erroneous strategy for constructing an enlarging-shrinking model of N holding for ‘all values of :r’. From this point students started to rethink the correlation between the two varying magnitudes in the Logo code.

**Researcher:** Since the one [i.e. the vertical segment] is 100 and the other one [i.e. the slanted segment]. How many times of 100 is 145?

**Alexia:** One and...

**Christina:** One and ... Yes, one and a half.

**Researcher:** Well, how one and a half can be expressed in a relation?

**Alexia:** Maybe, one plus :r divided by two. Let’s try it [i.e. on the computer].
In the above excerpt (Phase 4) the students had already started to experiment with the identification of an appropriate functional relation for the slanted length so as to construct an enlarging-shrinking model of N (45°) (vertical segments=100, slanted segment=145 in the original pattern). The researcher took the opportunity to intervene so as to challenge students move the focus of their attention on the multiplicative dependence between the two varying lengths by comparing the numerical values of them in the original pattern.

**Analysis from a constructionist perspective**

The above episode brings to the foreground two critical aspects of the construction of geometrical figures according to proportionality: first, how the students appreciated the inappropriateness of additive strategies and, second, how they identified and expressed multiplicative functional relations in formal notation. In our view, a critical step in this direction was the translation of the dependency between the vertical and the slanted side in symbolic notation through a process in which values, variables and everyday language were simultaneously interlinked. Although the correlation between the two magnitudes was initially perceived by the students as additive, the computational environment provided a structure which they used to express the corresponding function, tools for experimenting with it and further elaborating its formula. At that time we see that meanings were reshaped as the students moved the focus of their attention onto a relation which was a new object within the setting. Thus, the students moved from identifying dependence and co-variation between magnitudes to identifying and expressing co-variation between magnitudes represented through variables. The chain of functional meanings here involved (a) the idea of variable as representing a general entity that can assume any value and symbolise general rules, (b) the specification of domains of validity for an additive functional relation and the development of methods to take control of the distortion of the figure (e.g. design of straight line in the letter base), (c) experimentation with the symbolic form of the additive functional relation and (d) implications of the potential emergence of the multiplicative correlation between the two variables representing the vertical and the slanted side. The chain of functional meanings wasreshaped through the use of the variation tool. The dragging on the one and only slider of the variation tool refuted the proportional enlarging-shrinking of the geometrical figure for all values of the respective variable, thus providing a link between the graphical distortion and the symbolic aspect of the functional relation.

We consider this excerpt as an illustrative example of the dynamic nature of the functional meanings developed by the students. The researcher’s remark about the correlation between two specific numerical values triggered students’ focus on the functional relation between the two corresponding lengths. Christina describes the emergent relation by words while Alexia seems to be able to articulate the dependent length as situated abstraction with direct reference to the independent variable \( r \). Although it is not clear what is the symbolic form of the functional relation suggested by Alexia—i.e. \( (r + r/2) \) or \( (1 + r/2) \) or \( (1+r)/2 \)—the available symbolic component of the environment allowed students to test these relations, thus providing a basis for further elaboration based on the use of the variation tool and the graphical feedback. Here situated abstraction emerged facilitating students’ reshaping of functional meanings towards the identification and expression of the multiplicative co-variation between the two variables.

**Analysis from a TDS perspective**

Brousseau (1998) presents TDS as a way to model mathematical situations in a learning context. In this model, a central notion is the “milieu”, a device which justifies the use of knowledge objectively to solve a given problem. He establishes a list of conditions to satisfy including: (a) the mathematical knowledge aimed at should be the only good method of solving the problem (b) students should be able to start working with inadequate "basic knowledge", (c) students should be able to tell for themselves whether their attempts are successful or not, (d) the feedbacks should not indicate the solution, but they should be suggestive of ways to improve strategies, (e) students should be able to make a rapid series of attempts, but anticipation should be favoured.

Considering the example with Turtleworlds we can see that these conditions are important for the success of the situation. Students could start with inadequate conceptualisation and the multiplicative functional relation appeared to be the only good method for completing adequately the
construction. Dragging on the slider of the variation tool allowed them to realize by themselves whether their attempts succeeded or failed. The graphical distortion produced by additive relation brought suggestive information about the inappropriateness of this solution. Expressing the relation into the Logo procedure favoured anticipation. It seems to us that the constructionist researchers implicitly build their analyses upon a model of mathematical situations consistent with Brousseau’s TDS. However, the episode when the researcher intervenes so as to move students’ attention on the multiplicative dependence would be interpreted by TDS as a too direct indication towards a solution (an “effet Topaze”). TDS researchers would note that the students do not follow the intervention, but rather continue to explore addition based relationships. The exploration with the software is productive because it helps to distinguish between multiplicative expressions (like $r + \frac{r}{2}$) and semi-additive expressions like $(1 + \frac{r}{2})$.

Another difference is that constructionism takes care of “meanings” rather than directly of “mathematical knowledge”. The effect of students’ interaction with the “milieu” is described in terms of development of functional meanings. It helps to incorporate in the analysis the symbolic components brought by the “milieu” and their effects onto the identification and expression by the students of the multiplicative co-variation. Furthermore, as shown in the presentation of the task constructionism favours situations where students can appreciate the utility of mathematical ideas. This means that the “milieu” should not only favor the emergence of meaning and of symbolic components, but also help students appreciate their utility beyond the boundaries of school mathematics. In contrast, TDS privileges in the analysis of the situation the knowledge to be taught and stresses the connection between knowledge built by interacting with the “milieu” and the standard mathematical knowledge at stake. This means that in a TDS perspective a process of institutionalization should be organized by the teacher in order that students interacting with Turtleworlds access some standard knowledge on multiplicative functions, whereas constructionism insist on the emergence of situated abstraction in the interaction itself.

For us adding a TDS analysis of the experiment helps (a) to better understand the conditions that make the interaction with Turtleworlds work in a productive way, (b) to focus on another aspect of learning: in addition to the development of functional meanings useful beyond school, the Turtleworlds situation can help students reach standard knowledge about multiplicative functions.

**Analysis based on the model of algebraic activity**

In order to makes sense of the potentialities of new computational environments for learning about functions, Lagrange and Artigue (2009) classified the various activities about functions into a model of algebraic activity. We limit here the model to a classification based on the epistemological assumption that the notion of function is connected to the idea of dependency in physical systems where one can observe mutual variations of objects. The model distinguishes three levels: (1) activity in a physical system where dependencies are “sensually” experienced (Radford, 2005); (2) activity on magnitudes, expected to provide a fruitful domain that enhances the consideration of functions as models of physical dependencies; (3) activity on mathematical functions, with formulas, graphs, tables and other possible algebraic representations.

The model helps to analyse the students’ activity in the Turtleworlds experiment. The physical system is a path of the turtle in three segments with a given angle between them. In phase 1, the path is fixed with given length of the segments. In phase 2 it depends on two variables and in the next phases, the challenge is to program the path in order that it depends on one variable while conforming to the goal that it represents the letter N. At the level of magnitudes, angles and lengths are involved. What is at stake is formulating a dependency between the length of the vertical segments and the length of the slanted segment in a functional form allowing its expression in a Logo procedure. At the level of mathematical functions, the dependency between the “vertical” and the “slanted” length is mathematically a multiplicative function, whose coefficient depends on the given angle.

In phases 1 to 5, while activating the sliders and working on the Logo procedure, the students consider together the physical system and the dependency between magnitudes. Their task is actually to understand the constraints of the physical system as a dependency linking two magnitudes and to find an expression for this dependency in order to write the procedure using a single variable. This implies to choose one length as an independent variable and the other as a dependent variable.
before building a suitable algebraic expression. In phase 6, the students move to mathematical functions. Taking the functional expression of the dependency for one angle into account, they understand that the same multiplicative model holds for other angles. Their task is then to find the multiplicative coefficient for each angle. Further tasks could deal with comparing the functions for different angles by using tables or graphs, allowing students more activity at this level of mathematics functions.

The model of algebraic activity is based on an epistemology of functions. It has been built to make sense of the design of geometrical and symbolic environments different from Turtleworlds and of their use by older students (aged 16 and more) to learn about functions. The distinction between the three levels of activity helps here to analyse the progression of functional meanings, showing the importance of working with magnitudes as a bridge between sensual experience and mathematical functions and suggesting further tasks.

**Conclusion**

Our concern was the fragmentation of theoretical frameworks within the field of mathematics education and its negative consequences in terms of appreciating research results and validating the constructed knowledge. Being interested in investigating the potential of computer environments offering integrated geometrical and algebraic representations we considered one environment and an experiment with students aged 13. The constructionist perspective brought elements of analysis: a chain of functional meanings was observed involving the idea of dependency between variables, a functional relation and the symbolic form of this relation. Thanks to the activity with the software the students recognised the inappropriateness of spontaneous models and identified and expressed appropriate multiplicative relations in formal notation. The TDS offered an appropriate framework to analyse this activity in terms of interaction with a “milieu”, specially adequate to justify the use of the appropriate knowledge objectively to solve the problem. The model of algebraic activity helped here to analyse the development of functional meanings, highlighting the importance of working with magnitudes as a bridge between sensual experience and mathematical functions. This brings evidence that by combining constructionism with TDS and the model of algebraic activity, the cross analysis captures more efficiently the potential of Turtleworlds as compared to the use of a single framework specific to the experiment. This is clearly a first step towards coordinating these approaches in order to get an integrated framework to analyse the potential of computer environments offering integrated geometrical and algebraic representations.

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Interactive self-paced learning using Mathematica

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Mathematica is a computer algebra system with powerful graphical and computational capabilities and an impressive ability for performing symbolic calculations. Although the richness of Mathematica’s computational environment is widely acknowledged, very little has been done to utilise its potential as a platform for a self-paced learning system. This new generation of Mathematica-based learning support software opens new horizons in teaching mathematics and related subjects, but also imposes significant challenges, both technical and pedagogical in nature. The pedagogical challenge is to design an interactive learning environment, whereas the technical challenge is to alleviate the programming burden incurred in the process of creating this environment. The paper presents the outcomes of a project which addresses these challenges.

Teaching applied mathematics: difficulties and opportunities

The constantly increasing capability of modern computers continues to boost the development and implementation of applied mathematical methods. The scale of research and development in the field of applied mathematics is unprecedented and is reflected in the university mathematical curriculum, where more and more teaching time is assigned to applied topics that are often very demanding computationally. This phenomenon imposes a new pedagogical challenge and requires the introduction of new teaching methods. Thus, in order to learn complex applied mathematical methods a student should complement theoretical knowledge by practical experience covering all stages of the computational process. This cannot be achieved merely by using commercial software with its black box approach where all details are hidden from the user. Moreover, the nature of learning requires that a student should be provided with the opportunity to perform as many calculations as possible manually. Since computational procedures are often very time consuming, the careful selection of computationally tractable tutorial problems and assessment tasks is a challenge which faces every instructor. In many cases, it is practically impossible to generate computationally tractable examples and exercises manually. The problem is further exacerbated by the necessity to ensure that these teaching materials cover all situations that can be incurred in practice. Regardless of the amount of effort spent on the development of good teaching materials, very often, the required calculations cannot be conducted or are undesirable to conduct in class. The first reason for this is differences in the pace of students’ learning which is normally a serious problem in any tutorials of this kind. The second reason is the amount of work, which may simply exceed the tutorial time available. So, one of the key elements of the learning process is often removed from the class environment or, at best, is implemented in this environment inefficiently. This leaves a student alone without any, or with inadequate, assistance from the instructor. The advent of computer algebra systems opened a new opportunity for addressing this problem.

Mathematica (Wolfram, 2010) is a popular computer algebra system capable of performing symbolic calculations in exactly the same manner as they are performed manually. This highly developed feature of Mathematica is complemented by powerful graphical capability. The development of any learning support software requires an advanced programming environment — Mathematica offers unique flexibility and richness of programming environment, including procedural, functional, and rule-based programming styles, and supports object-oriented programming. Mathematica’s programming capabilities have been especially designed with the aim of facilitating mathematical manipulations and dealing with data in textual and symbolic form.

An extensive body of literature reflects the breadth of experience in teaching with Mathematica that has been accumulated in universities all over the globe (see http://library.wolfram.com/ for example). In saying this, it is important to stress that, in the majority of cases, Mathematica is used only as a tool for visualisation or as important software for any career in mathematics, science or engineering. In the latter case, Mathematica becomes not a medium for, but rather the object of, teaching.

The development of learning support software, oriented for individual use by students, presents a serious technological challenge, since the self-paced learning environment requires an enabling technology that is powerful enough not only to implement the algorithms and procedures of the
In the learning process, a student is guided from one stage to the next by these questions. Hence, components. The first component is a question requiring the stage result for the corresponding stage. This result (a scalar, a matrix, a function, an inequality, etc.) will be referred to as a *stage result*. The point in the program (module code) that corresponds to the start of a stage (portion of calculation) will be referred to as an *insertion point*. The purpose of an insertion point is to assess the student’s understanding of a particular concept and ability to carry out a particular calculation. For each stage but the first, the insertion point of this stage signifies also the end of the previous stage.

Each insertion point is associated with a specification in a standard form which includes several components. The first component is a question requiring the stage result for the corresponding stage. In the learning process, a student is guided from one stage to the next by these questions. Hence,
each insertion point initiates a communication between the student and the learning support system. If the student’s answer to the current question is correct, the corresponding portion of the module code is executed and then the question corresponding to the next stage is addressed to the student. Since the original module produces the output in the form that would be obtained in the case of manual calculation, a correct student answer invokes display of all steps of calculation corresponding to the current stage. This informs the student what form of the answer is desirable and provides the correct answer in the case when the correct stage result was obtained by an incorrect method (which happens from time to time).

The remaining part of the specification is concerned with the situation when the student answers incorrectly. Thus, the specification informs the system how many times the question should be repeated in order to establish that the failure in providing the correct answer has been caused by the lack of knowledge rather than a mere error in calculation. The instructor can specify what text (for example a hint) will accompany each repetition of the question.

If a gap in knowledge has been detected, the instructor has two options. The first option is the display of an explanatory text followed by another question or by direct progression to the next insertion point. The second option is the attempt to assess the background knowledge which underpins the calculation of the stage result. If this background knowledge includes the skills of calculating several mathematical objects (referred to as the background mathematical objects), the instructor may opt to invoke in succession the corresponding auxiliary programming modules. Each auxiliary programming module will require the calculation of one of the background mathematical objects and may itself be split into stages by its own insertion points. The programming system treats each auxiliary module in exactly the same way as the original one.

**An example of Mathematica-based self-paced learning procedure**

An extensive library of Mathematica modules has been developed as teaching support software for the cluster of subjects pertaining to optimization which are taught in the University of Technology, Sydney. As an example, we consider a “linear” version of the Zoutendijk method intended for problems with a nonlinear objective function but linear constraints. The Zoutendijk method is first introduced in its “linear” form, followed later by the general form. For students who have already mastered the ideas underlying the “linear” version, the general case is just a straightforward generalization of this “linear” version. The students learn the “linear” version of the Zoutendijk method by studying the detailed proofs and solving manually a set of specially chosen two-dimensional problems covering all potential situations that can arise in the solution process. The corresponding teaching support module produces all calculations in exactly the same form as in the course of manual calculations and illustrates each step graphically. Due to lack of space, the following description contains only a few snapshots of the output and does not cover the entire method.

As a preprocessing step, the Zoutendijk method converts all constraints into the form of less than or equal to constraints with zero right-hand sides. For an optimization problem with \( n \) variables, the “linear” version of the Zoutendijk method (as well as the general case) moves from one feasible point in the corresponding \( n \)-dimensional space to another point, with each transfer resulting in a reduction of the value of the objective function. The starting feasible point is given. The word “feasible” here refers to the fact that the given point satisfies all constraints.

In each iteration, the Zoutendijk method strives to find, for the current feasible point, an improving feasible direction. The adjective “improving” indicates that the objective function decreases if a point begins to move from the position of the current feasible point in the direction under consideration. The notion of “improving direction” is linked with the notion of the gradient — a vector of first-order partial derivatives for all independent variables. By the time of studying the Zoutendijk method, the students should already know that any direction which forms an obtuse angle with the gradient is an improving direction. They also should know that the angle between two vectors is obtuse when the scalar product of these two vectors is negative.

The adjective “feasible” in the term “improving feasible direction” indicates that when a point begins to move from the position of the current feasible point it remains in the feasible region, i.e. it satisfies all constraints. The feasibility of a direction chosen at the current feasible point depends on the
position of this point, which brings into consideration the notions of the interior and boundary of the feasible region. An understanding of the meaning of interior and boundary requires from students an understanding of the concepts of open and closed sets. If the current feasible point belongs to the interior of the feasible region, then any direction is feasible. If the current feasible point belongs to the boundary of the feasible region it belongs to at least one hyper-plane specified by the constraints. In this case, the corresponding less than or equal to constraint becomes, at the point under consideration, an equality constraint which is called a binding constraint. Given the preprocessing step, when a point begins to move from a position on the hyper-plane specified by a binding constraint, in order to prevent violation of this constraint, the corresponding constraint function should not increase. This observation leads to the following condition which should be satisfied by any feasible direction: the scalar product of the gradient of any binding constraint and a feasible direction must be non-positive.

Based on the above discussion, an improving feasible direction can be found by solving a minimization linear programming problem. In this linear programming problem, the objective function is a scalar product of the gradient of the original objective function at the current feasible point and the vector designating a direction at this point. Each binding constraint contributes to this linear programming problem a less than or equal to constraint the left-hand side of which is a scalar product of the vector specifying a direction and the gradient of the binding constraint. The right-hand sides of all such constraints are zero. The next step demands from the students a good knowledge of linear programming. This knowledge and associated skills are needed not only for solving the linear program but also for understanding that the linear programming problem outlined above is unbounded and therefore requires so-called normalization constraints. Furthermore, the optimal value of the objective function cannot be positive, since the zero vector as a direction gives the zero value of the objective function of the linear programming problem.

The specification associated with the first insertion point, i.e. the point preceding all calculations, contains a question requiring the student to find all binding constraints. If the student repeatedly fails to determine all binding constraints, then the self-paced learning support system displays the definitions of a boundary, an interior, and a binding constraint and repeats the question again. If, this time, the student answers correctly, the system acknowledges the correct answer by a corresponding message and executes the portion of the module code corresponding to the first stage. This code depicts the feasible region together with the current feasible point and the level curves graphically and either produces a list of binding (active) constraints or (in the case of their absence) displays a message that there are no binding constraints.

If the student fails to answer correctly, the system displays a corresponding message and executes the portion of the module code corresponding to the first stage. In contrast to this implementation, an instructor may choose, in the case of an incorrect answer, to invoke one or several auxiliary modules to test the student’s understanding of the notions of boundary, interior, and binding constraint. This in turn can lead to a module related to the notions of an open set and closure.

The specification associated with the second insertion point asks the student to calculate the gradient of the objective function at the current feasible point.
In the case of repeated incorrect answers, the system checks the student’s understanding of the notion of partial derivative by invoking the corresponding programming module. This programming module has only one insertion point and asks for calculation of a partial derivative of the objective function for a randomly selected variable.

The question associated with the third insertion point asks the student to produce the objective function for the supplementary linear programming problem. In the case of an incorrect answer, the specification of the insertion point invokes a programming module related to the notion of an improving direction. This module in turn may be linked to two modules which if necessary can be called in succession. One is the aforementioned programming module related to the notion of a partial derivative, whereas the other tests (and if necessary teaches) the understanding of a scalar product.

The questions associated with the fourth, fifth and sixth insertion points guide the student through the remaining steps of the formulation of the supplementary linear programming problem. The following display gives an example (left) and the figure on the right illustrates the graphical solution of this linear programming problem.

**Implementation**

The nature of the system described in the previous sections gives rise to a number of technical issues whose resolution is dictated by the architecture of the Mathematica system. Mathematica comprises two main components: a front-end and the kernel. The front-end looks like a word-processor and provides a graphical interface with which the user interacts by means of executable documents called notebooks. The kernel is the evaluation engine responsible for simplifying and evaluating expressions — it is this which makes the notebooks executable. The front-end communicates with the kernel via a proprietary protocol. The calculations required by a programming module are specified in executable notebooks, and may draw upon Mathematica’s rich library of predefined functions as well as additional user-defined functions stored in separate supplementary files called package files.

As has been discussed above, each insertion point initiates a communication between the student and the learning support system. This communication is achieved by means of opening a new notebook, which in turn can open another notebook, and so on. This is possible in Mathematica through its ability to create dynamically, from specifications embedded at the insertion points,
additional notebooks containing the questions, corresponding answers and explanatory text. Although Mathematica has all necessary facilities, the implementation of the above approach poses some programming challenges caused by the organisation of this computer algebra system — in particular, by the fact that notebooks are executed (by the kernel) and opened (by the front-end) by different parts of the system. Nevertheless, these technical problems are outweighed by the advantages of using a dynamic set of notebooks as the medium of communication.

A standard and concise form of specifications associated with the insertion points is crucial for the success of the Mathematica-based self-paced learning support system, since otherwise its adaptation to changing circumstances will be difficult at best, which may result in the system falling rapidly into disuse. A standard, easy to understand format of specifications is also important for providing an effective medium of communication between instructors. The specification format should permit the conversion of the original programming module into self-paced learning support software even by instructors who may not have the time or specific expertise required to maintain this type of system.

The adopted specification format satisfies these requirements. This format is based on a very simple markup language which in turn is implemented via a number of standard notebooks and packages. These provide a standard format for questions, explanatory texts, as well as methods for verifying students’ answers and navigating between the insertion points and auxiliary programming modules. In terms of the preceding discussion, this collection of notebooks and packages constitute a vehicle for communication between students and computer. The collection of insertion point specifications, together with the original programming module implementing the mathematical method being studied, form a so-called source notebook. Besides its technical purpose of defining the system functionality, this source notebook serves as a medium for communication among collaborating instructors.

Conclusions

In this paper we have articulated some of the pedagogical challenges and opportunities in teaching computationally intensive mathematics offered by modern computer algebra systems. We have described a conceptual framework for the design of a computer-based self-paced learning support system and illustrated its use by outlining such a system. This Mathematica-based learning support system has been designed to incorporate an already existing library of Mathematica programming modules which were successfully used at the University of Technology, Sydney in teaching optimisation-related subjects over several years.

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Dynamic and Interactive Mathematics Learning Environments (DIMLE): The case of teaching the limit concept

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This theoretical paper is an attempt to explore the potential of the dynamic and interactive mathematics learning environments (DIMLE) in relation to the technological pedagogical content knowledge (TPACK) framework. DIMLE are developed with intent to support learning mathematics through free exploration in a less constrained environment. A typical DIMLE software package has interactivity and dynamism as key affordances; these are especially suitable for enhancing learning and teaching with technology of the essentially dynamic mathematics concepts. Moreover, we propose that DIMLE and their affordances should be studied under the TPACK framework because this framework is explicit in considering technology-supported mathematics learning as a qualitative add-on as contrasted to what would be a simple totalling of technological, pedagogical and mathematical knowledges. As an example, we focus in our discussion on using a DIMLE in order to support learner in development of the limit concept.

The dynamic and interactive mathematics learning environments (DIMLE), such as Cabri, GeoGebra, Geometer Sketchpad, Fathom, and the like, provide unique opportunities for teachers to extend both the mathematics knowledge and understanding of their students, especially in the areas that are both dynamic in nature and otherwise difficult to understand. These unique opportunities could be better exploited if the teachers become better aware of the affordances of DIMLE. These affordances could be defined as: interactivity—an iterative action-reaction cycle that can be considered as an immediate feedback mechanism, while dynamism relates to the continual change in a process. The need to better understand the impact of the DIMLE technology on the content, progression and approach to the mathematics curriculum has led us to discuss it in conjunction with the technological pedagogical content knowledge (TPACK) framework (Koehler & Mishra, 2008). Here we follow Koehler and Mishra who suggested that in order to truly benefit from the integration of technology in their teaching, teachers should be aware of “what makes concepts difficult or easy to learn” (p. 17) for their students and adapt their pedagogy accordingly.

The thrust of this paper is to discuss use of the DIMLE in conjunction with the TPACK framework, in order to demonstrate development of the limit concept. Procedurally, limit of the rate of change of a function over a diminishing interval, leads to its derivative. This connection is natural to establish through DIMLE, as both the software and the concept are clearly dynamic. However, the DIMLE are still not widely used by the educators and the deficiency of representing the inherently dynamic concepts (e.g., the limit process) in the traditional ways (e.g., without technology, with an emphasis on formulas) makes their learning unnecessary challenging.

Mathematics Curriculum in Ontario

The 1997 Ontario Mathematics curriculum suggested that teachers should first engage students in the mathematically meaningful and purposeful situations which will enable them to develop interest and conceptual understanding, based on which, learning of facts and skills will be built (see Figure 1).

![Figure 1. Visualization of changes in 1997 Ontario mathematics curriculum.](image_url)

Coincidentally with these changes, a role of technology in teaching and learning mathematics gained in importance. In the latest Mathematics Curriculum for Grades 11 and 12 (Ministry of Education, 2007), it is highlighted that “In an effective mathematics program, students learn in the presence of technology [that] should influence the mathematics content taught and how it is taught” (p.5). This statement supports our use of the TPACK framework (Koehler & Mishra, 2008) in that it connects mathematics content with how it is taught (i.e., pedagogy) and by what means it is taught (i.e., technology).
Technological Pedagogical Content Knowledge (TPACK)

TPACK is defined as the intersection of three bodies of knowledge: technological, pedagogical, and content knowledge. In establishing TPACK framework, Koehler and Mishra (2008) argued that it is the basis of teaching with technology and that the substantial skills of teaching mathematics with technology need to take into account the interactions and relations between and among the three main domains. In other words, technological knowledge needs to be considered as an important factor in learning to teach mathematics. TPACK requires:

- an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students’ prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones. (Koehler & Mishra, 2008, p. 17)

Unfortunately, achieving the TPACK is not easy; deficiency in any of the three domains may prevent effective teaching of mathematics with technology. On the other hand, there exist some more systemic obstacles. For example, the knowledge of the limit concept and the procedural connection between the limit of the rate of change and the function derivative are usually underemphasized because of the prevalent “static” (as opposed to “dynamic”) representation employed in textbooks and sometimes in schools too. Also, what has been prescribed by curriculum developers not necessarily ends up implemented in practice and thus the divergence between the intended and enacted curricula.

In the further text we discuss teaching the limit concept through DIMLE, by using the TPACK framework. We start by describing content knowledge (i.e., CK in TPACK framework) related to the limit concept, connect it to the pedagogy of choice (i.e., PCK in TPACK framework) that focuses on the dynamic nature of the concept, and finally, relate it to technological pedagogical content knowledge (i.e., TPACK in TPACK framework), where we illustrate the application of DIMLE, as the learning environment of choice (see Figure 2).

![Figure 2. TPACK framework applied in DIMLE is the basis of teaching the concept of limit.](image)

Mathematics content knowledge: Connecting limit to derivative

Here, the mathematics content knowledge refers to understanding the procedural-dynamic attribute of limit and the connection of limit to derivative that could also be considered as a process of abstraction. Although abstraction is a process, it is considered as a concept after the process is completed. For example, the rate of change of a function at a certain interval is defined as rise over run, that is, $\Delta y/\Delta x$. Then, one starts decreasing the value of $\Delta x$ with an intention of making it infinitely small. This process of taking the limit of a function is the abstraction process because one has to use his or her own imagination to understand what is happening, to generalize accordingly, and to
succeed in representing the process algebraically. In that way the process of approaching is mostly dealt with conceptually and as a consequence, limit is considered to be a concept rather than a process. Tall (1991) describes this abstraction as a process-concept amalgam and defines it as procept. For more discussion of abstraction and procept, readers may refer to Tall (1991) and Hershkowitz, Schwarz, and Dreyfus (2001).

This course of action, which lies at the heart of understanding the limit-derivative relation, appears not to be emphasized enough in the secondary school curricula. For example, the Ontario Curriculum Guide (OCG, 2007, revised) suggests determining “through investigation using technology, the graph of the derivative … of a given sinusoidal function … by generating a table showing the instantaneous rate of change of the function for various values of x and graphing the ordered pairs…” (p. 102). This approach prefers a quantitative investigation of the concept, followed by a visual representation. In this sense, it seems that the curriculum developers prefer inductive approach as more appropriate for understanding of the derivative of trigonometric function, although derivatives (as well as trigonometry) were introduced earlier (see Figure 3).

![Figure 3. Using Excel spreadsheet to teach derivative of the Sine function in accordance with the Ontario Curriculum Guide.](image)

Similarly, the Calculus and Vectors book (Donato et al., 2009), suggests using calculators to investigate the limit-derivative relationship of, for example $y = \sin(x)$ function. The suggested investigation starts with using calculator to create a table with columns containing $x$, $y = \sin(x)$, and $\Delta y/\Delta x$ values. Once these calculations are done, it is suggested to create a fourth column with corresponding $\cos(x)$ values. While this approach is valuable in some cases, it may not be most appropriate for introducing the derivative of the Sine function. A student may rightfully ask, why is $\cos(x)$ in the fourth column? How could a novice anticipate the function $\cos(x)$ as the derivative?

Contrary to the aforementioned approach of guided discovery that is followed in secondary school, the university students are exposed to algebraic approach to derivatives, accompanied by some graphical representations. For example, they use a formula of the first principles, namely

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

where $f(x) = \sin(x)$ (Stewart, 2008, p. 190).

To summarize, the secondary school curriculum documents rely on quantitative exploration that may be followed by visualization (i.e., graphically representing information given in tables), while the undergraduate book relies more on an algebraic-symbolic approach to study derivatives.
Pedagogical content knowledge: Identifying the key areas

It is our belief that the pedagogical knowledge of the derivative concept (PCK) would benefit from a dynamic representation that would allow students to visualise and conceptualise the limit and derivative concepts in an embodied manner (Tall, 2000). However, it appears that the aforementioned Calculus and Vectors book misses the dynamic connection because it focuses only on the numerically lead investigation. Despite the fact that creating the sequence of numeric values getting closer to a certain value is meant to illustrate a process of continual change, a proposed way of comparison to the anticipated end product (a column of Cosine values) leads to a static investigation.

However, as discussed earlier in the content knowledge section, this investigation could be done in an environment which allows students to learn by exploring. Cognitive science literature (i.e., Marshall, 1995; Martindale, 1991; and Matlin, 2005) suggest explorative learning as the means to help students develop cognitive schemas of processes and concepts. That is, it is suggested that students who learn through exploration may better develop cognitive schemas of content knowledge, and connect these schemas to the ones previously developed and stored in their long term memories. At this point, we consider using some tools that provide for a dynamic and interactive exploration.

Technological pedagogical content knowledge: Evolution

Here we demonstrate a learning activity and use of one of the contemporary DIMLE because these learning environments have been specifically designed to address the stated pedagogical concerns. For example, teachers can provide students with a dynamic worksheet illustrating a function, 

\[ f(x) = \frac{\sin(x + h) - \sin x}{h} \]

where \( h \) is a parameter whose values are assigned by using a slider \( h \) (see Figure 4).

![Figure 4. A visualization of a problem in the DIMLE.](image)

By having both graphical and algebraic representations together, teachers have the opportunity not only to make the mentioned abstraction more visible, but also to make connection between representations. Given that the algebraic representation on the left clearly illustrates the function in algebraic form, teachers can confidently assume that their students recall their prior knowledge on functions and can build the learning task upon this assumption. This is what cognitive scientists call schema construction (Marshall, 1995; Martindale, 1991; and Matlin, 2005). The slider in the right upper corner of the graphical window helps students to interact with the software. Each action
applied by students gets a reaction provided by the software, and this iterative action-reaction process refers to the interactivity affordance of the DIMLE.

Moreover, students have the opportunity to follow the change in each of these representations while assigning different values to the parameter $h$ (see Figure 5). By making limit implicit in the software, as Tall (2000) suggested, students are supported in developing embodied understanding of the concept of limit and the dynamic change in the function through moving a slider, and therefore assigning various values to the parameter $h$.

![Figure 5. Dynamic representation of $f(x) = \frac{\sin(x + h) - \sin x}{h}$ when $h = 0.5$ and $h = 0.02$ using the “trace” feature in DIMLE.](image)

By using the “trace” feature in the DIMLE, teachers can help students compare the change in time while assigning different values to the parameter $h$, which can be also observed in gradually changing colors (dynamic colors) of the graph of the function and the equation of $f(x)$. The use of tracing feature of the software as well as implementing the use of dynamic colors to the formula demonstrates how the dynamic affordance of the DIMLEs plays its role in teaching of dynamic mathematics concepts.

**Conclusions**

In this paper we considered use of the DIMLE incorporated in the TPACK framework (Koehler & Mishra, 2008), and used the case of limit to identify the following themes:

**Type of representation of concept**

The representation of the mathematics concept in the DIMLE appears to the learner dynamically and interactively. Depending on the software used, usually there are several representational tools available, together with clear association between them. The teacher can easily emphasize one or the other representation (e.g., graphical, algebraic), or how one effects the other. The learners can compare, contrast, and analyze different representations and conjecture on how each is related to another.

**Pedagogical techniques used**

Different technologies provide for different levels of experimentation intensity (Borba & Zullatto, 2006). The DIMLE support pedagogical approach with high level of experimentation intensity. Moreover, DIMLE provide a great freedom to explore concepts and relations among concepts so that students may individually or collaboratively work to develop their own cognitive schemas.
**Ability to make learning process easier**

By making the process of change implicit in software and interactive, students learn the concept of limit in an embodied manner. Dynamic features of DIMLE shift the nature of questions and actions, putting emphasis on visual in the process of exploration (Sinclair & Yurita, 2005).

**Building upon the students’ prior knowledge**

DIMLE provide teachers with features to create horizontal and vertical connections between curriculum units; vertical connections to the prior knowledge and horizontal connections among simultaneous, multiple representations. Perception of continual change in mathematical objects may affect the users’ understanding of mathematics concepts and lead them to develop a new type of learning.

The current mathematics curriculum in Ontario encourages concept development through student engagement in mathematically meaningful situations. It also acknowledges an important role that technology has in mathematics learning and teaching. By emphasizing the affordances of DIMLE, such as dynamism and interactivity, and by providing examples that would utilize these affordances, curriculum developers would help teachers better understand how such technology could be used to help students learn difficult concepts. In that way teachers would more fully benefit from the integration of technology in their teaching and develop TPACK that would fully embrace the potential of DIMLE to enhance learning of mathematics.

**References**


Solving a contextual problem with the spreadsheet as an environment for algebraic thinking development

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In this paper we report and discuss a contextual problem solving task which was proposed to a class of 8th grade (13-14 years old) students. These students had been developing a reasonable experience in the use of the spreadsheet to model relations within contextual problems and chose to use this tool to solve the mentioned problem, engaging in the process of translating relations between variables and combining them in chained models, while working with fractions, multiples, and expressions. We intend to highlight the role of the spreadsheet in students’ processes of variable identification and translation of the problem conditions, their numerical approaches to algebraic models and their experimental forms of finding solutions to equations.

Introduction

The spreadsheet has been considered an educational resource with great potential for the construction of algebraic concepts, including the establishment of functional relationships, representing sequences or procedures of a recursive nature, all significant to be used in solving mathematical problems. Our aim is to understand how the spreadsheet supports the work of 8th grade students in solving a contextual problem, by focusing on their ways of representing the variables and the conditions stated in the problem.

Algebraic thinking

According to Kieran (2007) algebraic thinking is much more than knowledge of algorithms and techniques. To Zazkis and Liljedahl (2002) the term algebra encompasses two distinct aspects: algebraic thinking and algebraic symbolism, stressing that the presence of algebraic symbolism should be taken as an indicator in that the absence of algebraic notation should not be judged as an inability to think algebraically. This idea is in the spirit advocated by Radford (2000) whereby students are able to think algebraically even when they do not resort to algebraic symbolism in their written productions. The writing of symbolic numerical relations in school algebra has favoured the use of letters. However, the availability of technological tools allows other representations of such relations, as well as new forms of their exploration, which can be seen as algebraic activities of generation and transformation. Thus, it seems appropriate that these different representations of numerical relations, as well as the thinking that goes with it, is included in the field of algebra (Kieran, 1996).

Problem solving with the spreadsheet and the development of algebraic thinking

The spreadsheet is a powerful tool in mathematical problem solving and particularly in the development of algebraic thinking embedded in problem solving activities as highlighted by several authors (e.g., Ainley et al., 2004; Dettori et al., 2001; Rojano, 2002). One of the gains of connecting algebraic thinking and the use of spreadsheets is the creation of a significant environment to induce students into algebraic language that facilitates the construction of algebraic concepts, especially in what concerns working with functional relations, sequences and recursive procedures. Using the spreadsheet in the context of problem solving emphasizes the need to identify the relevant variables involved and fosters the search for variables that depend on other variables, resulting in composed relations. The definition of intermediate relations, by means of spreadsheet formulas in intermediate columns, meaning the decomposition of more complex relations in chained simpler ones, is a special feature inherent to the use of the spreadsheet that amounts to important results in solving algebraic contextual problems (Carreira, 1992; Haspekian, 2005). Moreover, as noted by Haspekian (2005) a spreadsheet also allows an algebraic organization of apparently arithmetical solutions and this kind of hybridism, where arithmetic and algebra naturally cohabit, becomes an educational option that may
help students in moving from arithmetic to algebra (Kieran, 1996). Spreadsheets can act as a bridge between arithmetic and algebra by helping students generalize patterns, develop an understanding of variable, facilitate transformation of algebraic expressions, and provide a space to explore equations (Tabach, Hershkowitz, & Arcavi, 2008). In addition spreadsheets allow students to focus on the mathematical reasoning by freeing them from the burden of calculations and algebraic manipulations (Ozgun-Koca, 2000). However as Dettori et al (2001) have noticed from their research on 13 to 14-year-old students’ work with algebra on spreadsheets, “spreadsheets can start the journey of learning algebra, but do not have the tools to complete it. Being able to write down parts of the relations among the considered objects, but not to synthesize and manipulate the complete relations, is like knowing the words and phrases of a language, but being unable to compose them into complete sentences” (p. 206). What still remains a research issue is to understand the scope of the spreadsheet contribution in going further than just the recognition and manipulation of relations among objects to a broader understanding of the algebraic foundations of the methods for solving algebraic conditions.

**Methodology**

This study follows a qualitative and interpretative methodology. The participants are four 8th graders (13-14 years), two of them working as a pair and the other two working individually. The students were given the freedom to choose whether or not they to work in groups or individually as was part of the didactical contract in the classroom. In both cases the teacher engaged in dialog with the students and asked questions whenever necessary to appreciate students’ reasoning and approaches. They had previously obtained some experience in solving word problems with a spreadsheet in the classroom, from which they acquired the basics of the spreadsheet functioning. Many of the problems that were explored with this class were selected from an online mathematical problem solving competition promoted by the University of Algarve, the Sub14, which addresses students of 7th, 8th grades. The possibility of participants sending their answers in different digital formats (including spreadsheet files) was seen as an incitement to engage students in working on contextual word problems with the use of the spreadsheet and an opportunity to develop students’ algebraic thinking.

The detailed recording of students’ processes was achieved with the use of Camtasia Studio. This software allows the simultaneous collecting of the dialogue of the students and the sequence of the computer screens that show all the actions that were performed on the computer. We were able to analyze the students’ conversations while we observed their operations on a spreadsheet. This type of computer protocol is very powerful as it allows the description of the actions in real time on the computer (Weigand & Weller, 2001).

The given problem entitled The Opening of the Restaurant “Sombrero Style”, presented characteristics that were seen as interesting to be explored with the spreadsheet, namely due to the fact that it may be solved by a numerical approach. To a certain extent it proved to be different from other problems solved by the students in the classroom. One of its features relates to the fact of being placed in the form of a narrative, which makes it pretty close to a real situation, where conditions are steeped in the story thus requiring a very careful reading to identify those which are relevant to solving the problem.

**The opening of the restaurant “Sombrero Style”**

The restaurant Sombrero Style was opened yesterday and I was there having dinner with three friends. The maximum capacity of customers – said the manager – is 100 people. Luckily I had booked a table for four, because when I got there several tables were already full with four people and one table had only three people. While I was waiting for the employee to take us to our table, I counted the women and men who were in the restaurant and the number of women was exactly twice the number of men. What could be the maximum number of people who were already at the restaurant when I came in?

A possible algebraic approach to the problem is presented in Figure 1. Solving this problem through a formal algebraic approach, namely using a system of equations such as presented, was beyond the reach of these students.
Results

Students expressed many difficulties in understanding the problem, particularly in the question about the maximum number of people sat in the restaurant before the group of four people arrived. There were other obstacles that relate to the simultaneous conditions pertaining to the distribution of persons by tables of 3 and 4 and to the division of clients by gender.

Since these students had already worked with the spreadsheet in other problems and knew some of their potentialities, they largely resorted to this tool over other processes to solve the problem.

Figure 2. Print-screen of Mary and Jenny’s representation

Mary and Jenny began by writing the condition on the number of people seated at tables of 4, as shown in the first two columns of Figure 2. In this process the students identify the variable “number of tables of 4” and in the next column show the number of people sitting at these tables. Jenny enters the column referring to the three people which were sat at one of the tables and calculates the total number of people by using the formula “= H11 + G11”.

Afterwards they separately represent the condition concerning the separation of clients by gender, as shown in Figure 2. The shaded row in each of the tables shows that the students sought to identify the solution by comparing the columns of totals in the two tables. The work of these students shows how the spreadsheet has helped them overcome the initial difficulties, in that it enabled them to work separately on the different conditions and afterwards relate the feedback from each one to get the solution. The establishment of relations proved to be an intermediate point which facilitated the expression of all conditions present in the statement.

The following is an analysis of the production of Carol, who like her colleagues Mary and Jenny also organized the two conditions by separating them, as shown in Figure 3.

The student begins by making the separation of customers by gender. The first column represents the integer values of the total number of people in the restaurant, in descending order, the second is the division by 3 for the calculation of one-third and the third column calculates twice the previous results. In the last three columns another condition of the problem is shown, namely, the distribution of customers by tables of 3 and 4. The successive multiples of 4 represent the various tables with 4 people and as there was only one table with 3 people, the number 3 is repeated in the column for
tables of 3. Comparing the two columns with the totals she found how many people were in the restaurant, that is, 87.

In solving the problem, the student uses the concept of fraction to “separate” the restaurant customers by gender, as mentioned in her answer “… as the number of women is exactly twice the number of men, it can be concluded that the sum is represented by three thirds, one third were men and two thirds were women…”

She also uses the notion of multiples of four to define the number of people sitting at tables of four.

![Figure 3. Print-screen of Carol’s representation](image)

When inserting formulas in the spreadsheet, she applies the concepts of variable and functional relations to find the number of men and women as well as to calculate the total number of people in the restaurant (in the last column).

It is apparent that using the spreadsheet stressed the need to identify all the relevant variables and encouraged the search of functional relations. In addition, it led to a strategy that allowed addressing the two conditions involved in the problem separately, and later making their connection by finding equal outcomes in both analyses.

Anne took a different approach from that of her colleagues (Figure 4).

This student began by considering the condition that relates the number of men and the number of women and after obtained the total number of people.

The student concluded that the totals were multiples of 3. Then she subtracted 3 people to the total of persons and divided the result by 4. In cell H3, the student entered the title “Number of tables of 4” and in the line bellow, the formula “= G4 /4” and then dragged the handle of the cell.

In her answer, Anne wrote: “The maximum number was 87 for it was before the 4 friends came, if I considered the 99 and added the 4 friends I would get 103 but the capacity of the restaurant is 100 people, which means it is not the solution.”
We can see how Anne expressed the conditions in the spreadsheet and how they were chained in a particular sequence without separating them as in the previous examples.

These solutions show the importance of the identifications of all the variables and conditions in the problem. The work with the spreadsheet enables to validate the equivalence of expressions and experimentally to determine the solutions of simultaneous equations. In the three cases presented there is a clear image of how it is possible to generate different equivalent equations that translate the given problem. This is a fundamental concept that can be grasped with the use of the spreadsheet and that can be developed in subsequent symbolic approaches. Different equations may represent the same problem and this means a fundamental path to engage students in realising how they can be transformed into others and to uncover instances of algebraic transformations in a set of conditions.

Concluding Remarks

In any of the solutions presented the students identified the conditions and expressed them in the specific language of the spreadsheet. They have recognized the relevant variables and through the definition of columns expressed the relationships between these variables. Furthermore it is possible to observe the correlation that exists between each of these conditions expressed numerically in the spreadsheet with the conditions of the system of equations shown in Figure 1.

We found that the spreadsheet helped the students to establish relations between variables, expressed through numerical sequences and with the use of formulas to produce variable-columns.

We claim that algebraic thinking was fostered by the affordances of the spreadsheet in generating the rules imposed by the problem. This result resonates with other research reports such as Ainley et al. (2004) but it also highlights the structure of students’ algebraic thinking expressed in a particular representation system. It provided a clear indicator of how students interpreted the problem in light of their mathematical knowledge and their knowledge of the tool. The analysis allows us to make inferences about what is gained in using the spreadsheet to solve algebraic problems, and helps to understand the relationship between the symbolic language of the spreadsheet and the algebraic language. The use of the computational tool can be seen as a means to fill the gap between the algebraic thinking and the ability to use algebraic notation to express such thinking. The lack of algebraic notation and formal algebra methods does not mean the absence of algebraic thinking. The kind of algebraic thinking that emerges from the use of the spreadsheet is the kind that belongs to global algebraic activities (Kieran, 2004). Our perspective of algebraic thinking stresses the distinction between algebraic notation and algebraic structures, separated by a gap that is often underestimated. We suggest that this gap can be gainfully filled with suitable spreadsheet activities.

Rather than insisting on any particular symbolic notation, this gap should be accepted and used as a venue for students to practice their algebraic thinking. They should have the opportunity to engage in situations that promote such thinking without the constraints of formal symbolism (Zazkis & Liljedhal, 2002).
The use of the spreadsheet in problem solving provides the establishment of connections between arithmetic and algebra. It strengthens the understanding of the functional relations involved, and the way they combine. The spreadsheet is an educational option to help students in the transition from arithmetic to algebra by making these two fields cohabiting.

References


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Didactic conception of using the interactive GeoGebra based educational assistance for introducing concepts connected with averages - preliminary research results

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There are many interesting concepts connected with different notions of averages in the curriculum. Part of them seems to be not quite difficult for pupils, but they use them rarely cause it's really serious problem for them. Main difficulty for pupils seems to be the transfer from numeric accounts and symbolical exemplars to the geometrical conception. The fact that the teacher does his best during classes can be not enough to overcome the problem. Here comes the idea of didactic assistance, that will be available “outside classroom”. Interactive material accessible to pupils by Internet may be helpful because it not only could explain but also widen topics by some additional materials. In my lecture I'll show such didactic conception of using the interactive GeoGebra educational assistance for discovering and introducing concepts connected with averages published on e-learning platform. This help focused is not only on traditional materials including classic tasks, but also it goes beyond range of the curriculum. In my research I concentrate on 16 - 17 years old pupils.

Characteristics of the experimental sample

The experiment was carried out in two secondary school teams in class I (students aged 16) and class II (students aged 17). In these classes there are students interested in Mathematics, Physics and Computer Science, who, after graduating from the junior high school consciously decided to continue their education in a class with such a profile. During the course they use the available resources of information technology, such as the Internet resources, interactive board, computer programmes (i.e. GeoGebra) as well as remote learning (moodle – the e-learning platform). Students belonging to these classes willingly deal with a problem, which would be, for many people, unsolvable in a traditional way. However, with a skilful usage of the information technology available, the task becomes easier to solve. They are aware that the information technology is designated to illustrate certain hypotheses, but it is not a justification, solution or proof of the basic problem. After analyzing the problem the students make an attempt to formally solve or prove the given task. In Polish schools, such a class team is a rare innovation, encountering reasonable resistance even among the teachers. For the students from class I the issues presented in the course of the experiment were introduced for the first time. However, for class II, which discussed these issues last year in the traditional form, the course was a repetition of knowledge as well as consolidation of skills with the addition of extracurricular content.

The theme of the experiment

In the Polish educational law for the school year of 2011/2012, there are two documents, which determine the high school students’ preparation for the matriculation examination; it is the core curriculum [2] and the examination standards. From 2012/2013 there will be a new core curriculum [3], which will replace also the examination standards. According to these documents students, graduating from secondary school and taking the matriculation exam on the basic and advanced level are to master the ability to use the arithmetic mean, the weighted arithmetic mean and the median. Among the abilities, mentioned by the above documents, attention should be drawn to efficient reading and processing of diagrams and charts. The student, taking the exam, is also to interpret, use and create mathematical text, definitions and theories, as well as proofs. According to the guidelines, mentioned above, the aim of the experiment was to get the students accustomed with the theorem of the classical and positional mean, which goes far beyond the framework of the curriculum currently in force, as well as the curriculum, which will apply from the school year of 2012/2013. The presentation of the course was to use the specific of the experiment sample, which deals with information technology on the everyday basis.

Purpose of the use of the interactive teaching aids

During the experiment tools like e-learning and interactive aids prepared in the GeoGebra programme were used. When using the platform the students had an opportunity to get familiar with new content, and then deal with the problems set. Materials prepared in the GeoGebra programme
illustrated the geometric interpretation of issues. After practicing each part of the material, the students took part in tests, which diagnosed the progress, made at each stage. The test closing the cycle was a test in a traditional form, covering issues of the advanced level of the matriculation exam, with optional content submitted during the course.

The research questions were: Can the information technology, used in such a way, help in acquiring knowledge and skills? Can the information technology, used in such a way, “be better” than traditional teaching?

The experiment

Materials

The students in both classes received the same materials and the same time was given for finding the solutions. The enthusiasm and eagerness to tackle the problems were equally high in both classes, but they began to decrease in class II as they were exploring new areas beyond the curriculum. Materials submitted during the course were divided into three parts (classical means, positional means and theorems), and there was a test after each part. The students were given the formal definition of the mean, and then they were to find solutions to tasks, related to the issue. Classical means were divided into 5 work sheets (located in the moodle platform): arithmetic mean, geometric mean, root mean square, harmonic mean and the sheet on the Cauchy Theorem, which discusses the connections between the means mentioned, closing this part. There were interactive teaching aids prepared for each of the work sheets, illustrating the geometric interpretation of the mean (in a segment, triangle or circle). After solving all e-worksheets, the students took part in the quiz, located in the moodle platform. Every student in a convenient time and place could take the test within the time limit provided for it. The next part of the material contained only one e–worksheet, with the issues related to the notion of positional mean. The students after getting to know once more the theorem on the median, modal and quartiles as well as solving the tasks in the e-worksheet, took the test, which this time included all the material covered (classical mean Cauchy Theorem, as well as positional mean). The last part consisted also of only one e-worksheet. It was concerned with the making of hypotheses concerning the relationships between the different means in plane geometry and stereometry, successive presentation of the geometric interpretation of the hypothesis and then providing proofs. After this set of 10 tasks, the students took another quiz, covering the material from the whole course. After each quiz the students could correct it. The last final element of the course was the traditional test, containing tasks from the advanced level of the matriculation examination, as well as the issues presented in the course, beyond the core curriculum.

Tasks

The tasks, put before the students, have been placed on the e–learning platform. As a task solution the student sent an e-worksheet or a file, illustrating a problem, made in the GeoGebra programme or a set of two files. E-worksheets on the classical mean contained an average of 7 such tasks, whereas the e–worksheet on various theorems was the biggest of the e–worksheets and contained as many as 14 questions posed.

The tasks, included in the e–worksheets, were very different. Starting from the typical tasks on providing a general formula for a certain amount of numbers, or calculate a number knowing the average value and other numbers in the set, ending with “prove” type of tasks.

The students were given problems such as a formal justification for why the length of the segment between the middle parts of the parallelogram arms is equal to the arithmetic means of the length of the basis. The students, using their knowledge, did not limit themselves to one correct solution, and presented their solution, using the resources of information technology (fig. 1).
GeoGebra was used also in situations where it has not been arbitrarily enforced. Just like it was done in the example above.

The students also received ready files, illustrating the problem. One of them was to prove that the length of the segment $x$ is equal to twice the geometric mean of the length of the circle radii (fig. 2a),

or Prove that $|AG| = \frac{1}{2} \sqrt{\frac{|AB|^2 + |AC|^2}{2}}$ (fig. 2b).

During the problem solving the students from class I did the tasks more precisely and more correctly than the students from class II. It would seem that it should be quite the opposite as for the students from class II it was the strengthening of knowledge, gained by means of a traditional method.

**Quizzes**

Prior to the quiz the students were informed on: the extent of tested material, how many tasks the test will consist of, the type of tasks used (multiple choice with more than one correct answer, true/false, computational, short answer, matching), as well as how much time will be provided (fig. 3). Analyzing the results gained in the three quizzes class I turned out again slightly better than class II. The students of class II, feeling more confident, did not use the entire time, which was provided for the task solving. Whereas persistent class II used almost the maximum amount of time, designated for the task solving, which helped them achieve a better result.
This mechanism is not without flaws. The experiment showed that three of the students from class II tried to “outwit” the moodle system. After the quiz has been finished by one person, the solution, which gave a satisfactory amount of points, was given to other people. Still, the platform tools revealed the scam, which has been captured, and additional “face to face” examination of the persons involved, verified their lack of knowledge and abilities.

**Test**

The students received a set of 11 tasks, and class II had an additional task, where they should incorporate the knowledge of quadratic functions (as class I have not yet discussed this issue). The analysis did not include the additional task for class II. The percentage scores are as follows (fig. 4):

All the students, who did not take the test within the first scheduled period or received a result of less than 55 % (marked as red line on fig. 4), had a possibility to work independently on the platform. There they could find a set of tasks in a traditional form as well as a quiz, preparing for the test correction. All student in class I received results above 55 %, whereas in class II 5 students did not get
a satisfactory result. Tasks difficulty presents figure 5. There are 5 grades of the difficulty in Poland. Their value are equivalent to proportional realization by pupils. The level of difficulty was 0,63 (0,72 - the correction), for students of class I, whereas the level of difficulty of the same set for class II was only 0,53 (0,61 – the correction).

Conclusions

On the basis of the experiment, it can be hypothesised that the transfer of knowledge using information technology, in the broad sense of the term, makes it easy to absorb new knowledge compared to the traditional forms of communication. The material presented in such a manner, based on blended learning, interact with Aronson’s theory [1] of social facilitation. The students eagerly made use of the available materials, according to the above theory. Issues that caused difficulties, were learned independently (at the students’ initiative), and based on available materials. Studies have shown that this possibility is very beneficial.

As for class II the material was not new, but the consolidation of the knowledge gained previously, we would expect their results to be very high. In the course of the experiment it turned out that the deepening of knowledge did not lead to the expected high didactic results. Therefore, one can put forward the hypothesis that complementing of knowledge, gained in the traditional way does not allow to achieve such results as learning the new material from the beginning, with the usage of IT.

The tool used in the course of the experiment is not perfect and there is a possibility of glitches. However, the mechanisms available help guard against such illegal scams.

The constant use of IT changes the form of solutions presented by the student. The student reaches for the interactive visualization in the form of, i.e. a GeoGebra file, like others reach for a figure.

References:

Analysing the Solution Files of T-algebra to Improve the Support for Students

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T-algebra is an interactive learning environment for basic school algebra. The student solves tasks step by step. The program verifies each input and displays error messages. The student should correct any mistakes before the next step. T-algebra contains an automatic solver and is able to give hints in any situation. T-algebra saves in solution files the student solutions together with error and help request situations. The paper describes how we used the reviewing facilities and additional scripts to identify the difficulties caused by T-algebra itself.

Introduction

T-algebra is an interactive learning environment for four areas of school mathematics: calculation of the values of integer expressions; operations with fractions; solving of linear equations, inequalities and linear equation systems; operations with polynomials. We have tried to design an environment where the student can simultaneously train two important skills: finding an appropriate path of solution steps and execution of concrete steps. We have also tried to support both of these aspects in T-algebra through help and feedback facilities.

The student solves tasks step by step. Each solution step in T-algebra consists of two substeps:
1) selection of an operation from the menu and marking the operand(s) in expression;
2) entering the result of operation.

The program verifies each substep and displays error messages, if needed. The student should correct any mistakes before the next substep. The program contains an automatic solver and is able to scaffold any concrete action: selection, marking or input.

The support mechanism in T-algebra is quite sizeable. For example, T-algebra supports 61 expression conversion rules and the collection of respective error messages contains over 1000 lines. It would be too optimistic to suppose that all thousand error messages are mathematically correct, well-understandable, describe all possible situations with sufficient level of completeness, and guide the students quickly to the right actions.

T-algebra saves in solution files the student solutions (both finished and unfinished) together with all error and help request situations. The student program has facilities for reviewing this information. We have also created additional teacher tools for reviewing the work of the group of students on an assignment.

The paper describes how we used the reviewing facilities to identify the difficulties caused by T-algebra itself. As a result, we made some changes in the operation of the student program, reformulated instructions for steps and error messages, but also developed additional recommendations for teachers on how to obviate certain difficulties. Before presentation of this main material, the paper gives a brief overview of the solution dialog, student support modes and reviewing tools of T-algebra.

Solution dialog and support for students in T-algebra

We give, first, a very brief description of the solution step dialog in T-algebra. More detailed descriptions are published in (Issakova, Lepp & Prank, 2006; Prank, Issakova, Lepp, Tõnisson & Vaiksaar, 2007).

The student solves expression manipulation tasks step by step. The step dialog of the majority of conversion rules consists of two substeps:
1) selection of an operation from the menu and marking the operand(s) in expression;
2) entering the result of the operation.
The first substep contains two actions. The student can perform them in any order. Figure 1 demonstrates essential parts of the solution window during execution of the first two stages of a step.

![Solution window during the first two actions.](image)

Figure 1. Solution window during the first two actions. For the next step in the solution, the student has selected the operation Combine like terms and has marked two terms for combining. T-algebra starts to check the operation and marking after the student clicks the green check button on the virtual keyboard. The upper frame displays the text of the task; the lower frame contains instruction for current work.

If the operation and operands are acceptable, T-algebra copies the unmarked parts from the previous expression onto the next line and asks the student to enter the result of application of the selected operation to the marked operands. T-algebra has three input modes for entering the results of operations: free input, structured input and partial input. The actual input mode used is defined by the teacher in the task file and cannot be changed by the student. Figure 2 presents the three different modes of input, resulting from the situation in Figure 1.

![Three input modes.](image)

Figure 2. Three input modes.

When the expression has been converted to the required form, the student should click the button “Solved – give answer”.

T-algebra provides several types of support for students:

1. Instructions for selection of the task, selection of the operation, marking, entering the result of the step, etc., are given in the lower frame of the solution window.

2. In structured and partial input modes, T-algebra pre-defines the form of the result.

3. T-algebra contains an automatic solver that implements textbook algorithms for all task types. The student can request help for selecting an operation, marking the operands and entering the result of the step. The program can also generate a full solution. Availability of each of the help facilities is set up in the task file.

4. Error messages are the most detailed part of the student support. At the first substep, T-algebra checks whether the marked parts are syntactically correct subexpressions and whether they are suitable for selected operation. At the second substep, T-algebra checks syntactical correctness of entered expression(s) and equality between entered and solver-computed result of the step. For many operations, T-algebra also checks whether the structure of the result corresponds to the performed operation. At the end of solution, the program checks whether the expression (equation, inequality, system) is really in the final form (required for the current task type). In case of mistakes or deficiencies, the program issues an error message, highlights a part of the student’s marking/input in red, and requires correction of the mistake.
Reviewing facilities of T-algebra and additional tools

Different programs for doing algebraic conversions have very different facilities for recording and reviewing of student activities. Many small programs but also mathematically sophisticated MathXpert (Beeson, 1998) do not record student’s steps. On the other hand, Aplusix contains ‘video tape recorder’ that allows to record and replay every action of the student (Sander, Nicaud, Chachoua & Croset, 2005). But the authors own that analyse of this detailed information requires ‘several hours for one hour of one student problem solving session’. ASSISTment (Feng & Heffernan, 2007) asks from the student only the final answers but records also successive corrections made after one or more hints given by the system in case if the answer was wrong.

In the solution file, T-algebra records student solutions (complete or incomplete) of all tasks, all instances of error and help usage, all uses of the Autosolve function, as well as information on the time spent on each task. The student program of T-algebra enables to open the solutions as they appeared on the screen when the student solved the task, and offers five additional views of recorded information:

1) table of errors in tasks (where errors are classified in 20 types);
2) list of errors (with a possibility to open each error situation);
3) table of help usage in tasks (help requests are classified by the action in solution step);
4) list of help requests (with a possibility to open each situation);
5) table of general statistics by tasks (numbers of errors and help usage, solution time, etc).

Figure 3 presents a solution and one error situation

![Figure 3. Review of a solution and an error situation therein (in structured input mode). Error message “Wrong member” is displayed in Estonian as recorded in the solution file.](image)

We also implemented some auxiliary teacher tools for collecting information about the work of an entire class or a group of students. These tools can create:

1) student x task tables of solved/unsolved tasks, step counts, solution times and error counts;
2) error message x task table of errors made in tasks;
3) list of sticking points where the student has made multiple errors at the same place;
4) list of appearances of user-defined strings (for example, subexpressions or elements of error message) in solutions or in the parts of solution files that refer to errors.

Further details about the reviewing facilities are presented in (Prank & Lepp, 2010).
Student errors that point to problems in error handling

In this section we describe how the data from solution files and our tools for analysing them were used for monitoring and evaluation of the guidance provided for students.

1. Impossible operations. We start with a series of discovered difficulties that compelled us to change some aspects of error checking and formulation of error messages. Even in our initial plans for the step dialog, we were already hesitant about separate checking of selected operation. In many cases, the steps are quite independent and it is not important to follow the ‘official’ order of operations. On the other hand, some students follow the learned solution algorithm mechanically. They would submit an operation for checking before thinking whether the expression (or equation) contains parts for which this operation is required. If the dialog also required marking of operands before the program checks the operation, such students would, in most cases, understand that the operation should be omitted. In our first experiments with students, we also saw that some students wanted to mark operands before selecting an operation.

We decided to combine selection of operation and marking of operands in one “mixed” substep. We also decided to skip checking the operation itself and to check only its consistency with operands. If the selected operation could not be applied to the marked operands, an error message would indicate which property was missing, for example, ‘Selected terms are not members of a sum’. Such messages worked normally when we tried them with students of a few teachers, who worked with us on the project. However, in our first experiment on linear equations with students of seven schools, we saw that many students of weaker teachers selected irrelevant operations because of problems with terminology. They confused ‘Reduce’ with ‘Combine’ and ‘numerator’ with ‘denominator’. They wanted to apply ‘Add/Subtract numbers’ to expressions that were not numbers but contained variables or tried to apply ‘Multiply/Divide numbers’ instead of ‘Reduce’. Such misunderstandings often caused repeated errors, while the student was thinking about a completely different action and was not able understand the error message. To eliminate at least a part of such repetitions, T-algebra now first checks whether the selected rule can be applied to any subexpression of the current expression. If not, a corresponding message is given. Another conclusion was that we recommended teachers to demonstrate a larger range of operations and to invite the students to ask more questions.

2. Unfinished solutions. Auxiliary programs for group views generate tables of errors and step counts, highlighting the cases where the student has not finished the solution. If the unfinished solution is not the last of the session, the student usually has had a problem and has not found support from the program.

There is one recurrent source of interrupted solutions: situations where the student should enter the result of the step but is not able to perform the necessary calculation correctly. The program does not accept the wrong result and requires new input. Sometimes students use a ‘trial and error method’, sometimes they give up.

![Figure 4. Trial and error method for opening parentheses (left column) and an interrupted solution with two error descriptions (right side)](imageURL)

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Our examples of calculation difficulties are from the experiment with linear equations. The tasks in this experiment were given in the structured input mode and, correspondingly, the results were entered in a set of pre-defined boxes. The left column in Figure 4 presents consequent inputs of a successful application of the trial and error calculation method. The right part shows an interrupted solution where the student tried to convert the improper fraction to a mixed number but gave up after two attempts. Another issue for further thought is the fact that, in the last situation, the recommendation given by the solver of T-algebra is not reduction or conversion to a mixed number, but use of ‘Add/Subtract numbers’.

The problem of support with calculation difficulties is more pedagogical than technical. The help system of T-algebra enables to get the result of application of the operation from the program by clicking the button with computer monitor (Figure 1). However, teachers are usually afraid that unmotivated pupils do not perform the calculations and simply click the button instead. Teachers disable the automatic insertion of the result (although they permit help for the first two actions of the step). A partial solution could be addition of an intermediate option where the right result would be available after a certain number of trials. However, what would be the right number of attempts? It is easy to imagine a teenager, who prefers to enter five arbitrary answers in quick succession and get the correct result from the computer. In some occasions we could offer an hint instead of typing the result. For example, in the case in Figure 4 (right side), advice to reduce the fraction by 5 could be helpful.

The concrete cases of abandoning a task can be surprising. Sometimes help is available but the students do not use it. In our experiment with linear equations we saw interrupted solutions where the student had already selected the right operation but was unable to perform the marking (although help in the form of automated marking was not disabled). In case of substitution of a variable with its value (a number or an expression), students receive the instruction to mark exactly one variable in the expression/equation. Nevertheless, many students (in different schools) tried to mark larger parts of the equation and repeated this in different forms.

3. Most common error messages. In all real-life classes where T-algebra has been used, ‘Calculation error’ is the most frequent error message, often followed by ‘Incorrect sign’ (if the problems enable making sign errors). For example, we analysed data from one 45-minute session in a large school in Tartu. The students had previous experience with T-algebra. The task file contained 30 tasks on addition, subtraction and multiplication of fractions and mixed numbers (only positive). The average number of solved tasks was 15.2. The message ‘Calculation error’ appeared 487 times, ‘Reducing is impossible’ 123 times, a reducible fraction was proposed as an answer in 103 times, an improper fraction in 66 times. There were also 104 failed calls of the rule ‘Multiply/divide numbers’. They were caused mostly by marking of an entire fraction that contained a product in numerator or/and denominator (T-algebra requires precise marking and allows counting only one product at any one step). We believe that this marking problem can be solved by instructions given by the teacher.

However, the solution files of our linear equation experiment contained several cases with a large number of error messages, which were clearly related to the interface. The first hours of work frequently brought the message ‘Nothing is marked’. It usually appeared in situations where the student received an error message at the first substep and decided to change the operation. This usually also requires a complete change in marking. To remove marking (green) from a box, the student should select this box (or a part of the expression that has a nonempty intersection with it) with the usual blue selection colour and click the button with the green box and red cross (see Figure 1). However, when the cursor was positioned on this button, the program displayed the hint ‘Unmark objects that are selected’. The students’ interpretation was that simply clicking this button was enough to remove previous marking. We changed the hint text to ‘Remove green boxes that are marked with blue or red’. We also recommended the teachers to provide a better demonstration of marking.

A quite strange source of systematic errors concerned operations that apply to the whole equation (reversing of sides, multiplication/division by a number, etc.). T-algebra enables to select them without marking or with marking the whole equation, but many students, for some reason, marked only certain part(s) of the equation.
4. Data mining results. The large number of calculation mistakes in the abovementioned session on fractions directed us to extract situations where students made more than one mistake at the same step. The tasks were solved in the free input mode where the student gets one input box for the result of the step. Weaker students were obviously not ready for performing operations with fractions in one step. They entered different wrong answers and tried to guess the result. It could be better to use the structured mode where the program asks first for the common denominator, then for the factors to multiply each addend, etc. Actually, the free mode of T-algebra does not require the final result of the operation, either, but accepts working in small steps. For example, in case of \( \frac{1}{6} + \frac{3}{8} \), the student can enter the final answer \( \frac{13}{24} \), present the sum as one fraction \( \frac{4+9}{24} \), convert to common denominator \( \frac{4}{24} + \frac{9}{24} \) or \( \frac{8}{48} + \frac{18}{48} \), etc.

There was yet another reason for calculation mistakes. The file contained tasks where the initial addends were reducible fractions. Data mining showed that the students almost never reduced the fractions before operations. In addition to this, they usually used the product instead of the smallest common denominator for addition and subtraction. As a result, the numbers in the numerator and denominator became too large for their calculation abilities or for checking whether the fraction can be reduced or not.

We also found interface-related mistakes connected to handling of special cases of fractions (numerator 0, denominator 1, etc.). The menu of T-algebra contains a special group of rules for such patterns but their marking instructions were not accurate enough. For example, we had ‘Select fraction with numerator 0’ instead of ‘Select fraction or mixed number with numerator 0’.

Conclusions

The reviewing facilities of T-algebra were originally designed as support for students to re-examine their work and for teachers to check the results (including computer-aided assessment). However we quickly discovered that they are also an extremely efficient tool for discovering and communicating mistakes and deficiencies in T-algebra itself. Using this tool, we found many deficiencies of the program that were never reported by the teachers.

References


Computer didactic games as a tool for discovering reduction reasoning

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Reduction is a very effective type of reasoning, especially for solving mathematical problems. One of the primary aims of a school is to teach pupils how to think and how to argue. However, it is difficult to teach reduction in a natural way at school. It could be that special kinds of mathematical problems could help with this. A well-chosen educational computer game can provide problems to be solved. It can discreetly provoke a situation, where pupils discover reductive methods to win a game.

In this lecture I will show the results of my research using educational computer games to help form reductive reasoning in pupils aged 10 – 13 years. I have tested a few specially prepared PC games based on Flash technology “outside the classroom”. The results show not only the positive aspects of using such a tool, but also how, especially, students must change their way of thinking from visual perception to numerical inference.

Introduction

We live in a world that makes us more and more addicted to the Information Technology that surrounds us. Computers are already being used in every field of life. Their role in education has also become crucial. The teaching of mathematics, with the use of computers, is the subject of many discussions conducted by both teachers and educators. There has been some research on various problems connected with the function of computers in the teaching process. My research is concentrated on the discovery of the role they have in the process of forming reductive reasoning [9], [10], [11], [12].

Didactic game as mathematics tasks

In the process of teaching mathematics one can distinguish, with considerable simplification, four components: the forming of mathematical terms, task solving, mathematical reasoning, establishing mathematical language [8].

Mathematics is associated with task-olving. There is a lot of truth in this sentence, because tasks have played, play and will play a fundamental part in the teaching of mathematics.

There are many different classifications of mathematical tasks in literature [1], [2], [7], [14]. In all of them “didactic games” are displayed as separate types of tasks which, however, can realize the functions of other types of mathematical tasks. This is one of the basic properties of didactic games which can be used as training tasks, mathematical problems or educational “provocation” … .

What is a game? We can use the notion of a game as a series of actions (moves) executed by individual players or teams (of at least two persons), peaceably, with the rules set in advance, of which victory is the aim of one of the players (one of the teams) [4].

A didactic game is a specific kind of game, which bases its basic function on the child’s psyche, on the need to play, and this game consciously influences his intellectual actions [3].

The rules of a didactic game characterize that: the realization of moves peacefully within the rules of the game requires the realization of anoperation, which captures the aim of teaching. Each evaluation of the strategy of the game is connected to the discovery of property or dependence, which is perceived as the aim of teaching [13].

The utilization of the games is one of the methods that can make pupils more interested in mathematics, which impacts on their approach to this “difficult” subject making it a more positive experience. This positivism is the essential element of didactic success.

Currently, didactic games as a method of education, are becoming more popular in schools. The usage of such games has three main functions: motivating, encouraging intellectual effort; didactic,
teaching of the contents and methods of mathematics; educational, teaching the rules of teamwork [5].

Mathematical didactic games have well-chosen mathematical contents and well-constructed principles which lead to mathematical activities. Additionally, they introduce an element of rivalry.

Examples of didactic games focusing on reduction

All of the didactic games presented below (focusing on reduction) are Flash based computer games for two players. If there is no opponent the computer can play as the second player.

My research on the use of all educational computer games for forming reductive reasoning was conducted with the same group of pupils aged 10 – 13 years (IV – VI class of secondary school, about 45 pupils in each level). This is the critical period in Piaget’s theory at the stage of concrete operations and the stage of formal operations [7]. That is why observation of the growing pupils in this period is very interesting. Each pupil played every game and everything was recorded (on film) for further analysis.

Everyone who plays a game wants to win. The intention of winning the game creates a good situation for discovering reduction reasoning. Why? Using deduction to discover strategy in the games presented is hardly useful, because there are too many paths to analyse. But if we use reduction we will solve this problem more quickly. Reduction is a very attractive and effective method in this situation. Take a look at the first game.

Maths Meadow

![Image of Maths Meadow Game]

The Maths Meadow Game consists of flying over a meadow. There are two different boards (both named Meadow – one is depicted in figure 1). Both players control one pawn - a bee, and each make one move (one jump to the nearest flower) per turn. A bee can be seen on the flower at the very bottom of the screen at the beginning of the game. Arrows indicate all possible moves. The winner is the player who reaches the hive (at the top of the board).

There are two different boards and each has a different winning strategy. The winning strategy is a procedure which provides a win independently on the movements of the opponent.

There are too many paths to analyse in deduction to discover the full strategy this game requires. But if we use reduction we will notice some strategic flowers (marked in figure 1). The player should place the bee on the marked flowers to win. Reductive reasoning is the key to success.

However perception of the full reductive method is difficult and not every pupil is able to do it. It is reduced to a graphical recognition of positionally strategic points, or sometimes the winning path. This game allows such graphic analysis and recognition and that is why it appears to be easy. However, the teacher must be conscious that this is just the beginning of the path. The pupil has to pass from the graphic perception to abstract reasoning.

Do pupils follow this path?

The results of my observations are presented in figure 2 (for both boards of Math’s Meadow).
The graphs show in each game the percentage of pupils who solved the given task (how they played each game). Conventions from the graph: “Full strategy” means that the pupil discovered the full winning strategy (made reductive reasoning). “Almost full strategy” – the pupil discovered the whole winning strategy, but didn’t notice the last reductive step. “Lucky points” – the pupil observed the positions of strategic points, noticed the winning path and discovered part of the strategy. “No strategy” – pupil didn’t notice any important fact that would help him win.

Figure 2 – Math’s Meadow - results

It is easy to observe that most of the pupils discovered at least part of the strategy in both cases. This game seems to be easy to “solve” for them.

Many conclusions can be formulated based on the results of my investigations. Unfortunately, there is not enough space for deeper analysis of all the graphs. However, summary conclusions are placed at the end of the article.

**Matchtaking**

Another game is Matchtaking. This game consists of taking matches from a table, however, the principles are specifically defined. Players establish before the beginning of the game, how many matches there are on the table and the maximum number that can be taken by each player in one turn (fig. 3). For example there are 30 matches on the table and each player can take up to 5 of them at once. The winner is the player who takes the LAST match. The results of my observations are presented in figure 4.

Conventions: “Full strategy” means that the pupil discovered the full winning strategy in every situation (made reductive reasoning based on symbols). “Almost full strategy” – the pupil discovered the whole winning strategy, but did not realise the global situation or found a strategy for some situations but not one based on symbols. “Lucky matches” – the pupil found a strategy for specific concrete situations. “No strategy” – the pupil did not notice any important fact that would help him win or make any hypotheses.
This game is based on a graphic representation, just like the previous one. The idea for both of them is also similar, however, aspects of abstract reasoning based on numbers is more important in Matchtaking. It could be the reason, that more pupils had problems with this game.

Maths Laboratory is another didactic game, very similar to the previous one. The game consists of filling up a large laboratory cylinder, however, the principles are specifically defined. Players establish before the beginning of the game how much liquid is needed (required measurement) and how much liquid can be maximally added by each player in one turn (fig. 5). For example there are 150 volumes and each player can add up to 10 volumes at once. The winner is the player who fills the cylinder – reaching the required measurement.

The essence of the game is the concentration on abstract reasoning based on numbers. It is possible to analyse graphic representation of the problem (but this aspect is less important).

The results of my observations presented in figure 6 follows:
Conventions: “Strategy for all variants” means that the pupil discovered the full winning strategy in every situation based on symbols. “Full strategy” – the pupil discovered the whole winning strategy in every situation (but did not prove it when it came to symbols). “Almost full strategy” – the pupil discovered the whole winning strategy in all situations he played, but did not realise the global situation. “Lucky numbers” – the pupil found strategies for specific concrete situations but not for all. “No strategy” – the pupil did not notice any important fact that would help him win or make any hypotheses.

The signification of graphic representation in this game was less essential. Pupils had to appeal to numerical dependence and make abstract reasoning. This problem appeared very difficult for most pupils – almost 75% from the youngest class, and almost 50% of the rest were unable to make any hypotheses.

**Thinkers**

This game is based on analogous principles similar to the previous, but it is “fully abstractive”. The game depends on exchanging numbers in turns, to receive an earlier established number. Players establish before the beginning of the game the winning number (required number) and how many can be maximally added by each player in one turn (fig. 7). For example the aim is 10, and each player can add up to 3 at once. Example of the “dialogue”: “2”; “2+2=4”; “4+1=5”; “5+2=7”; “7+3=10 I win!”.

The winner is the player who reaches the required number.

There are two modes to the game: first - easier – the player just has to indicate the number he wants to add, second - more difficult – the player has to calculate the result of his addition (the number which is received after adding). Graphic presentation in this game is some kind of notation - actual situation - but it is only the artistic part. The main game is in the pupil's mind. The results of my observations presented in figure 8 follows:

Conventions of the graph is similar to the previous case (see fig. 6). This game appeared to be the most difficult of all. A considerable number of pupils were unable to solve this problem (as in the previous case). Only some of the older pupils (almost 15%) made any symbolic reasoning.
Conclusions

The games described above are dedicated for children. A well-made educational game can become an object of interest even for the most demanding player. In colourful, breathtaking graphics, with interesting music and sound effects, this is a relatively easy way to “smuggle” mechanisms responsible for formatting logical and creative thinking as well as the skill of discovering the strategy.

At the beginning of the process of forming reductive reasoning, the better game is the one with an abstract mathematical background and the support of nicely-developed graphical representation (such as Math’s Meadow). Analysis of a graphic situation can help us to recognise the essence of the problem for younger, less advanced pupils. Pupils appeal to the graphic aspect of the game very often. Their first observations are based on the perception of graphic dependences.

The next step is to generalize, and it is essential this process appears. Pupils should not stay only on the “graphic level”. Here is an important role for the teacher to take care of pupils proceeding along this path. My investigations show that this is a very difficult and complicated process, and not all of the pupils, 10 – 13 years old, are ready to pass it. However, even if this abstraction is too difficult for some pupils, it is essential to show the idea of this reasoning to them. Such games as Matchtaking for example, are able to make it possible for them to discover part of the strategy, to conduct at least some local reasoning, to collect new experiences which can “yield fruit” in the future. It is easier to pass from the graphic aspect to formal reasoning with them. Clever utilization of didactic games can help full comprehension of the idea of reductive reasoning. More difficult didactic games (such as Maths laboratory or Thinkers) allow pupils to stay within the convention of entertainment, while they struggle with difficult, full abstract mathematical reasonings.

My observations show, that it is possible, and also may be effective, to teach reduction from 10 years old. The desire of victory is a natural helping factor for uncovering winning strategies. It seems, that such didactic games are proper for pupils, independent of age, and it could be a natural way to teach reduction, thus leading to wider mathematical knowledge. Educational games help to develop ways of thinking, which should be the basic aim of education at school, but it is not possible without the general introduction of computers. Computers allow us to teach in a way that was unavailable up to present times.

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Using Sport to Engage and Motivate Students to Learn Mathematics

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This paper describes how technology has been used to motivate the learning of mathematics for students of Sports Technology at Loughborough University. Sports applications are introduced whenever appropriate and Matlab is taught to enable the students to solve realistic problems. The mathematical background of the students is varied and the required pre-requisite is a GCSE grade A in mathematics. Group projects include modelling the velocity of a downhill skier, the effects of lift and drag on the length of drive of a golf ball and the size of parachute required to ensure a smooth landing. All of these require the use of the Matlab. In-class engagement is enhanced by the introduction of electronic voting systems. Questions involving sports applications can be posed in-class and immediate feedback received. The effect of introducing such material, on attendance and progression rates, and student engagement is reported.

Introduction

Throughout the UK, there has been concern with the level of engagement of students in the teaching that takes place in universities. Sometimes this manifests itself in poor attendance. At other times students attend but play a passive role in the process. This paper describes the introduction of sports based problems into a first year mathematics module and reports on the outcomes of the initiative.

Loughborough University is renowned as being one of the leading universities for sport. Top sportsmen and women are attracted to Loughborough to study and train. Many wish to study sports related subjects and in recent years a course in Sports Technology has been developed. This course focuses primarily upon the design and manufacture of sports equipment. It is supported with a background of manufacturing technology, engineering science, mathematics, statistics and experimental design.

The School of Mathematics is responsible for the teaching of mathematics to the majority of engineering students at the University and teaches mathematics to the Sports Technology students. Prior to 2003-4 these students were taught mathematics alongside Manufacturing Engineering students in a class of approximately 100 students. However there was poor attendance by Sports Technology students at mathematics tutorials and a high failure rate in mathematics. The reasons were varied. Some did not see the relevance of mathematics to their course and therefore were not motivated to study the mathematics module. Others found the transition from school to university mathematics difficult.

It was decided that a new mathematics module, for first year Sports Technology students only, would be introduced for the academic year 2003-4. The author was assigned to teach the new module.

This paper describes some initiatives which feature in this module. These include the introduction of sports related problems into the syllabus and the inclusion of Matlab, a computer algebra system. Moreover, from 2008 onwards, electronic voting systems (EVS) were also introduced to, amongst other things, facilitate student interaction and engagement in lectures.

Some of the sports-based problems, used in group projects, are described. There are also examples of sports-based EVS questions and information about a website developed by the author with resources for EVS in mathematics. Examples of the students’ work are provided. The outcome of the initiatives is then reported. Attendance by the students on the module and their end of year results are compared with these figures prior to the changes. Motivation of the students is also discussed. Finally the paper provides comments upon some of the issues to be addressed if others wish to adopt some or all of the features introduced in this course.

Sports-based Projects

Sports-based group projects were introduced for a variety of reasons. Group projects and computer software enable inclusion of realistic problems and prepare students for final year projects and
industry. The ability to function as part of a team is a crucial skill for engineers and the introduction of assessed group projects allows students to learn the skill of teamwork. Moreover, group work provides the chance for students to learn from each other through discussion. MacBean et al (2001) provide an overview of the advantages and disadvantages of introducing group work in undergraduate mathematics. Moreover the group projects centre on applications of mathematics in sport. It has been noted (Yates, 2003) that a sufficient supply of discipline related problems is one of the factors leading to success in teaching mathematics to non-specialists.

The students undertake two group projects, one in each semester in their first year, and each is worth 10% of the 20 credit mathematics module. Typically the students work in groups of three and can choose from a selection of projects. As the mathematics entry requirement for the course is GCSE grade A, this has to be borne in mind in the setting of the projects. In practice, about one third of the students have this minimum mathematics qualification as their highest mathematics qualification, and the other two thirds have studied mathematics for one or two years post-GCSE. The first semester projects require knowledge of only basic algebra and functions. Calculus, which is introduced in the second semester, is used for the later projects. Copies of the projects described below can be obtained from the author on request.

**Downhill Skiing**

This project is concerned with a mathematical model which is used to predict the velocity and terminal velocity of a skier and the length of time for a downhill ski run. It is based on a model developed by Townend (1984) and further details are available in his book. In fact his book contains many excellent applications of mathematics in sport. The forces considered are the skier’s weight $mg$ (where $m$ is the mass of the skier and $g$ is the acceleration due to gravity), the aerodynamic drag force caused by the cross sectional area $A$, which the skier’s body presents to the air, and the frictional force between the skis and the snow.

An expression for the magnitude of the terminal velocity, $\bar{V}$, can be derived and is

$$
\bar{V} = \sqrt{\frac{2mg(\sin \alpha - \mu \cos \alpha)}{\rho AC_D}} \tag{1}
$$

where $\alpha$ is the angle that the slope makes with the horizontal and $C_D$ is a constant known as the drag coefficient. The expression for the velocity as a function of displacement is also derived in Townend (1984). In the project, students are provided with typical values of the parameters and then asked to use Matlab to vary certain parameters and evaluate the effect on $\bar{V}$. They also investigate whether the terminal velocity is a good approximation to the actual velocity predicted by the model and calculate the time taken for the ski runs. Figure 1 is a poster produced by one of the groups working on this project and demonstrates the results obtained (not necessarily all correct!). The students take pride in their work and make great efforts to produce posters, as well as written reports, of a high standard.
In fact this project was developed for the undergraduates by a final year mathematics student, Andrew Wong, who was working under the supervision of the author. Developing sports-based projects at the correct level for undergraduate students requires skill to ensure the mathematics is at the correct level and yet is challenging and it can be a very time-consuming process. Involving final year project students in this has developed their understanding of mathematics and assessment and has provided new areas of application for the Sports Technology students.

**Soccer**

This project is concerned with a mathematical model which is used to predict the effect of air resistance on soccer balls (see Figure 2). The model is derived in Daish (1972 pp134-137). It investigates shots at goal in a soccer match in Mexico City and predicts the velocity of the ball when it reaches the goalkeeper and the time the ball is in the air. The results obtained are compared with shots at goal at sea-level where the density of air is considerably higher than the density of air in Mexico City.

![Figure 2. Soccer Ball with initial velocity $V_0$ and subsequent velocity $V$ when drag force, $D$, acts on it.](image)

With knowledge of how to solve first order separable differential equations, students can verify that the velocity at displacement $s$ (for example, after it has travelled $s$ metres to the goal) is given by equation

$$ V(s) = V_0 \exp \left( -\frac{D}{m} s \right), $$

where $m$ is the mass of the soccer ball and $D$ is the drag force and the motion is assumed horizontal. Students can then compare velocities of the ball when different air densities are present, can calculate the different reaction times the goalkeepers have and can investigate the effect of varying the initial velocity, the mass of the ball, etc. Figure 3 is the poster demonstrating the findings of one group.

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Electronic Voting Systems (EVS)

In 2007, Loughborough University, via its Teaching Support Unit, purchased an Electronic Voting System. The author introduced EVS into the teaching of Sports Technology students in 2008.

Caldwell’s (2007) review of existing literature on EVS use is a comprehensive and detailed work that covers every aspect of EVS use including description of the technology, use of questions, effect on student performance and association of EVS with ‘peer learning’. The study also includes guidelines for writing good questions and best practice tips. The single, most important benefit of EVS use, identified from literature review, is its capacity to enhance, catalyse or increase student engagement during lectures. This was reinforced by the finding from surveys of students at Loughborough University who were introduced to EVS in the study of engineering mathematics. 145 students completed a questionnaire on the use of EVS in class. The results showed that the majority of students were extremely positive about the usefulness and overall advantageousness of EVS use in classes. Results also showed that EVS use did increase the likelihood of students participating and engaging in class. Students identified the main benefits of EVS use and their two most important benefits related to feedback. These were, ‘Checks whether I’m understanding course material as I thought I was’ and ‘Allows learners to identify problem areas’. Further details are available in King and Robinson (2009).

Examples of EVS questions related to sport

Figures 4 and 5 give examples of some of the questions used by the author in class. The two questions are related to the Vectors part of the course. The first question (Figure 4) is an example of a straightforward calculation and requires students to use text-entry handsets to input a numerical answer. The second question (Figure 5) is a multiple-choice question and the correct answer can be selected using a standard handset. (Note that this latter question is an adaptation of a question developed by the Mathquest project in America (http://mathquest.carroll.edu/). The second question requires much more thought by the student. In this type of question one could ask the students to respond initially and then, given the responses indicated, could ask them to discuss their answers with a fellow-student and then vote again. This peer-instruction can be a very valuable pedagogical tool to encourage deep learning in class, see for example Mazur(1997).
EVS Resources

There are many people starting to use EVS and one of the first things they need to do is to start to write questions for the topics they teach. Many are unaware that there have been projects, particularly in the USA, which have resulted in question banks being developed. The author thus successfully applied for sigma-cetl (http://www.sigma-cetl.ac.uk/) mini-project funding to collate information about existing resources for using Electronic Voting Systems (EVS) for mathematics and also to develop questions. A project website was developed (http://mec.lboro.ac.uk/evs) which provides information and links to relevant material. The website also provides over 300 mathematics questions which have been developed in the Mathematics Education Centre at Loughborough University. Also, the visitor to this website will find links to some key papers, and websites which review papers, on the use of EVS in general and EVS in mathematics in particular. The website was launched in March 2010, with a one-day conference. Key-note speakers at this conference were Steve Draper, Glasgow University, and Mark Russell, the University of Hertfordshire, UK. Their presentations are also available.

Questions developed at Loughborough University cover topics typically found in a first year university course in engineering mathematics. These include differentiation, integration, differential equations, complex numbers, matrices and vectors. It should be noted that most of the questions are not related to sport, but there is scope for adapting them for this purpose.

Outcome of the Initiatives

In this section the success or otherwise of the changes introduced is discussed. Firstly, attendance is considered, then motivation of the students and finally end of year results.

As stated previously, attendance had been an area of concern prior to the changes discussed in this paper. In 2002-3, the average tutorial attendance was 21%, with zero attendance in six of the eleven
weeks in semester 2. Since then average attendance has been in the range of 58 – 87%. It should be noted however that the form of the tutorials was quite different prior to and subsequent to the changes. In 2002-3 the tutorials took the form of a traditional mathematics tutorial where the students were expected to work through exercises and help would be available to those who required it. In following years the tutorials took place in a computer laboratory and the students worked through exercises using Matlab and/or by hand.

It is not easy to measure the motivation levels of students. Attendance, standard of work and student feedback can all be used as indicators. The increased levels of attendance have been noted. It has also been found that the students work hard on the sports based projects and submit work of a high standard, as is evidenced by Figures 1 and 3. Feedback obtained via the University’s standard module feedback questionnaire from 2003-4 onwards has had consistently high scores, indicating high levels of satisfaction with the course.

After just one year, the module pass rate increased from 55% to 94% and has continued at this high level.

Conclusions

The use of sports-based group projects and EVS questions in the teaching of mathematics to Sports Technology students has been reported. The introduction of Matlab provided students with a new challenge and also allowed more demanding sports-based problems to be set than would otherwise have been the case. It has been found that there have been significant improvements in attendance levels and in pass rates. Moreover the students appear more motivated to study the mathematics module. The overall picture is that of students who see the relevance of mathematics to their degree and future career and hence are motivated to study it.

For other practitioners wishing to adopt some or all of these changes, there are many issues to consider. Much time needs to be invested in providing context-based problems. As noted here, the involvement of final year mathematics students in developing projects has proved very valuable for the students themselves and has led to new projects for the Sports Technology students. However even these students need considerable advice and guidance in the development of the projects and so the work of the author was not reduced by their involvement. Developing EVS questions is also very time-consuming and it is hoped that the website developed by the author (http://mec.lboro.ac.uk/evs) will enable practitioners to search for relevant questions in the areas they teach. Finally, in the author’s opinion, it is worthwhile investing the time required if the effect is to motivate and enthuse engineering students in their study of mathematics.

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The problem of the digital divide for (math) teachers in developing countries

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In this digital era, there is a need to incorporate technologies to educational practices. In our country (Mexico), there have been many policy and educational reforms with this aim. However, many factors create a gap between this political will and the reality of school teachers (Ruthven, 2008), particularly in non-developed countries, where resources, access to digital technologies and training are scarce. Our aim was to look at how Mexican mathematics teachers use technologies in their practice. We drew data from a community of 71 middle-school mathematics teachers who were asked to sign up in an online forum, as well as through a survey of 140 high-school mathematics teachers and classroom observation of a subset of these. The results show some deep limitations that teachers have and point to the gravity of the digital divide in Mexico: digital competencies and use are scarce, and create an obstacle towards harnessing technologies for enriching the mathematical teaching and learning.

Introduction: The educational needs of the 21st century and our concern

In order to incorporate digital technologies (DT) to educational practices, and face up to the challenges of the 21st century, teachers need to enhance their professional knowledge and skills, and adapt their practice. In the United States, the National Council of Teachers of Mathematics (NCTM, 2000) recommends that technologies should be used widely, since these technologies can facilitate visualization of mathematical ideas, save computational time, and allow to focus on decision-making, reasoning and problem-solving. In our country (Mexico), there have been many policy and educational reforms with this aim, such as Enciclomedia which was a massive, large-scale Mexican government programme that began in 2004, devised with the purpose of enriching primary school teaching and learning by providing multimedia resources and applets linked to the official textbook lessons (see Sandoval, Lozano & Trigueros, 2006), and the Teaching of Mathematics with Technology (EMAT), also sponsored by the Mexican government from 1997 to 2007, for incorporating DT tools into middle-school mathematics classrooms (see Sacristán & Rojano, 2009). However, even in some of the most carefully conceived programs, such as EMAT, research results show that teachers have great difficulties in changing their practice as conceived by the designers and in harnessing the potential of DT tools (Trigueros & Sacristán, 2008). It is clear (see Assude, Buteau & Forgasz, 2010; Ruthven, 2008) that there is a contradiction between the political will for integration of digital technologies in mathematics and the realities in mathematics classrooms.

So we asked ourselves: how aware are teachers in our country (Mexico), of the ways in which they can use DT to enhance mathematical learning and in their teaching practice, and of the aims of the recommendations for DT use? Our aim was to look at how mathematics teachers use technologies in their practice, the factors and limitations affecting that use, and what awareness do they have of the potentials of DT for mathematical learning. Thus, in this paper, we present data drawn from nearly 200 Mexican mathematics teachers of different school levels that participated in two related studies.

Middle-school teachers limitations for participating in a virtual community of practice

In 2010, we worked with a community of 71 middle-school mathematics teachers and asked them to sign up in an online platform that we had set up for them, and that provided:

- Resources for their mathematics teaching practice and professional development: blogs, documents and presentations related to the face-to-face sessions of the community; repositories of specially designed resources and activities for these teachers; and useful links for downloading software and other external resources, as well as to other mathematics education sites and communities.

- Communication tools: discussion forums, blogs and activity calendars.

We programmed virtual activities to be carried out in the month between face-to-face sessions. They were meant to complement the work of specific mathematical topics that were at the time being
analysed by the community. For example, the virtual activities, which were posted in the community’s online discussion forums, included problems that would help teachers confront their own mathematical content knowledge of a particular theme, as well as help them in the design of teaching sequences that they would test with their students. The idea was for teachers to post their experiences and queries on the discussion forum. We also provided specifically designed resources, such as applets, using different digital tools (e.g. dynamic geometry, spreadsheets, etc.) that would help teachers see the different approaches to a mathematical topic through various technological tools, and analyse the benefits and limitations of each. It is worth noting that these resources were also provided at the request of members of the community who wanted to know how to use DT tools for their classroom activities.

Despite the fact that the virtual tasks and resources were an important part of the community’s activities, and that the teachers’ themselves had requested some of these resources, we found a low level of participation. When we first set up the online platform, and we saw that hardly any teachers had registered, we were forced to setup a special face-to-face workshop to assist the teachers in the use of the website, the discussion forums and for downloading resources or uploading their own files.

However during the workshop, we were surprised how new it was for teachers to use a computer. Many teachers had difficulties using the hardware (the mouse, the keyboard, etc.) and most teachers admitted not knowing how to use a web browser. Though some teachers had used computers for word-processing activities, even they did not understand how application windows and browsers function. Over a third of them did not even have an email account. And of those who did, half of them had accounts that had been setup by their children, so they did not know how to use them properly. When we observed them using email, we saw that several of them didn’t know where to write a message, sometimes doing it in the subject box. Therefore, simply registering for access to the site was a big obstacle: not only did it require an email account, but we also witnessed first-hand during the workshop how teachers had difficulties filling out the simple registration form that is shown in Figure 1.

![Figure 1. The registration form for the teachers’ community online platform.](image)

Despite the remedial actions on our part, many difficulties continued. After one year, still, only 54 out of 71 participants who participated in the face-to-face sessions of the community, registered on the website; and then, over that one-year period, the online participation was very limited, notwithstanding the fact, as mentioned above, that many activities, resources and discussions in the community were carried out online: over a third of the participants never participated in any of the virtual activities and only 16% of them posted on the discussion forum and uploaded some of their files.

Though these teachers admitted their limitations, they also expressed their fear of using technologies, and of exposing their lack of digital competencies and of asking for help: a teacher stated “some of us have difficulties using computers and we are embarrassed that our colleagues find that out”. Another teacher said: “I am afraid of making mistakes and of being exposed by making a mistake”.

Also related to the latter point, but in relation to the virtual activities that had been proposed, it became clear that the fear of being exposed also made teachers uncomfortable in sharing experiences and concerns through the Internet. In interviews some said that they didn’t want to participate in the discussion forums because they did not want to “expose” themselves to their colleagues (and added that they preferred instead to simply receive information on teaching strategies). This also points to the lack of familiarity that these teachers have with the Internet, since they could participate in the forums without revealing their identity and that would prevent the
“exposure” that concerns them. (Not surprisingly, the use of the online platform for reading posts and maybe downloading resources was much greater than that for contributing).

All of the above fears and insecurities create a vicious circle, since they prevent teachers from developing the competencies they need.

**High-school teachers' use of technologies**

In another part of our research, we have been analyzing how Mexican high-school teachers use DT in their classroom practices. For this exploratory and descriptive study (Gay & Airasian, 2000), we’ve used as methodological tools, surveys, interviews, and videotaped in-classroom observations. This is an on-going study and so far, we have surveyed 140 high-school mathematics teachers from many regions in Mexico. A large majority of them, 72.5%, claimed to use DT to support their mathematics teaching practice. And 65% said they used Internet to search for theoretical information related to the topics studied in their classes, to search for formulae, or to send homework to their students.

So we’ve tried to directly interview and observe, in their classroom practice and in their supposed use of DT for that, as many as possible of those who claimed this. At the time this paper was written, we had observed 15 teachers during at least two classroom sessions each (thus, over 30 observations); however the only use of technologies that we witnessed, was the case of three teachers who each used a beamer to project some graphs of functions. Through interviews we had some of the following explanations:

- I only use it once per term or school year to teach students how to use some software and then I only give them some homework.
- The school doesn’t have the necessary resources such as connections, electrical extensions, beamer, etc.
- I only use it for preparing my class.

However, all 15 teachers mentioned that they had taken professional development workshops on the use of digital tools, mainly on the use of graphing tools such as Winplot – which was also the most common use mentioned; the use of information and communication tools (i.e. Internet), and some dynamic geometry software (i.e. Cabri, SketchPad and Geogebra).

When asked why they would use DT in their classrooms, most of the answers were for comparative purposes or to save time, such as using tools (e.g. Winplot) that would facilitate the construction of graphics. One teacher mentioned that in order to save time, she would give her students as homework the task of producing a graph with a graphing tool. Interestingly, another mentioned that he had shared with his students a graphing tool sent to mobile phones, but this was so that they could compare graphs drawn with paper-and-pencil with those produced by the tool.

A surprising response was that of a teacher who said he only used DT because it was a requirement of his school.

Though the high-school teachers in the study contrast with those of middle-school, in the sense that the majority do have basic digital skills, the findings point to a very limited and scarce use of technologies (if they use it at all, it seems they only once or twice during the academic year), with little awareness of how to harness these tools for supporting teaching, learning and exploration in classrooms.

**Discussion and final remarks**

Twenty years ago, Kerr (1991) wondered why technology was not more widespread in classroom instruction, stating “after eighty years of efforts to apply technology in ways that would ‘revolutionize’ education, most teaching practice today looks remarkably the way it did at the beginning of this century” (Kerr, 1991, p. 114). This seems to still be particularly true today in countries like ours; but in general, even for developed countries, the accumulated evidence at the beginning of this century pointed to a slow incorporation of technology into classrooms (Cuban, Kilpatrick, & Peck, 2001). Since then, the role and involvement of teachers in the process of technology-integration into (mathematics) education has been increasingly emphasised. Hennessy,
Ruthven & Brindley (2005) state that it would serve to elevate the role of practitioners in effecting classroom change. Moreover, they point to a shift away from a technologically-driven model of technology integration, towards one based on teacher involvement; but they also point that this involvement is undoubtedly influenced by the teachers’ working contexts.

The above results illustrate some of the present realities of teachers in developing countries, such as Mexico, that may hinder the incorporation of technologies in teaching practices as well as the involvement of teachers. In this sense, we consider that there are two types of limitations and obstacles. One is related to digital divides; the second is related to professional development of teachers and the educational system itself.

The results, particularly the ones related to the middle-school teachers, point to the gravity of the digital divides in Mexico, and of teachers as digital immigrants (Prensky, 2001). When referring to digital divides we think both in terms of generational gaps but also in terms of the geographical digital divide; that is, it seems that in developing countries like Mexico, in certain sectors of society and for certain generations (which is the population that most teachers belong to), the everyday use of basic technologies and ICT, like email, which seems to be taken for granted in developed countries, is still not commonplace. If teachers are ill-prepared to use DT even in their own lives, how can we expect them to develop the skills necessary for a meaningful incorporation of these tools in their practice?

In terms of professional development and the educational system, interviews of teachers in both groups show that they lack the preparation, and sometimes even lack motivation to develop competencies for using technologies in their teaching practice in meaningful ways. One factor in this, is the lack of adequate training opportunities: some of the high-school teachers complained that the only type of training they had received were on the basic use of office software suite packages, and not in more specific tools for mathematics education (such as dynamic geometry or CAS).

Another important factor is time: lack of time to develop technological and pedagogical skills; lack of time to analyze how to exploit DT tools for improving mathematical learning; or simply lack of time to participate in professional development activities of those skills. In fact, some of the middle-school teachers in the community of practice presented above, expressed that they did not participate in the virtual activities because they considered them a waste of time, particularly when they wouldn’t receive anything in exchange (referring to some type of official participation recognition; it further seems that they did not consider their own development and learning, as rewarding enough).

Papert (1993) explained that innovation in schools is slow due to the resistance that teachers have towards the unknown. This is particularly the case in countries where resources, access to digital technologies and training are scarce, and therefore teachers are still unfamiliar with some of the basic uses and possibilities of technologies, as evidenced by the data presented in this paper. Hitt (1998) pointed out that teachers will only feel the need to incorporate technologies to their practice when they experience the effectiveness of a tool or resource in dealing with a problem. Teachers tend to teach the way they’ve been taught (Selden & Selden, 1997; Thompson, 1992); the little use of technologies we observed in the studies related here, is because teachers have not experienced other uses. Thus, rather than focus on the delivery of technical skills, it might be helpful if teachers can participate in professional development models that immerse them – and support them – in the experience of dealing with mathematical situations through technology (up to even participating in virtual activities and exchanges, such as the ones we proposed for our community of teachers; these, however, are often still too foreign processes in countries like Mexico). But the first challenge is how to help teachers simply plunge into a daily use of technologies so that these won’t be so foreign to them, and so that the digital divides that prevent them from living up to the digital needs of the 21st century, begin to be closed.

Acknowledgement

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References


Teachers engage in peer tutoring and course design inspired by a professional training model for incorporating technologies for mathematics teaching in Mexican schools

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Two years ago, at ICTMT 9 (see Sacristán, Sandoval & Gil, 2009), we presented some results from a 3-year professional development program, linked to a 3-year master’s course for Mexican in-service teachers; that program focussed on the incorporation of digital technologies (DT) for mathematics teaching and learning. In the paper from that previous conference, we discussed the participants’ experiences and transformations during the development program. Here, we present a schematic of the pedagogical training model that we used during in that program, and describe how, inspired on our own model, the participants engaged, by their own initiative, in peer training and course design for the use of DT.

Introduction

In previous papers (e.g. Sacristán et al., 2009), we presented some results derived from a professional development program that aimed to train six in-service mathematics teachers in the use of digital tools (DT) for mathematics teaching, by trying to assist them to develop a meaningful way of understanding how to incorporate these tools to their practice, thereby transforming it, as well as their school culture. That program was part of a 3-year master’s course and was linked to a research study that lasted over 5 years (until 2010), and that aimed to research and understand how changes in school practice and culture are brought about by the implementation of those tools into the classroom. During those 5 years, we were able to document the methods, techniques, resources and strategies that the participant teachers used and developed when incorporating new technologies into their practice. Each of the participants also had to carry out a long-term development and research study that they reported in written dissertations, and which also provided a wealth of data. In this way, we observed (as advocated by researchers such as Ruthven, 2008) the educational reality and daily practices of teachers when using technology; as well as the impact of our professional development program on the participants practice.

Related to the latter, in this paper, we describe a tangential finding that was not part of our research aims, but that we nevertheless found was very interesting and worth sharing: how four of the participants engaged in course design and training programs for other teachers (not involved in the development program) in the use of digital technologies, motivated by their enhanced appreciation of those technologies and inspired by the training model that we used. We will begin by presenting a brief description of that training model.

The professional development model

Our approach during the three-year professional development program, was for our participants to carry out classroom practices using DT and reflect on the changes in their teaching practice (following models such as that of Artzt & Armour-Thomas, 1999). Many authors have stated (e.g. Goldenberg, 2000) that some of the key factors for success and for transforming school practices are gradual implementation of DT with continuous teacher training and support. Thus, in the pedagogical design of our professional development model, we assumed, in accordance with a statement by Thompson (1992), that teaching must be a constructive process that requires reorganising and reinterpreting the subject matter and the practice as a result of experience. That is, we considered that the knowledge that is derived from social interactions in a (real-life) context is more valuable and significant for the teacher (Liu & Huang; 2005). It is worth noting that, although all six participants had previously taken some brief workshops on the use of some technological tools (Dynamic Geometry or Spreadsheets; or on the interactive resources for primary schools provided by the government-sponsored Enciclopedia program – http://www.enciclopedia.edu.mx), the use they had carried out of these tools in their practice, was extremely limited, if any at all, and they had no pedagogical knowledge of the possible applications of those tools for mathematical teaching and learning.
In Figure 1, we provide a schematic of the pedagogical model that we used during the professional development training program; this model involved several, almost simultaneous, activities by the participants: a) training and developing abilities (content knowledge and software competency), for the use of DT in the classroom (mainly open and universal tools such as Spreadsheets, Dynamic Geometry, CAS and Logo; as well as some applets); b) designing and planning of teaching strategies and activities to integrate DT; and c) engaging in observation and reflection-on-action (Bjuland, 2004) of the changes in their own teaching practice with the new tools. These activities were accompanied by collaborative and reflexive work between the participants (teachers-in-training; peers), and with us, the tutors (experts). In particular, the participants analysed and reflected upon the potentials, limitations and changes brought forth by the incorporation of DT into their practice, and that of their colleagues, from various perspectives (discussed in Sacristán et al., 2009), and identified problematic situations. The teachers’ observations and reflections were communicated through written, multimedia and oral reports, collaboratively discussed, and complemented by the study of theoretical frameworks and pedagogical models for a meaningful incorporation of DT into the (mathematics) classroom.

From participants to peer training and course design

As is described in other papers (e.g. Sacristán et al., 2009), half-way through the professional development training, the participants developed an appreciation of the potential educational benefits for mathematical learning of DT tools when used appropriately. This led four (those presented in this paper) of the six participants to be self-motivated to engage in the training of other teachers (not involved in the program) to help them incorporate DT in their practices. We will now try to illustrate how – in the peer training and course design that these teachers implemented with their colleagues – they apparently appropriate themselves of the reflective and collaborative professional development model that we had been using with them.

*The appropriation of the model by Jane and Sarah for peer training*

We begin by presenting the case of Jane and Sarah, who were primary school teachers in their forties with over 20 years of in-service practice; they were also pedagogical advisors to nearly two thousand teachers in 649 primary schools in their state. When our program began, Jane and Sarah had almost no experience at all using DT. After their own initial training in the master’s course, they gradually discovered some of the potential of DT for mathematical teaching and learning, and became interested in observing and training (both in mathematical and technical contents) fellow primary-school teachers in the use of DT in their teaching (as they each describe in González-Martínez, 2010; and Granados, 2010): Starting in the second year, Sarah worked with 9 fellow teacher-advisors and designed a project called “Support for the strengthening of mathematics” that had as aim for the participants to each observe and assist one 5th grade primary-school in-service teacher. In the same way as we had done in our development program, she held and conducted monthly meetings and workshops with her 9 peers, for the course of an entire academic year; in these meetings they could...
explore and discuss (from both technical and didactical perspectives) DT resources that could be linked to the curriculum and share their experiences from their field-work. On her part, Jane tried to work with 17 primary-school teachers of 5th and 6th grades in 5 schools; she designed activities to promote both individual and collective reflection by the teachers, on the possible benefits and limitations for the teaching and learning processes, of different DT tools (Spreadsheets, Logo, and the Enciclomedia program).

Then, in the third year, Jane and Sarah, due to their positions as advisors, participated in the development and implementation (in cooperation with the Mexican Mathematical Society and a local state university) of a state-government-sponsored training program on the teaching of mathematics. Jane and Sarah helped develop a module called “The use of technologies in the teaching of mathematics” and trained 900 primary school teachers in their area (Note: In Sacristán et al., 2009, we inadvertently omitted a digit, reducing to a tenth, the actual number of teachers involved). Inspired by what they learned in our training program, Jane and Sarah developed their own constructivist and collaborative model of working (Figure 2), to promote critical reflection on teaching with DT. Their model included analysis, reflections and dialogues on explorations of DT tools; the development of technical, didactical and content knowledge and competencies; practical (including planning) activities and observations; and the possibility of transformation of school cultures. In their module’s guidebook, it states:

The facilitator must propitiate collaborative work with a high dose of reflection on the teaching practice [...] All of this in order to promote communication and contact with peers, to come together through the socialization of knowledge (internal and external dialogue). (Granados et al., 2008; p.7. Translated from the original Spanish)

![Methodological model and structure](https://via.placeholder.com/204x345)

**Figure 2:** Methodological model and structure (as given in their module’s guidebook) for the course designed by Jane and Sarah for peer training (Granados et al., 2008).

In this way, the purpose of their module was to develop in teachers an awareness of how to use different DT tools in order to enhance mathematical learning, comprehension and develop problem-solving and other abilities in children. These purposes and methodology matched perfectly with those of our own professional development model. One of the challenges that Jane and Sarah faced, was to convince fellow design-team members of this methodology. They also had to insist on the inclusion of universal DT tools such as Spreadsheets, Logo, and Dynamic Geometry (which they had worked with during the our program), alongside the proposal, by their module’s contributing authors, of several Mexican mathematics teaching resource websites.

In 2008, their model (Figure 2) was put to the test when they implemented their course module with 900 teachers, divided into 30 groups of 30 participants each; each group had six sessions of training on the use of different DT resources, that also included the exchange of experiences and didactical situations. That is, in each session, the exploration of resources was carried out through didactic situations that helped the participants reflect on the use of DT for mathematical teaching and learning; they also addressed questions (similar to ones in our program) such as: “What is mathematics?”; “What is the role of the teacher?” After each session (and at the end of the entire course module) their participants had to design and implement a didactical planning for the incorporation in their practice of some of the DT resources seen in the module, where they had to make clear the topic, abilities to be developed, mathematical content, and tools to be used. The topics selected had to be linked to the curriculum, and the tool selection had to be well justified.
the following meeting, and at the end of the course, they would present, reflect upon, and discuss with their peers the experiences and observations they had had (bringing evidence such as their design, videos, photos, students coursework and observation forms). In these discussions, attention was given (as had been done in our own program) on the teacher’s role, the tool’s role, and possible assessment techniques. In a collective way, they would analyse the strengths and weaknesses of the implementations and come up with recommendations for the future.

What Jane and Sarah found (a finding shared by the other participants in our program) was some initial resistance in their colleagues towards the use of DT due to fear and ignorance of the use of these tools. Some participants even had difficulties creating email accounts, a requirement of the course module. But they observed that teachers can face and overcome their fears – of the technology, of their own technical and mathematical weaknesses – by directly engaging with DT in their classroom practice, and also assisted by their students or colleagues; in this sense, the support of the methodological structure of the module, of collaborative reflection, was crucial. An important finding from Jane and Sarah, is that many participating teachers changed their perception on the use of DT (as had happened with them in our course) “now seeing it as an aid for the child to observe, play with, program and, in general, interact with the computer tools” (González-Martínez, 2010); no longer using DT as supplementary means to “teach the same”, but as possible tools for exploration and construction of mathematical knowledge. They also had evidence (ibid; Granados, 2010) that the participating teachers began transferring, as our own participants had done, what they had learned of DT use and the pedagogical training model, to other colleagues. Moreover, their participants benefited in incorporating DT tools to their lives, such as email and Internet.

Both Jane and Sarah point out that they consider the following elements – of training, reflection and collaboration – as crucial for achieving changes in teachers: 1) “An adequate and permanent training of the teachers on the (technical and pedagogical) use of DT as an aid in primary-school mathematics classrooms.” 2) “The creation of a space for reflection and collective work, where experiences, teaching and learning strategies and other programs involving DT as an aid, can be shared” (González-Martínez, 2010). Jane adds (ibid) that the collaborative approach in teacher training “generated analysis, reflections, and attention to issues that teachers face when teaching with DT”; and that “by recognising alternative solutions, [teachers] are motivated to continue using [the DT tools] and sharing them with their colleagues”.

From scepticism to peer tutoring: the case of Adrian and Raymond

Here we present the case of another two participants, Adrian and Raymond, who were teachers, in their late thirties, in the “Tele-Secondary” (Telesecundaria) school system, with approximately eight years in-service experience. Telesecundaria – which began over 40 years ago – is an educational model of the Ministry of Education that aims to reach students (over a million students in the country, mainly in rural areas) that may not have access to regular lower secondary schools: in this model, learning is structured through learning and content guides, television programs and multimedia resources; with only one teacher-promoter for all subjects (this is a main difference with other secondary schools and is a big challenge for the one teacher). In 2006, a reform began to include computer media labs in this system, with the premise that ICT can enrich and transform teaching and learning.

As explained in Sacristán et al. (2009), when Adrian and Raymond joined our program, both of them were very sceptical on the possibilities of DT for mathematical teaching and learning, and felt uncomfortable about their use. But as narrated in that paper, through the immersion in our program they gradually changed, realising the potentials of DT, for mathematical learning, yearning to change their teaching methodologies, and being inspired to train fellow teachers. The latter, they felt, was particularly important because in 2006 (the second year in our program) the Telesecundaria reform was meant to be implemented, and yet teachers – though they were provided with textbooks that suggested uses of DT in each topic – did not receive any didactical or technological training for putting into practice those suggestions, and very few resources. By interviewing colleagues, Raymond found out that none used the digital resources (most said they didn’t have any knowledge of DT; the few who did, had only technical knowledge; and others said that they could not find the resources in Internet suggested in the textbooks); and thus, the reality of the Telesecundaria reform was very
distant from what it had promised (González-Pérez, 2010). Thus, it is difficult for teachers to incorporate DT in these schools, particularly for mathematics.

Adrian decided to begin by training another colleague and also tried to create awareness of how to use DT, in the larger community of school officials and parents. He became aware of how teacher-centred his practice had previously been and recognised the need for deeper mathematical competences; a knowledge of software potentials beyond the technical; and the importance of a good pedagogical model (Caballero, 2010).

On his part, Raymond was inspired to design and implement teaching courses for his colleagues who were supposed to put into practice the reform, focussing on the use of Dynamic Geometry (with Sketchpad) for the study of basic geometric concepts. From 2006 to 2009, he attempted to train three different groups of teachers (9 teachers in the first year, 8 in the second, and 5 in the third). The teachers that he worked with (averaging 50 years in age and with at least 20 years of in-service practice) had never used technologies and many were set in their ways. His three course designs evolved as he himself evolved: In his first course, he used a traditional teaching model and failed to motivate his colleagues; he realized that didactical and pedagogical discussions had been lacking. In the subsequent courses, he felt that he had succeeded in motivating the participants; however, only one of the teachers put immediately into practice with his students what he had learned. Raymond identified some factors in teachers that, despite their recognition of possible benefits of DT, impede their use in their practice, such as fear (e.g. of loosing control of the class, of showing mathematical and technical deficiencies); and passivity (to change, or implement DT, before they are forced to). These impediments seem particularly strong in the Telesecundaria system, where teachers have to teach many subjects besides mathematics, and therefore lack time and mathematical competencies. Raymond recognised that change is a long process; that “a teacher can only improve his practice with time and self-reflection”; that it requires a change in the conception of teaching and learning; and that “a methodology is needed that helps link collaborative work, the use of technological tools and mathematical knowledge, in order to harness the possibilities offered by technology in dealing with mathematical concepts” (González-Pérez, 2010). He emphasised that in order to change and enhance mathematical teaching, it is important to focus, analyse and reflect on three aspects: technical knowledge (of the DT tool), conceptual and mathematical knowledge, and pedagogical knowledge:

By reflecting on the relationship between these three components, I was able to have a different perspective of how to teach mathematics, to recognize my deficiencies in mathematics and in the use of technology for didactical purposes, and to implement these new ideas into my own practice. Looking at these with students, helps to consider that the integration of these new ideas requires a process of adaptation by the teacher, the students and the educational community in general. The impact on teaching, and therefore on learning, is not immediately evident; it is a process that requires time. ... Knowledge of different tools in a basic way, turns them into an end, [but] when this process [of knowing DT tools] progresses they can be used to construct mathematical knowledge. (González-Pérez, 2010)

Final remarks

Similarly, Sarah also appreciated the professional training model by stating (Granados, 2010):

It led me to become aware of my own limitations related to mathematical knowledge and specially in the use of that should be given to DT as tools to aid the work of the student and the teacher. [...] What was valuable from the project was exploring the tools in our practice, in classrooms and [observe] what happened in class. This helped me identify the tool’s possibilities and the difficulties we faced, without ignoring the mathematical knowledge involved.

In fact, after four years, all six participants recognised the impact of our model, in their own professional development and practice, while also acknowledging the difficulties they had in changing from a traditional, teacher-centred and instructional way of teaching (particularly, Raymond and Adrian); they recognised that a meaningful incorporation of DT into their – and other teachers – practice, is not easy (with additional obstacles from fears and habits) and is a slow and gradual process. However, what is interesting, is that the learning experience in our program was such that the participants appropriated themselves of the training model and led to transfer the basic principles of that program into their peer training designs. We consider that teachers tend to teach the way they have been taught (Selden & Selden, 1997), with their beliefs about teaching and learning shaped by their own experience as students (Thompson, 1992). As Thompson (1992) explains, much of the
knowledge and rules in effect while teaching and learning, is not taught explicitly in schools or in teacher preparation programs: much of it must be learned from experience in the classroom. As shown here, some of the factors (for change and for peer training), included in our professional development model, that seem fundamental, were the direct experience and engagement in their own practice of the DT resources and implementation strategies covered in the program; the individual and collaborative processes of reflection and self-observation that we promoted on all aspects: technological, mathematical and pedagogical; and continuous training and support (in mathematical content knowledge, and in DT use).

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References


Technological enhanced mathematics curriculum in teacher education: an exploratory study

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With the changes in the existing curricula of teacher education under the Bologna process it is necessary to assess the need for change in mathematics education models. We designed a mathematics curriculum for Basic Education course at an institution of higher education in Portugal who want to combine three levels of intervention. This exploratory study aims to identify two key points: academic performance of students and interactions made in the Moodle platform based on data collected, first by their final grades (study 1) and other by the access to Moodle during the course (study 2). Study participants were students who attended Mathematics I at 1st year and Geometry Topics in the 2nd year before and after the introduction of the new curriculum and the use of Moodle. Preliminary results indicate statistically significant differences for students who use the new curriculum with technology and those without this support in Geometry.

Introduction

With the changes in the existing curricula of teacher education under the Bologna process (the process of restructuring higher education) it is necessary to assess the need for change in mathematics education models. Contextualized in the recent policy to promote scientific education (especially in Mathematics, Science and Technology) where initiatives stand out as the Technological Plan for Education and School 2.0, from various government and private initiatives.

The introduction of a new mathematics curriculum for basic education is reflected in teacher training, either by the particular definition of a new kind of student or the need for new methods of teaching and learning of mathematics.

This opportunity to change the mathematics curriculum for teachers comes at a time it joins a combination of several factors: amendments to study plans in higher education; the growing concern with the teaching of mathematics in particular from the poor results of international studies like PISA (OECD); a significant change in the mathematics curriculum in elementary education; the focus of governmental bringing technology to schools, especially through computers, interactive whiteboards and broadband internet.

The questions of mathematical preparation of future teachers has been investigated with a view to training and teaching on education and not have as much importance as the subject of study for conceptual knowledge of these professionals. Studies on this topic have shown signs of concern, because this kind of mathematical knowledge is not present in many teachers (Veloso, 2004).

Considering all these factors we designed a mathematics curriculum for Basic Education course at an institution of higher education in Portugal who want to combine three levels of intervention: A solid mathematical foundation for all pre-service teachers; A comprehensive training for teaching mathematics connection between knowing and teaching mathematics; Promoting the technological skills of pre-service teachers in technology enhanced learning environment.

It is necessary to assess the changes to the mathematics curriculum in pre-service teacher training, in terms of content, methodologies and pedagogy; this paper presents the rationale and structural changes to the curriculum in a basic education course at a College of Education in Portugal by a technological enhanced learning environment. Focused on a preliminary study of the use of Moodle as tutoring environment using forums, demonstration videos and problem solving in teaching mathematics to pre-service teachers.

Background

In the current context of higher education in Portugal there was a (r)evolution in terms of structure,
especially with regard to training of teachers and educators.

Since the beginning of the Colleges of Education (Polytechnic School) in the ‘80s, and the publication of the Law of the Education System that regulates the Portuguese Education System, teacher training was separated into three major areas, three types of degree course covering all areas of education from Nursery school, Kindergarten school, Primary School to the Elementary School. In 2004 the need arises on the restructuring of teacher education in light of the Bologna Process that advocates separation in two teaching cycles, a cycle of broadband general knowledge and a second cycle of speciality (Ponte, 2004).

The College of Education where the study was conducted has made a substantial bet on the use of technological tools, including the use of Moodle to support the disciplines in the various courses.

**What mathematics should be taught to pre-service teachers**

The aim of this investigation is to understand the extent to which this methodology encourages pre-service teachers in-depth knowledge of mathematics especially from the point of view of Advanced Mathematical Thinking (AMT) in the last twenty years has developed as well as new trends theories of mathematics education expressed by Sriraman and English (2010).

Tall (1981, 2002) connected the AMT in the formal mathematical concept definition and concept image that has been studied by Domingos (2003) in characterizing the academic performance of students at the beginning of higher education.

Edwards, Dubinsky and McDonald (2005) focus on the definition of the phenomenon that seems to occur during the mathematical experience of students as they begin to relate to abstract concepts and deductive statements, recognizing that the mental structures that ensured academic success has not resulted thereby linking the concept of AMT in this transition period.

Another aim is to bridge the gap between the self-regulated learning (Wolters, 2010) and the development of learning skills, linking aspects of mathematics teaching and learning to use technological tools in the follow-up study conducted by Carneiro, Steffens and Lefrere (2007) at European level.

**Using technology to teach mathematics**

The curriculum has been developed taking into account the argument that the pre-service teachers need to have a better understanding of mathematical concepts because it helps them gain a greater understanding of the connections between various areas of mathematics.

It is argued that the curriculum should provide a sufficiently strong mathematical background and flexibility so that pre-service teachers can handle and create conditions for students to learn mathematics based on the three problems facing the mathematical training, to identify content relevant to the mathematical education, to understand how knowledge should be learned and what we need to teach mathematical concepts to children. Hence the focus on theoretical issues and data in mathematical proofs and demonstrations of various properties. The structure of the curriculum was set up as follows:

<table>
<thead>
<tr>
<th>CH1</th>
<th>FH2</th>
<th>ECTS</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math I</td>
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<td>160</td>
<td>6</td>
</tr>
<tr>
<td>Geometry Topics</td>
<td>60</td>
<td>160</td>
<td>6</td>
</tr>
<tr>
<td>Math II</td>
<td>60</td>
<td>150</td>
<td>6</td>
</tr>
<tr>
<td>Math III</td>
<td>40</td>
<td>150</td>
<td>6</td>
</tr>
<tr>
<td>Didactics</td>
<td>60</td>
<td>130</td>
<td>5</td>
</tr>
</tbody>
</table>

280  
750  
29

1. Contact hours; 2. Full hours; 3. European Credit Transfer and Accumulation System

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Table 1: Mathematical content addressed throughout the course.

The structure of all the disciplines of mathematics in the course is identical: 30% of the workload is theoretical and 70% is practical/laboratory. In addition the second group in both courses had also teaching hours with virtual support on the course page in Moodle.

This support is structured around three main lines: tutorial (explanation of theories, problems and exercises using video conferencing, short videos explaining the exercise or interactive applets) usually organized via forum; including subsequent assignments via discussion forums and quizzes (training) for the student to independently check their progress in learning a particular topic or concept.

With this addition of the online support structure, a number of technological tools to support learning based on issues raised by Oldknow, Taylor and Tetlow (2010) in the context of mathematics education. We used small instructional videos in Mathematics I, in particular on truth tables in Propositional Logic and animations on Set Theory.

In the discipline of Geometry Topics we introduced small interactive applets to intuit the properties of geometric figures and solids and the students were introduced in dynamic geometry environments with GeoGebra.

Methodology

This exploratory study aims to identify two key points: the academic performance of students studied and the interactions made in the Moodle platform based on data collected, first by their final grades (study 1) and other data collected by the access to Moodle during the course (study 2).

In Study 1 descriptive statistics were performed in order to organize the data collected and allow to assess differences between the final scores of the groups.

In study 2 there was a very elemental analysis based on hits students in courses in Moodle and graphics collected with the SNAPP software version 1.5 (Bakharia & Dawson, 2009) of social network analysis.

Study 1

Study participants were students who attended the Mathematics I at 1st year and Geometry Topics in the 2nd year immediately before and after the introduction of the new curriculum and the use of Moodle.

The descriptive statistics are shown in Table 2.

<table>
<thead>
<tr>
<th>Curriculum</th>
<th>N</th>
<th>Max</th>
<th>Min</th>
<th>SD</th>
<th>δ</th>
<th>% of approval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Old</td>
<td>42</td>
<td>18</td>
<td>1</td>
<td>9.1</td>
<td>3.9</td>
<td>15.2</td>
</tr>
<tr>
<td>New</td>
<td>45</td>
<td>14</td>
<td>1</td>
<td>8.8</td>
<td>2.8</td>
<td>7.4</td>
</tr>
<tr>
<td>Geometry Topics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Old</td>
<td>38</td>
<td>18</td>
<td>1</td>
<td>8.4</td>
<td>4.9</td>
<td>23.4</td>
</tr>
<tr>
<td>New</td>
<td>50</td>
<td>19</td>
<td>3</td>
<td>12.7</td>
<td>4.5</td>
<td>19.8</td>
</tr>
</tbody>
</table>

Table 2: Descriptive statistics calculated with R software

Study 2

The interactions made during the forums were also analysed by SNAPP software by obtaining the graphs presented in Figure 1.
The frequency of the course page in Moodle, either in hits (preview documents, personal messages, posts in forums and answers to quizzes), contributions (submission of assignments, quizzes and forums) and interactions in the forums are shown in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Page hits</th>
<th>Contributions</th>
<th>Students in forum</th>
<th>Posts in forum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics I</td>
<td>45</td>
<td>18,295</td>
<td>3,824</td>
<td>26</td>
<td>166</td>
</tr>
<tr>
<td>Geometry Topics</td>
<td>50</td>
<td>20,623</td>
<td>4,178</td>
<td>14</td>
<td>57</td>
</tr>
</tbody>
</table>

Table 3: Students hits on course page on Moodle and forum participation

**Discussion**

In study 1 the results of descriptive statistics of students’ educational achievements of Mathematics I suggest no major changes to the introduction of Moodle and the technological tools, there is even a marginal decrease of 3% in approvals, but the variance of the values suggest a greater dispersion of results. In data collected in Geometry Topics differences are evident with the introduction of technological tools and with Moodle, since the average is positive to the difference in percentage of approvals with a difference of slightly more than 30%.

Analysing the data presented in Table 2 points to the decrease in average ratings and the percentage of approvals between the group that did not use Moodle thus indicating an alert to the factors of this curricular change. On the other hand, the group that used Moodle significantly increased their average grade (from 8.8 to 12.7 on a scale of 0 to 20), obtaining the first positive average ratings and the percentage of approvals rose by 23%.

Data collected in study 2 from the use of Moodle points to the decrease of interactions (in numerical terms) of questions in the forum, which contradicts somewhat the relationship between the other values and analysis of interaction maps (Figure 1).

Both interactions are teacher-centred but these data may therefore require further progress in the direction of the conclusions of Domingos (2003) in which formation of new mathematical concepts are formed from a chain of events that necessarily pass through the implementation of procedures to evolve, increasing their sophistication and while the more advanced understanding, in line with the works of Tall (2002) with the AMT. Thus technological tools can serve as a complement to the AMT, including symbolic manipulation but with explicit conceptual purposes in order to explore concepts in a meaningful way.
The technological environment can serve to explore ideas as in the case of geometry with the use of GeoGebra, opening the field to the student to conceptualize mathematical ideas. This gap points to a close description of the findings of Edwards, Dubinsky and McDonald (2005) in which the AMT occurs only under certain specific conditions, we as teachers need to provide them with these conditions.

SRL (Self Regulated Learning) models can be developed, taking into account that provide a viable theoretical basis for educational intervention in order to understand how the student takes an active and reflective role in their own learning, and in this particular instance in learning mathematics significantly. There were more posts in Figure 1A possibly because they are first-year students, and be the first to have contact with the technological tools, they felt the need to request more aid. In the graph is no longer so obvious the teacher’s role as centralizing information noting more interactions between students themselves, even in a very simple way but this may indicate some elements in the skills of cooperation in line with the SRL that requires more analysis.

The increasing number of hits and contributions points exactly two in this direction, because a larger field of technological tools, allows students to be more autonomous in performing the required tasks, including exercises and problems that are on the course page in Moodle.

Concluding thoughts

The increasing use of technological tools and the steady expansion of the Internet provide a set of technological tools increasingly accessible to all school levels and all age groups.

The pre-service teachers cannot be alienated from the meaning of this (r)evolution, much less in the areas of STEM (Science, Technology, Engineering and Mathematics) education.

Data from this preliminary study are interesting but need further clarification that a statistical approach cannot achieve, it is necessary to understand the preparation (and initial shock) of students in 1st year potentiate their performance in the discipline of the 2nd year, or was simply because it is a discipline more practical and more visually interesting.

It seems not, since the former group (that did not use the technological tools or Moodle) has decreased marginally from the average ratings from the 1st to the 2nd year.

These data suggest a more thorough investigation of the factors that influenced this increase since the only change has been a curriculum change (teachers remained the same) is also necessary to check the group later if the results remain or if the results of the study group are a special case.

References


The Redesign of a Quantitative Literacy Class: Student Responses to a Lab-Based Format

Nicole Scherger
Elgin Community College, USA

The purpose of this study was to observe students’ retention, success, and attitudes towards mathematics in a community college quantitative literacy course, taught in a lab-based format. The redesigned course implemented the daily use of Microsoft Excel in the classroom demonstrations, group activities, and individual assignments, and utilized data from many fields of study. Results showed statistically significant growth in attitudes towards real world application problems, the use of computers in mathematics, and the consideration of taking additional mathematics courses. There was also marginally significant growth in students’ attitudes towards the relevance and utility of mathematics. Higher retention and success rates in the redesigned course were also observed, although those rates were not found to be statistically significant.

Introduction

The 2007 report commissioned by United States Congress, Rising Above the Gathering Storm (NAS, 2007b), states that the United States’ triumphs in science and technology are currently eroding and that in order to improve U.S. status, a widespread national effort is immediately required. The report is full of many eye-opening statistics related to Science, Technology, Engineering, and Mathematics (STEM), such as 38% of all undergraduates degrees in South Korea, 47% in France, 50% in China, 67% in Singapore, and a meager 15% in the United States are in natural science or engineering. In fact, the report also reveals that approximately one-third of the U.S. students who initially set out to major in engineering will change majors prior to graduating (NAS, 2007b). “The urgent need for STEM workers presents enormous challenges to our nation’s future productivity and to its educational systems” (Tyson, Lee, Borman, & Hanson, 2007, p. 248). A paramount concern is that “American high schools and universities do not produce sufficient numbers of students who pursue and persist in STEM careers” (p. 248).

The nations’ community colleges play a crucial role in these STEM areas. In a report for the National Science Foundation, Tsapogas (2004) states that “community colleges are important institutions in the educational lives of science and engineering graduates. Open admissions, proximity to jobs and family, and low tuition and fees make community colleges attractive to a large number of students” (p. 6). According to the American Association of Community Colleges (2008), community colleges serve nearly half (46%) of undergraduate students. Further, the National Science Foundation (NSF) data reveal that “almost half of the more than 740,000 S&E [science and engineering] graduates with bachelor’s degrees attended a community college” (Tsapogas, pp. 1-2). Recognizing the unique role of the community college, President Barack Obama (2009) recently set a goal to be the world’s leader in college degrees by 2020 and has himself recognized the role of community colleges, stating the following:

We believe it’s time to reform our community colleges so that they provide Americans of all ages a chance to learn the skills and knowledge necessary to compete for the jobs of the future. Our community colleges can serve as 21<sup>st</sup>-century job training centers, working with local businesses to help workers learn the skills they need to fill the jobs of the future. We can reallocate funding to help them modernize their facilities, increase the quality of online courses and ultimately meet the goal of graduating 5 million more Americans from community colleges by 2020.

While President Obama is now bringing the importance of community colleges to a national stage, the NSF has already been investing in the community college as a way to increase STEM participation in our nation’s colleges. “NSF-funded programs such as STEM Talent Expansion Programs (STEP) and Advantaged Technological Education (ATE) Programs specifically support implementation strategies that have led to an increase in the number of community college students studying in STEM, transferring to four-year institutions, and graduating with a STEM baccalaureate degree” (Starobin & Lannan, 2008, p. 38).
Thus, as the country looks for ways to meet the growing national need of more STEM students and with community colleges serving an important and growing role in this arena, this study redesigned an existing community college mathematics class to include the regular use of technology as well as more collaborative pedagogical techniques. The purpose of this study was to observe students’ retention, success, and attitudes towards mathematics in a quantitative literacy course taught in this lab-based format, utilizing Microsoft Excel.

**Overview of the Study**

**Quantitative Literacy**

The course chosen for this study was Quantitative Literacy. On a national level the importance of quantitative literacy is gaining attention in higher education. Quantitative literacy is broadly concerned with the ability to use mathematical and quantitative concepts in areas of problem solving and decision making within real-world applications. The seminal work in the field of quantitative literacy, *Mathematics and Democracy: The Case for Quantitative Literacy* (Steen, 2001), calls on colleges to re-examine how they prepare their students to be citizens capable of dealing with the numerical and quantitative needs of the present and the future. The national attention on quantitative literacy contributed to the decision to use this course as the setting for the study.

The specific course itself is designed to satisfy a general education outcome at the university level. It provides a foundation for basic numeracy and quantitative literacy. It develops skills in problem solving, logical analysis, use of mathematical models and functions, statistical and graphical representation of data, and decision making. This course is taken primarily by students pursuing the two-year Associate of Arts degrees or who will be transferring and pursuing Bachelor of Arts degrees at four-year institutions. These students elect to take either Quantitative Literacy or General Education Statistics to meet their general education math requirement; therefore, this course often serves as their capstone mathematics course.

The redesigned course implemented the daily use of Excel in classroom demonstrations, group activities, and individual assignments, and utilized data from many fields of study. The teaching techniques used in the course included daily collaborative groups, practical content and real data, open communication and dialogue, and the use of writing. Students spent approximately two-thirds of their class time collaborating in small groups and engaging with real-world application problems in the form of carefully crafted group activities, almost always utilizing Excel. Students were regularly reading, analyzing, discussing, and writing about their quantitative activities and then reflecting on and writing more about those experiences.

**Sample**

The study took place in an accredited two-year comprehensive community college, located in a suburban area of a major midwestern city. The sample for this study was a convenience sample which consisted of all students who registered for Quantitative Literacy during the spring 2010 semester. No advertising or recruiting was done for the redesigned course. In addition, because both the redesigned and traditional sections of the course met the same course outcomes and differed only in their delivery and pedagogy, the students were not made aware of the different formats of the course when they registered.

Students who registered for Quantitative Literacy in the spring semester of 2010 were the primary participants of this descriptive study. Three sections of the redesigned course were taught by two different instructors and two sections of the traditional version of the course were taught by two different instructors. The traditional sections began the semester with approximately 28 students and the redesigned sections began the semester with approximately 25 students, due to the size of the computer labs in which these sections were taught. Because only two traditional sections were taught during that semester and substantially fewer surveys were gathered from those traditional sections, data were also collected from students in the one traditional section that was offered in the summer semester of 2010. The final sample size for the analysis of the paired pre- and post-math-attitude surveys was 55 students from the redesigned sections and 37 students from the traditional sections.
A pilot study was conducted in the fall of 2009, in which three sections of the redesigned course were taught by two different instructors and three sections of the traditional course were taught by three different instructors. A pre- and post-math-attitude survey was given in the redesigned sections during this semester; however, this survey was significantly revised and thus, this pilot math attitude survey was not included in the data analysis. Therefore, the only data from this pilot semester that was used are those comparing the retention and grades data of the redesigned sections to the traditional sections of the course. The final sample size for the analysis of the retention and grades data was 154 students from the redesigned sections and 236 students from the traditional sections.

Results

Retention and Success

Retention and grades data were gathered and analyzed from the fall 2009 through summer 2010 sections of the course. For retention, a retained student was one who stayed enrolled in the course, regardless of the grade they earned. Success was defined as students who earned a C or higher, compared with students who earned a D, F, or dropped the course. Retention and success were the only variables that included data from the fall 2009 pilot semester. All of the statistical testing was conducted at the significance (alpha) level of .05.

The proportion of students retained in the redesigned course was 90.3%, compared to 84.7% retention in the traditional sections, as shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Student Retention: Fall 2009-Summer 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Redesigned</td>
</tr>
<tr>
<td>Withdrew</td>
<td>15</td>
</tr>
<tr>
<td>Retained</td>
<td>139</td>
</tr>
<tr>
<td></td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>15.3</td>
</tr>
</tbody>
</table>

Table 1: Student Retention

A logistic regression was carried out to determine whether this difference was significant and it showed no such statistically significant effect. Then, post hoc power analysis was conducted and the observed power was .35, indicating that, if the effect size in the population was as large as observed in the sample, there was only a 35% chance that the logistic regression would detect it. Thus, it is possible that non-significance here may have been a result of the lower power of the test.

The proportion of successful students in the redesigned course was 72.1%, compared to 66.1% success in the traditional sections, as shown in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Student Success: Fall 2009-Summer 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Redesigned</td>
</tr>
<tr>
<td>Unsuccessful (D, F, W)</td>
<td>43</td>
</tr>
<tr>
<td>Successful (A, B, C)</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>33.9</td>
</tr>
</tbody>
</table>

Table 2: Student Success

Again, the logistic regression that was carried out showed that this difference was not statistically significant; however, the post hoc power analysis conducted found the observed power was .24, again indicating a low probability that the regression would detect an effect as large as was observed in the sample. Thus, it is again possible that non-significance here may have been a result of the lower power of the test.

Attitudes Towards Mathematics

A pre- and post-math-attitude survey was given to determine change in students’ perception of the relevance and utility of mathematics as well as to investigate any change in their math self-confidence. Both the math self-confidence subscale and the perceived relevance and utility of math subscale consisted of five items, measured using a Likert response scale with items both positively and negatively worded. There were also four additional items on the survey that were of specific interest to the redesigned course and were created to measure student attitudes in preference for...
working on real-world applications in math, working in groups in math, using computers in math courses, and receptiveness towards taking additional math courses.

All subsequent analyses regarding the pre- and post-math-attitude surveys were conducted out using a mixed-design (randomized-repeated) ANOVA, with time as the single within-subjects factor and course type (traditional versus redesigned) as the between-subjects factor.

Although there was no significant difference in students’ mathematical self-confidence, there was a marginally significant difference in students’ attitudes towards the relevance and utility of mathematics ($F(1,92) = 3.84, p = .05$). In particular, the mean relevance/utility subscore total for the traditional sections increased from 19.13 to 19.37, whereas the mean for the redesigned sections increased from 17.55 to 19.24 (Figure 1).

Figure 1: Students’ Attitudes Towards the Relevance and Utility of Mathematics.

Responses to the single item, “I prefer working on real-world application problems in math,” showed a statistically significant effect ($F(1, 92) = 9.12, p < .01$). In particular, the mean score on this item for the traditional sections decreased slightly from 3.70 to 3.57, whereas the mean for the redesigned sections increased from 3.37 to 3.91 (Figure 2).

Figure 2: Students’ Attitudes Towards Real-World Applications.

Similarly, responses to the item, “Using computers helps me learn math,” also showed a significant effect ($F(1, 92) = 15.6, p < .01$). In particular, the mean score for the traditional sections decreased from 3.07 to 2.78, whereas the mean score for the redesigned sections increased from 3.27 to 4.03 (Figure 3).

Figure 3: Students’ Attitudes Towards the Relevance and Utility of Mathematics.
The final item from the math attitude survey that showed significance was the item, “Based on my experience, I would strongly consider taking another math class after this one (even if not required).” The effect was statistically significant ($F(1, 92) = 4.35, p = .04$), with the mean score for the traditional sections decreased from 2.73 to 2.64, whereas the mean for the redesigned sections increased from 2.74 to 3.10 (Figure 4).

**Students Value Practicality in Content and Delivery**

It is my belief that students often equate real-world application problems in math to what they have traditionally experienced as word problems in math textbooks. Unfortunately, these problems often involve such contrived situations that they have little actual meaning in students’ lives. But, when given an opportunity to work on meaningful application problems, utilizing real data, students’ attitudes towards applications are positive.

Similarly, it was my supposition that many students’ beliefs regarding computers and math have been shaped by the growth of the use of online homework systems in both online and traditional face-to-face classes as opposed to the types of relevant software (such as Microsoft Excel) that are used in real-life situations. Again, when given an opportunity to work with relevant software that they know they may see and use in their futures, students respond positively.

The results from qualitative open-ended survey support the quantitative findings. Students were asked if this course had changed any of their attitudes or opinions on the subject of math and if they found this course more useful and/or rewarding than previous courses. The responses to these questions were overwhelmingly positive and comments revealed that students placed genuine value
in studying authentic real-world applications. They compared their experiences in this course with what they saw as a lack of practicality in their previous math courses. Julie’s comment summarizes this point well. She stated,

Yes, this course was much more useful to me because it used real-world situations. In previous math courses, I hardly paid attention because there were pointless formulas that I knew I would never use outside of the class but with this course, there are situations that I could be able to use what I have learned. I found this course much more interesting and useful than previous math courses I have taken.

Students also specifically commented on the use of Microsoft Excel. For example, William stated that,

This class felt “modern.” We were doing all the work on computers and learned how to use Excel which is a very good program to know. I just thought that using Excel was practical and made the math easier to learn. It was a ‘hands on’ way approach to learning that sometimes gave you a couple different ways to solve a problem.

Comments like Julie’s and Williams’ support the quantitative findings that showed a significant increase in students’ views towards real-world application problems and students’ beliefs that using computers helped them learn math.

**Recommendations**

As was previously discussed, Quantitative Literacy is general education course and is not a course in the traditional STEM pipeline. However, if positive results of a more practical, technology-based mathematics course can be seen here, then perhaps similar gains could be seen in other more traditional STEM mathematics courses, which would make a good area of future research. Although these courses often have a stricter curriculum and thus the same daily level of technology implementation may not be possible, it could perhaps still be utilized and studied with a lower level of frequency, appropriate for the content of the particular course. If we can begin to improve the attitudes and experiences of students in our collegiate mathematics courses, then perhaps we can begin to grow the number of our students who are interested in STEM majors and fields.

**References**


Order and chaos: interactive computational activities for the classroom

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It has long been believed that typical students learn better through contemporary approaches to questions originated by physics problems that allow experiments. This belief motivated us to develop interactive computational didactic materials about contemporaneous mathematics that can be used both in the classroom and in mathematics clubs in school. Dynamical Systems, the study of how physical systems evolve with time, inspired the activities developed. They share a key goal of understanding the order/chaos relationship in natural phenomena, human behaviour and social systems. Another goal to achieve is to give mathematics an experimental/laboratorial component, which rarely is present. In fact, all the interactive computational didactic materials developed include simulations and the capability to generate wonderful pictures, from which students can enjoy the beauty of mathematics.

Introduction

With today’s available web page technology, it is no difficult to recognize the limitations of traditional text-based instructional techniques: not only it is now possible to complement a lecture session with some multimedia support, so often better in communicating a concept, but also, with its asynchronous and distance character, is changing forever the school’s traditional role. Moreover, the use of Java applets is giving us, educators, the possibility to introduce the student to an experimental or laboratorial vision of mathematics, that, it is our belief, will complement its traditional learning process. With this work, we present a web page constructed around three contemporaneous ideas of science, whose development were strongly supported by mathematical or computational models. Since all of them use topics of secondary school mathematics curriculum, we think it will motivate secondary school students to use and even explore some of its concepts.

Naturally Complex

The times they are a changing
Bob Dylan

In a report, published in 1989, by the Mathematical Sciences Education Board, [1], one can read that “at the end of the nineteenth century, the axiomatization of mathematics on a foundation of logic and sets made possible grand theories of algebra, analysis, and topology whose synthesis dominated mathematics research and teaching for the first two thirds of the twentieth century. These traditional areas have now been supplemented by major developments in other mathematical sciences - in number theory, logic, statistics, operations research, probability, computation, geometry, and combinatorics”. Furthermore, “much more than arithmetic and geometry, mathematics today is a diverse discipline that deals with data, measurements, and observations from science; with inference, deduction, and proof; and with mathematical models of natural phenomena, of human behaviour, and of social systems”, i.e., “the process of doing mathematics is far more than just calculation and deduction; it involves observation of patterns, testing of conjectures, and estimation of results”. Being unquestionable that, since its beginning, mathematics has been a science of patterns and order, one can say that, in the last decades, these two words are used with a totally new meaning: complex fractal patterns and chaos, now found copiously in nature, become exciting subjects, studied by mathematicians, scientists, and philosophers alike.

Shapes

Explanation: Looking for fossils can be a quite unique and exciting outdoor experience, most of all due to the expectation that you can always run into a record left behind by an ancient living creature. However, there are many features in rocks that, for centuries, were misidentified as fossils. Dendrites, usually more complex and less regular in shape and without the vein structures found in leaves, are
purely mineral growths whose branching pattern gives them an organic appearance; they are pseudo fossils.

Figure 1. A dendrite with its typical branching tree-like form.

Built on a model introduced twenty years ago, today we have computer models that mimic the complex and disorderly pattern formation due to the process of self-assembly of small particles to form larger structures via Brownian motion. Yet, the original diffusion-limited aggregation (DLA) computational model of T.A. Witten and L.M. Sander is considered one of the most striking examples of complex pattern generation by a simple algorithm. Being developed at a time when interest in fractals was growing rapidly, the DLA model turned out to be the paradigm for pattern generation far-from-equilibrium. Under this theme, the students will use regression analysis and the logarithm function from their mathematics curriculum.

Mathematical/Computational modelling: In the simple model proposed by Witten and Sander the growing structures are represented by clusters of filled sites on a lattice: starting from a large empty lattice, a simulation begins simply by filling a site, usually chosen at the lattice’s origin; then, the growth process takes place with the launching of a random walker far from the occupied site and allowing it to follow its path until it reaches a site adjacent to the occupied site, which is then filled and becomes part of the growing cluster. The process is repeated with a new random walker and its path until a perimeter site of the new two-particle aggregate is selected and filled. Since a random walker can always find its way to the aggregate, the growth process will never stop (although it is highly recommended to stop a random walker whenever its path is gone too far from the region occupied by the cluster, and start a new one).

Experimental mathematics: The students will use an applet to simulate the growth process over and over again and watch the complexity of the obtained clusters. Even though, due to the obvious random character of its growth, every single cluster is different from any other, it is fundamental to emphasize its similarities.
At this point, it is important to invite the students to choose different values for the cluster size, in order to appreciate the scaling irrelevance of this DLA growth process.

**Measuring complexity:** Since it is fundamental to give the students an idea of how intricate these organic-like structures are, it will be indispensable to introduce them to the concept of box dimension of a set. After an initial brief explanation, the students will be invited to use an applet to get a few points on the plane "length of the box" versus "number of boxes necessary to cover the cluster" from which a regression line will be drawn, and the box dimension of the cluster computed. Again, it should be noted how close these complexity measures are for different simulations, being therefore a characteristic of this kind of pattern formation.

**Social Network**

**Explanation:** The complexity observed in the inanimate world, it is also evident in living organisms. In this second theme, we suggest the students to study the time evolution of a very simple social network, from a mathematical model analogous to the previous DLA model. The mathematics curriculum topics that it aims to introduce, develop and explore are graph theory, probability and combinatorics.

**Mathematical/Computational modelling:** The Passion Madness Network: consider a group of people.

For the sake of simplicity, we will say that they are seated around a table and that someone is either not in love, or madly in love. Furthermore, we will say that everyone follows the same time evolution rule, depending solely on the love state of both his/her left and right neighbours:

- someone not in love between two people also not in love, then the passion remains switched off and he/she will be not in love the following moment;
- someone not in love with one neighbour madly in love, then the passion inflames and he/she will be madly in love the following moment;
- someone not in love between two people madly in love, then it is too confusing, the passion remains switched off and he/she will be not in love the following moment;
- someone madly in love between two people not in love, then the passion turns off and he/she will be not in love the following moment;
- someone madly in love with one neighbour also madly in love, then the passion remains light and he/she will be madly in love the following moment;
- someone madly in love between two people madly in love, then it is too confusing, the passion burns out and ... extinguishes and he/she will be not in love the following moment.

The question we ask is about the collective behaviour of such network of people, that is, given a group of people, what is the future of each one's love state? To answer this question, we invite the students to consider the following mathematical/computational model: consider a set of cells positioned in a circle. Since each cell can only assume one of two states, we choose two colours to represent each of the states: light grey for the not in love state, and orange for the madly in love one. In Figure 1 (left), we represent a group of 40 people with everyone's love state clearly identified by its colour. Then, all we have to do is to follow the rules. In order to see the changes of the love state of the cells, it is convenient to represent both configurations of the system: it is usual to represent the next moment system configuration under the initial one, as shown in Figure 1 (center). If we continue to apply the rules, we get the time evolution of each cell love state. In Figure 1 (right) we show the time evolution of the system for 6 moments.
Figure 3. Everyone’s love state time evolution, from a chosen initial configuration.

Since this 3-dimensional graphs is time and memory demanding, we invite the students to consider its 2-dimensional version: do the maths with the cells positioned in a circle, but choose a position, cut the circle, and show the set of cells as positioned in a line segment. Since we represent the next moment system configuration under the initial one, they should be careful and do the cut at the same position as before.

**Experimental mathematics:** Given the model and its graphical representation, next we allow the students to use an applet to explore it: for systems with small number of elements, can we identify its most probable future? And what happens if we choose a not so small number of elements? How can we describe its collective behaviour?

**Creative disorder:** By slightly modifying the rules describing the interaction between different elements, but still depending only on its own and its left and right neighbours previous state, the students will be able to build systems whose most probable collective behaviour is very different from the love madness, i.e., systems which collective behaviour is clearly characterized by its total organization, Figure 4. (left). Yet, for a few choices of the rules, the system will have a collective behaviour completely different from those two, highly organized but with some components of disorder, Figure 4. (right). Is this the kind of behaviour one can observe on an ant colony? And can we say the same about the activity of our brain, so much complicated, and yet, highly organized?

Figure 4. The different collective behaviour showed by this computer model.

This third type of collective behaviour, point out by computer simulation, has sparked an enormous interest at least because of its implications on business organization. Until now, was more or less common sense that a large company could only survive based on a rigid hierarchy, a solution to prevent that, to an increase in the number of employees and activities, inevitably follows a lack of coordination and the company's collapse as a supplier of products. Today, we can see social networks, as Facebook, and companies, like Google, adopting different strategies, by putting their efforts on an organization without the previous stiffness, attempting to keep a kind of ordered collective behaviour, but where minor changes are allowed and even stimulated. Google's success as a creative organization reveals that it is possible to take lessons from a computer simulation to manage a company. Except that, unlike our simple computer models, it is extraordinarily difficult to recognize the organization characteristics that are crucial to reach that desirable creative disorder behaviour.
**Dynamics**

**Explanation:** In his book, "An Essay on the Principle of Population; or, a View of its Past and Present Effects on Human Happiness; with an enquiry into our Prospects respecting the Future Removal or Mitigation of the Evils which it occasions", the revised second edition of a previous work, published in 1802, the Reverend Thomas Robert Malthus alerted the reader to the unequal nature of food supply to population growth: while food supply, he argued, is subject to an arithmetic/linear growth, population growth had a geometric/exponential nature. This divergence, that played a key role in the development of the theory of natural selection, by both Charles Darwin and Alfred Russell Wallace, was solved with a nonlinear model that takes into account the consequences of food shortage when the population grows over a certain number. But, what seemed to be a slight modification of Malthus law, brought to population dynamics the new and exciting mathematics of chaos theory. With this third theme, the students will be able to work extensively with the parabola, to draw its graph and to interpret some of its features from it.

**Mathematical/Computational modeling:** A perfectly reasonable model for the time evolution of the number of individuals of a population comes as \( P(t+1) = f(P(t)) \), where \( P(t) \) stands for the number of individuals at time \( t \), \( P(t+1) \) for the number of individuals at the next moment, \( t+1 \), and \( f \) some number of individuals of a population comes as \( P(t+1) = f(P(t)) \), where \( P(t) \) stands for the number of individuals at time \( t \), \( P(t+1) = f(P(t)) \), with \( P(t) \) the number of individuals of a population at time \( t \), we can never have something other than zero, at time \( t+1 \): therefore, our parabola is required to satisfy \( f(0) = 0 \). A second condition for the parabola \( f \) is obtained when we demand that, if we have a critical overcrowded situation at time \( t \), then the population faces extinction at time \( t+1 \). Since this critical value depends on the population and its habitat, it is preferable to work with the percentage of the population relative to the critical extinction value, instead of the number of individuals. Thus, our nonlinear model is given by \( X(t+1) = f(X(t)) \), with \( X(t) \) and \( X(t+1) \) the percentage of individuals of a population relative to its critical extinction value, at time \( t \) and at time \( t+1 \), respectively, and \( f \) a parabola satisfying both \( f(0) = 0 \) and \( f(1) = 0 \), which leads us to \( X(t+1) = A X(t) (1 - X(t+1)) \), with \( A \) a parameter to be chosen between 0 and 4.

**Experimental mathematics:** The students will be invited to use a first applet that will give them the graphical representation of \( X(t) \), for \( t = 0 \) to a certain \( t_{\text{max}} \), once chosen a value for the parameter \( A \) and the initial value \( X(0) \). After being familiarized with the concept of time series, a second applet will be presented, in which the students will have both the time series and the corresponding graphical analysis associated with the iteration of the parabola. In this case, we think it will be better to allow the students to choose only among a few values for the parameter \( A \), being \( X(0) \) arbitrary, corresponding to asymptotic situations for the time evolution of \( X(t) \) easy to identify either as constant or as repeating a set of values.

**Can we trust it?** It is no surprise for a population biologist the claim that, under certain circumstances, the number of individuals of a species can change periodically: it has been observed, both in laboratory and nature. But our model hides a huge surprise: if one chooses a parameter value over 3.87, the time series \( X(t) \) can exhibit a rather strange behaviour. Having fixed the parameter at \( A = 3.825 \), the time series \( X(t) \), taken from a given initial choice \( X(0) \), shows a lot of fluctuations, perhaps corresponding to a periodic cycle of a very long period. But, more interesting than that, is
what happens when we compare it with a second time series, \( X'(t) \), obtained from a different initial choice \( X'(0) \) as close as we want to \( X(0) \).

![Graphical representation of two time series from very close choices for its initial condition.](image)

Figure 6. Graphical representation of two time series from very close choices for its initial condition.

Although both \( X(t) \) and \( X'(t) \) change similarly for the first 25 time steps, reflecting the closeness of \( X(0) \) and \( X'(0) \), then they seem to be changing in completely different ways, a clear evidence of the sensitivity of the model to the initial conditions' choice. Despite the fact that there does not exist yet an unequivocal example of this type of behaviour in a natural population, this intrinsic mathematical characteristic of the parabola has raised fundamental questions about the limits of predictability. The marvelous scientific and engineering achievements in this area of predictability led us all to an enthusiastic generalization which we now recognize as false: there are models, and simple ones, practically unpredictable.

**Discussion**

Following the idea that teaching and learning mathematics in context has enormous benefits, we propose a theme that, in our view, has four main advantages: first, it is transversal to both natural and social sciences, that will fit the wide range of interests of secondary school students; second, knowing that The American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics announced that the theme of Mathematics Awareness Month 2011 is Unraveling Complex Systems, we expect secondary school students to be able to reveal how the systems of the world that surrounds us evolve with time; third, being so graphical, there is a notorious growing interest of the media on complex studies. Finally, it is not a closed subject! We have chosen three topics about complexity in nature but there are more available information from which the teacher and the students can build their own topics.

**Bibliography**


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The use of technology in the teaching of secondary school mathematics

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\(^2\)International Centre for Classroom Research,

Technology is recognised as valuable world wide in teaching in general and teaching of science and mathematics in particular. Research conducted in this field, has focused on pedagogical use of technology in schools, access to technology as well as the impact of technology on academic achievements of learners. The purpose of this study is to find out how different types of technology are used by mathematics teachers in teaching secondary school mathematics, as well as how technology is integrated into the teaching of mathematics curriculum. To this end, a sample of secondary school mathematics teachers was selected in Empangeni district, KwaZulu-Natal, South Africa.

Theoretical background

Research on the use of technology in the world has started. This comes as no surprise as technology is in great vogue in all spheres of life. Thomas (2006) conducted a ten-year longitudinal study in New Zealand on the use of computers in the teaching of mathematics. The findings of this study is that while there was a tremendous increase in computers in schools, their use is increasing slightly (1.2%). On the other hand the study conducted in South Africa by Mofokeng and Mji (2010) revealed that teachers did not use computers in teaching mathematics and science. In addition to computers, Ball and Pierce (2006) reported that teachers used scientific calculators extensively across all year levels, while Goose and Bennison (2006) reported that schools used mostly graphic calculators. Studies on the use of software (Ball & Pierce, 2009; Keong, Horani & Daniel, 2005) reported similar findings that spreadsheets were most used software.

Since the roll out of technology has started in South African schools it is compelling to investigate this area. The access to information and communication technology (ICT) in schools is an important innovation in South Africa (SA). Research indicates the importance of technology in the teaching of mathematics. To show that learners can improve performance in mathematics Salman, Esec, Omotosho, Abdullahi and Onayangi (2011) had an experimental and control groups of 120 students in Ilorin Metropolis secondary school. The experimental group received skills training on cognitive restructuring principles. The results indicated significant difference in the mathematics improvement performance of the treatment than untreated participants.

Research has also shown that use of technology in teaching and learning brings about positive results in the performance of learners (Amory, 2007; Becker, 2008; Thomas & Emercole, 2002; de Villiers, 2002).

This is a government priority as evinced by the active role played by the National Government, Provincial Governments and Municipalities. (Isacc, 2007; Mostert & Ntetha, 2008). The National Department of Education embarked on the development of the National Policy of ICT in education, called White Paper on e-Education. This policy stipulates that every South African learner in General Education and Training (GET) and Further Education and Training (FET) bands must be able to use ICT. The aim is to help develop ICT skills and knowledge among learners. They need to participate in the ICT global community by 2013 (White Paper in e-Education, 2004). At present about 60% of the schools in the nine provinces of South Africa have ICT infrastructure as compared to about 45% in 2002 (Mnisi, 2011; White Paper in e-Education, 2004). The progress in this regard is made through two phases.

Poor performance in mathematics is caused by many factors. Sibaya and Sibaya (1996) conducted a study in KwaZulu-Natal involving grades 11and 12 learners. The results showed that out of 449 learners who were learning mathematics through the medium of English language, 9 learners were designated as high achievers, 49 classified as average achievers and the rest as under achievers. On the other hand Mji and Makgato (2006) investigated the cause of failure in mathematics and physical science from learners and educators. They interviewd 350 grade 11 Tshwane learners and 10
educators from seven schools in which poor pass rate was noted. Results indicated that factors that contribute to failure of learners are categorized into direct and indirect influence. Factors with direct influence related to teaching strategies, content knowledge, motivation, laboratory use, and non-completion of syllabus content. Indirect influence was attributed to role played by parents in their children’s education and general language usage.

To address this problem of underperformance and others such as under-qualified teachers and too few students taking mathematics and science related subjects, a number of initiatives and programmes have been developed at national and provincial levels as well as by Higher Education Institutions (HEIs). From government side, a typical example is the setting up of Dinaledi schools. The Dinaledi Focus Schools Project is part of the National Strategy for science, mathematics and technology, to increase the number of learners studying the mathematics and physical science in grades 10 to 12; to increase the number of higher grade learners in these subjects especially girls and disadvantaged learners; to increase the the pass rate and achievement in mathematics and science in these grades; to develop the capacity of the mathematics and physical science teachers (Western Cape Department of Education, 2005). HeyMath is one of the government intervention programmes where Dinaledi schools and few selected schools based on their good mathematics results were supplied with laptops with the mathematics programme called HeyMath which has mathematics of all school levels.

According to Mnisi (2010) the Phase 1 Implementation of computers in schools is covering about 1555 schools in the 9 provinces. In KwaZulu-Natal there are 204 schools which are involved in this Phase 1 Implementation. The chosen area of this study is the only one in the northern part of the KwaZulu-Natal province that is partaking in this project. It is against this background that this present study is conducted. This study attempts to assist teacher trainers about the availability, use and functions of technology in schools.

The types of technology for teaching mathematics are computers, calculators, graphic calculators, smart boards television, DVD players and overhead projectors( Zimmerman, 2011; Cobb 2003). The present study will focus on these types of technology.

Problem statement

Environmental scanning in KwaZulu-Natal has not been done to determine the availability of technology in the mathematics classrooms and how this technology is utilised by FET teachers in schools involved in Phase 1 Implementation of the ICT structure. It is compelling to conduct this study to respond to Government National Imperatives.

This paper addresses the following research questions:

- What technology is available in the schools for teaching mathematics?
- What software is available for teaching mathematics?
- How often do mathematics teachers use the available software?
- What are the teachers’ training needs to be able to integrate technology in their teaching of mathematics curriculum?

Methodology

Sampling

Cluster sampling design was used. This is a single-stage cluster sampling design. A probability sample of clusters/districts were selected. A table of random numbers was use to select a sample from listed districts of KwaZulu-Natal. All elements in the selected district are included in the sample.

Why we used this sampling design

A complete list of respondents is not readily available.

The respondents are scattered throughout the KwaZulu-Natal province.
When the cost of construction a frame of elements is high, cluster sampling may be more efficient. When it is more expensive to sample respondents scattered throughout the population than items that are close to one another, cluster sampling is ideal.

There are 12 districts in KwaZulu-Natal. There are 17 high schools in the area where the research was conducted. Attempts were made to visit all schools in the area demarcated for the study. The participants were 35 FET (i.e. grade 10.11 and 12,) mathematics teachers. There were 23 males (66%) and 12 females (34%) teachers. To facilitate interaction between the researchers and respondents the number was kept small.

Instrument

The questionnaire starts with biographical information of the respondents. The questionnaire explores these areas a) availability of technology at the school for teaching mathematics, b) software availability for teaching mathematics, c) frequency of usage of technology, d) software used in mathematics classroom, e) training needs for the integration of technology in the curriculum. The questionnaire focusing on use of technology and software in the classroom was adapted from Thomas (2006). The researchers felt that the questionnaire was relevant to what is being investigated since the New Zealand school mathematics curriculum is similar to the South African school curriculum.

The face validity for this instrument was assured by giving it to two senior academics who are involved in ICT in schools, to ascertain whether the scale measures what it is supposed to measure. The Cronbach coefficient alpha was used to determine the reliability the questionnaires. This was found to be 0.85 for availability of technology at the school for teaching mathematics, 0.78 for software availability, 0.86 for frequency of usage of technology, 0.85 for software used in mathematics classroom and 0.75 for training needs.

Results

Availability of technology and software in schools for teaching mathematics

<table>
<thead>
<tr>
<th>Technology</th>
<th>PERCENTAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Available</td>
</tr>
<tr>
<td>Computers</td>
<td>46</td>
</tr>
<tr>
<td>Calculators</td>
<td>80</td>
</tr>
<tr>
<td>Graphic calculators</td>
<td>3</td>
</tr>
<tr>
<td>Smart boards</td>
<td>51</td>
</tr>
<tr>
<td>Television</td>
<td>54</td>
</tr>
<tr>
<td>DVD</td>
<td>74</td>
</tr>
</tbody>
</table>

Table 1: Percentage distribution of teachers endorsing availability-non availability of technology for teaching mathematics

The percentage of teachers endorsing availability of teachers in schools is consistently higher then that indication non availability and no response categories. The significantly lower percentage of graphic calculators confirms non availability of this technology in our education system.
Proceedings of the 10th International Conference for Technology in Mathematics Teaching (ICTMT10)
Portsmouth, UK, July 2011

Table 2: Percentage with which teachers endorsing availability of software for teaching mathematics

<table>
<thead>
<tr>
<th>Technology</th>
<th>Available</th>
<th>Not available</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph drawing package</td>
<td>17</td>
<td>54</td>
<td>29</td>
</tr>
<tr>
<td>Statistics package</td>
<td>9</td>
<td>63</td>
<td>28</td>
</tr>
<tr>
<td>Spreadsheet</td>
<td>40</td>
<td>37</td>
<td>23</td>
</tr>
<tr>
<td>Mathematics software</td>
<td>40</td>
<td>36</td>
<td>24</td>
</tr>
<tr>
<td>Internet</td>
<td>32</td>
<td>51</td>
<td>17</td>
</tr>
</tbody>
</table>

Combining not available and no response categories reveals that the percentage for these are consistently higher than the percentage for the category of software availability. The software is not available in our schools.

The use of technology and software in the teach of mathematics in schools

Teachers were asked to indicate whether they participated in the professional development activities on the use of technology. The teachers were also asked whether they use technology in their teaching mathematics lesson, and also indicate the classes they use the technology.

A cross tabulation of professional development and the use of technology showed no statistical difference (two tailed Fisher’s exact \( p = 0.671 \)). Teachers who indicated that they were trained in using technology in teaching mathematics did not differ from those who were not trained in using technology when teaching mathematics.

Table 3 Percentage distribution with which teachers endorse technology usage in mathematics curriculum.

<table>
<thead>
<tr>
<th>Mathematics curriculum</th>
<th>Most often</th>
<th>Sometimes</th>
<th>Do not use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>49</td>
<td>20</td>
<td>31</td>
</tr>
<tr>
<td>Geometry</td>
<td>31</td>
<td>29</td>
<td>40</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>40</td>
<td>29</td>
<td>31</td>
</tr>
<tr>
<td>Data handling</td>
<td>37</td>
<td>23</td>
<td>40</td>
</tr>
<tr>
<td>Calculus</td>
<td>40</td>
<td>23</td>
<td>37</td>
</tr>
<tr>
<td>Transformation geometry</td>
<td>26</td>
<td>40</td>
<td>34</td>
</tr>
<tr>
<td>Analytical geometry</td>
<td>31</td>
<td>20</td>
<td>49</td>
</tr>
<tr>
<td>Financial mathematics</td>
<td>46</td>
<td>17</td>
<td>37</td>
</tr>
<tr>
<td>Graphs</td>
<td>43</td>
<td>23</td>
<td>34</td>
</tr>
</tbody>
</table>

It is pleasing to note a consistently higher percentage of teachers endorsing the use of technology for different aspects of mathematics curriculum. The topics such as Geometry, Data handling, transformation geometry and analytical geometry present challenges for teachers. Data handling and Transformation geometry are recently introduced to mathematics curriculum. The use of technology in these topics or aspects of the mathematics curriculum is equally challenging.
The study found that the majority of teachers do not use software. This problem could be attributed to a number of factors. One of them could be lack of skills and finance to buy the software. This position could also be brought about by absence of continuous professional development of teachers. Should the Department of Education obviate cost of software by providing central file server of software, teachers could easily access programmes through internet.

**Discussion**

The important finding in this study concerns availability and non availability of technology in schools for teaching mathematics. It was found that different types of technology are available for this purpose. The highest percentage of availability of technology was noted for calculators (80%). Graphical calculators are the least available technology (3%).

This finding is peculiar to previous studies. Generally researchers report that technology is found in abundance in schools (Thomas, 2006).

The study found that spreadsheet and mathematics software are in preponderance as compared to other types of software. This finding is consistent with the findings of Ball and Pierce (2009).

The study found that although technology is available in schools the software to propel these are not available.

The study found that there is a relationship between the use of technology and various fields of mathematics curriculum. The introduction of the new topics in mathematics curriculum presented challenges for the teachers. There is an integration into the mathematics curriculum and technology.

The study found that the majority of teachers do not use software. This problem could be attributed to a number of factors. One of them could be lack of skills and finance to buy the software. This position could also be brought about by absence of continuous professional development of teachers. Should the Department of Education obviate cost of software by providing central file server of software, teachers could easily access programmes through internet.

**Implications**

This study is topical in view of the fact that the Department of Education is rolling out technology to schools. It is crucial therefore for the Department of Education to be guided by the findings of the study. The study informs that it will be cost effective to make use of a file server for different software.

The study recommend the resuscitation of in-service centres. Computer skills and technology in general are better developed through these centres. The technology and software are ever changing and therefore dynamic.

The Department of Education has a programme of phases in the introduction of technology. This is applauded. The rolling out of technology and software should be continuous beyond phases.
References


3D Modelling in Teaching and Learning Geometry

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This article addresses the application of computer modelling in teaching and learning geometry. Our aim is to increase the interest of students in studying geometry at secondary schools and colleges. The main field of our interest is study of synthetic and descriptive geometry – geometric constructions, projections, geometry of curves and surfaces. One possible approach of improvement in studying geometry is the integration of computer software in the teaching process. Here we will demonstrate the usage of Rhinoceros program (Rhino) for visualization, for proving geometric problems in the plane and in the space or for the demonstration practical uses of geometry. Nowadays we have a great collection of pictures which illustrates examples of surfaces in building practice and architecture. We show the usage of software on some concrete examples. The outputs can be used in publications and also for home schooling and e-learning.

Introduction and motivation

Geometry, the study of properties and relations of geometric figures, is an important and essential branch of mathematics. Geometry can be conceived as an independent discipline with many branches – Euclidean geometry, differential geometry, algebraic geometry, topology, no-Euclidian geometry and so on. Geometry is useful for learning other branches of mathematics and it can also be used in a wide range of scientific and technical disciplines. Some scientific branches require direct knowledge of geometry.

The main field of our interest is the study of classical geometry and descriptive geometry – geometric constructions, projections, geometry of curves and surfaces. Mainly it is geometry which allows the representation of three dimensional objects in two dimensions. Descriptive geometry which studies the properties of projections has become the language of designers and engineers. Classical geometry is geometry of the Euclidean plane and space. Our aim is to show practical uses of this geometry. Mainly we will focus on examples of using geometry of surfaces.

Geometry is important for everyone, not only for technicians, designers, architects, builders or civil engineers. We all need good visual imagination in our everyday life as well. The two and three dimensional shapes which surround us are originated in geometry. The world we live in is influenced by geometry. If we know how to understand and apply the relationship between shapes and sizes we can use it more efficiently. Some people think in images and shapes so they need the understanding of geometry to be able to do that.

Without the use of geometry the great works of artists, painters and builders would only have stayed in ideas and dreams.

According to (Hilbert, 1999) the study of geometry develops logical reasoning and deductive thinking which helps us expand both mentally and mathematically. If we learn to use geometry we also learn to think logically. It’s very important in everyday life – many difficult problems can be erased and the simple solutions can be found. Students can often solve problems from other fields more easily if they represent the problems geometrically.

The rest of this paper is organized as follows: the next section “The study of geometry” is devoted to current problems of the unpopularity and the difficulty of studying geometry. “How to increase the interest of students in studying classical geometry; use of computers in the teaching process” is the main subject of the following section. Finally in the section “The examples of 3D modeling” we will demonstrate the advantages of modelling computer systems especially Rhinoceros on concrete examples from the field of descriptive geometry and geometry of surfaces. The models which illustrate examples of surfaces in building practice and architecture will be shown. In conclusion we will discuss the advantages of geometric modelling systems and the responses from our undergraduate students.
The study of geometry

The study of geometry can be very difficult. This branch of mathematics isn’t popular among students; see for example (Schwartz, 2008a; Schwartz, 2008b). Drawings (the results of geometric projections of some 3D model) are sometimes very difficult to understand. For that reason geometric problems must be provided with clear examples. Intuitive understanding plays a major role in geometry. With the aid of spatial imagination we can illuminate the problems of geometry. It is possible in many cases to show the geometric outline of the methods of investigation and proof without entering needlessly into details. The problem can be more understandable without strict definitions and actual calculations. Such intuition has a great value not only for research workers, but also for anyone who wishes to study and appreciate the results of research in geometry. Of course, if we understand the main principles of a problem then we can use exact definitions.

The currently predominant view among students and the general public is that classical geometry is not important and useful. Drawings of classical geometry can be replaced by the outputs of modern computer software. Of course, computers can help us solve geometric problems and increase the efficiency of our work but we still have to know the basic principles and rules in geometry.

Is it possible to learn geometry? Yes, but it would be easier for students if they had encountered classical geometry, constructions and geometric proofs earlier. Sometimes students of technical specializations experience geometry only at college. That is too late. We work mainly with undergraduate students, so what can be done to make college geometry more comprehensible? How to increase the interest of students in studying classical geometry at secondary schools and colleges? This is the main subject of this article.

How to increase the interest in studying geometry; use of computers in the teaching process

Our aim is to increase the interest of students in studying classical geometry at secondary schools and colleges. One possible approach of improvement in studying geometry is the integration of computer software in the teaching process. This way seems to be interesting, attractive and motivational for students. Indeed the usage of computers in education is very current. Computers influence our everyday life including geometry. We have to follow the general trend.

Nowadays, computer-aided design (CAD) is commonly used in the process of design, design documentation, construction and manufacturing processes. There exist a wide range of software and environments which provide the user input tools for modeling, drawing, documentation and design process. These software and environments can be used to design curves and geometric objects in the plane and curves, surfaces and solids in the space. According to the applications more than just shapes can be involved. In modern modelling software we can also work with rotations and other transformations; we can change the view of a designed object. Some software provides dynamic modeling. Technical and engineering drawings must contain material information and the methods of construction. Computer-aided design is used in numerous fields: industry, engineering, science and many others. The particular use varies according to the profession of the user and the type of software.

These modern methods which are widespread in various branches can be useful in the teaching process, too. We can prepare students for their future employment. We do not advocate the usage of computers at any price. Of course we still place emphasis on the understanding of the principles used in geometry. Sometimes it is of significant importance to work without technical support and use only our own mind.

I have my own experience of teaching classical, descriptive, and computational geometry at universities – Charles University in Prague – Faculty of Mathematics and Physics and Czech Technical University in Prague – Faculty of Architecture. College mathematics and geometry is very difficult for many students. It is necessary to motivate and to arouse their interest in geometry. As was mentioned above, it is necessary to improve the teaching of geometry at elementary schools and at secondary schools.
In my lessons I use computer software for visualization, for the proving of geometric problems in the plane and in the space, for the demonstration of the application of geometry in practice, for the creations of study materials for home schooling and e-learning or for the transformation geometric problems into mathematical form. I work mainly with Rhinoceros - NURBS modelling for Windows (Rhino), Cabri II Plus, Cabri 3D, MATLAB, Maple and with GeoGebra. Use by teachers and students is always free of charge, it is the great advantage of GeoGebra. Consequently it can be used by students for home schooling and e-learning. Other software is available for teachers and students at our school. Nowadays we have extensive database of geometric tasks, images and 3D models – the outputs of these software.

I use these software for creation of stepwise guides through geometric construction which can help my students understand the problem in intuitive and natural way. Moreover I show special constructions applied in descriptive geometry and due to included functions and tools in these software students can discover proofs more easily. In this contribution we will demonstrate the advantages of the use of geometric modelling systems on examples from the field of descriptive geometry. Several examples of constructions used in descriptive geometry will be shown, too. Good geometric imagination and perception is very important for understanding spatial constructions in geometry. It is not possible to memorize the constructions; we have to understand geometric problems. Classical geometry is very useful branch of mathematics in practical applications. Geometry of surfaces, the main field of our interest in this contribution, is very important in building practice and architecture. The models which illustrate examples of the usage of special surfaces will be shown.

**The examples of 3D modeling**

The aim of this paper is to show applications of classical and descriptive geometry especially geometry of surfaces. See the following examples of geometric surfaces used in architecture. These figures were made in Rhinoceros - NURBS modelling for Windows and I use these outputs as study materials in my lessons at Charles University in Prague and Czech Technical University in Prague. The outputs can be also used for home schooling and e-learning. Rhinoceros (Rhino) is commercial NURBS-based 3D modelling tool. This software is commonly used in the process of design, design documentation or construction. Rhinoceros has many tools and functions for graphics designers. Rhinoceros can create, edit, analyze, document, render, animate NURBS curves, surfaces, and solids.

If we want to work with similar computer modelling software (it is not necessary to work just with Rhino), we have to know geometric principles not only the special tools and functions which that software provides. On one hand these software are useful aid in the popularization of geometry in general but on the other hand we need to understand geometry if we work with them. Mainly mathematical and modelling computer software can motivate our students to discover the beauty of geometry. Of course, then students have to learn geometry in classical way. Finally students can create the outputs with some software themselves and apply their knowledge gained from studying. The construction of the models is not just a computer amusement. See the figures – the demonstration of the usage of Rhino. Theory of these surfaces can be found in (Pottman et al, 2007), (Farin, 2002), (Gerald et al., 2002).
Figure 1. Illustration of the generating helical surfaces by helical motion a curve about an axis. Every point of the curve describes a helix with the same axis as the axis of surface.

Figure 2. Illustration of the generating ruled surfaces by a moving straight line. This is the special type of ruled surface which is called a parabolic conoid. Straight lines on the surface are parallel with the depicted plane (green) and intersect parabola and straight line (yellow). On the left input elements, on the right the final shape of the conoid.

Figure 3. Intersection curves of two surfaces of revolution; on the left top view, on the right front view.

Figure 4. Perspective projection of intersection curves of two surfaces of revolution. The use of appropriate perspective or parallel views to illustrate spatial situations has the main advantage that we can imagine spatial situation in clearly way.
Figure 5. Examples of practical uses of surfaces of revolution; on the left, one-sheet rotational hyperboloids as cooling towers, on the right, rotational paraboloids as radio telescopes.

Figure 6. Examples of practical uses of helical surfaces.

Figure 7. Examples of hyperbolic paraboloids in practical use.
Conclusion and future work

We discussed possible approaches how to increase the interest in studying geometry by using computers. The usage of 3D modelling we demonstrated on some concrete examples from the field of descriptive geometry mainly geometry of surfaces. If we can work with 3D model of the object in modelling software and move with it we understand more its properties.

The responses from students to using mathematical and modelling computer software (for example Rhino) in teaching geometry are very positive. Students are satisfied because the computer software is very motivational and attractive for them. Classical and descriptive geometry are more understandable and geometry in general becomes modern discipline.

We also have web pages with database of geometric tasks, images and 3D models (Surynkova, 2011) – the outputs of various software which are suitable teaching aid can be used in publications and also for e-learning not only for our students. In future work we will focus on further methods which can improve the teaching process. We plan to extend our gallery of 3D models and geometric tasks.

References


Essay about future directions – New Technologies in the next decade

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Advantages and disadvantages of the use of digital technologies (DT) and especially of computer algebra systems in mathematics lessons are worldwide controversially discussed. What will be the meaning of DT in the next years or even decades? What is the basis for an answer to this question and how might it be possible to get a vision of the possible development?

A first aspect might be an evaluation of developments in the past. How was the situation, especially in the middle of the last century, when we introduced computers into mathematics lessons? Can we learn from the development of the past? A second aspect is the evaluation of the present situation. In the last year, the 17th ICMI Study “Mathematics Education and Technology – Rethinking the Terrain” was published by C. Hoyles & J.-B. Lagrange. It gives an evaluation of the present situation concerning the use of DT and it wants to give a basis or a vision for the development of DT in the upcoming years. The word “vision” is used quite often: a vision for the development of the software, the hardware, the pedagogical landscape, mathematics in the classroom, the learning and the teaching.

In the following, a critical reflection of the past and the actual development will give some hypotheses of possible, gainful developments in the future.

Visions

In the book „The World in 100 years“ (German: Die Welt in hundert Jahren), published in 1910, the editor Arthur Brehmer asked important scientists of that time to describe a vision of the world in 100 years. One article in this book is about „The wireless Century“, and its author Robert Sloss describes „The Telephone in the Vest Pocket“ (S. 35ff).

„The citizen of the wireless century will walk everywhere with his „receiver“ ... On his way to work, in the underground, everywhere, he will listen to the „spoken newspaper“ and he will get all the news he wants ... And if he wants, he will be able to connect with every theater, every church, every concert hall and he can take part on the lecture, the sermon, the music session. .... The events of the whole world will be open to him ...”

The vision of Robert Sloss became reality – in form „of the i-phone – 100 years later!

The NCTM standards of 1989 (and in the revised version of 2000) have been similarly visionary – concerning the field of mathematics education – by representing a vision for the future of mathematics education. This is especially true for the use of new technologies in mathematics classrooms, expressed in the “Technology Principle”:

“Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.” (S. 24)

and:

“... Calculators and computers are reshaping the mathematical landscape ... Students can learn more mathematics more deeply with the appropriate and responsible use of technology. ...” (S. 25)

Also, the first ICMI study in 1986 “The Influence of Computers and Informatics on Mathematics and its Teaching” (Churchhouse) has been affected by a great enthusiasm concerning the perspectives of
mathematics education in view of the availability of new technologies. Many mathematics educators, for instance Jim Kaput, forecasted that new technologies would change all fields of mathematics education quite quickly.

“Technology in mathematics education might work as a newly active volcano – the mathematical mountain is changing before our eyes” (1992, p. 515).

**Expectations - Calculators**

In 1972, the first pocket calculator – the HP 35 – appeared on the market. Between 1976 and 1978, it was permitted in most (or many) states all over the world. In Germany, its use in general started in grade 7. In the late 1970s and in the beginning of the 1980s, there have been many expectations going along with its use (e. g. GDM 1978): E. g. Calculators

- allow experimental activities in the frame of discovery learning and problem solving;
- give a numerical basis for concept formation;
- allow the integration of authentic real-life problems into the classroom;
- release students from algorithmic calculations, which do not have a central meaning for the solution of the problem;
- allow problem-adequate exercise phases.

However, the question “How might the aims of mathematics education to be better reached?” was posed in the centre of the discussion at that time. And already in those days, some or many (mathematics educators) thought that calculators will bring far-reaching changes of the goals of mathematics into the classroom (e. g. Winkelmann 1978).

Surely, it can be stated today that these hopes, demands and objectives were too extensive and have probably only been reached to a small part. Different reasons can be given for this development: Missing concepts for the use or introduction of calculators, the lacking of professional development for teachers, the resistance of many teachers, the belief of many teachers that paper and pencil abilities will be lost by using calculators are different reasons for the mentioned development.” (see Weigand 2003).

**Expectations and Disillusion**

There are only speculations upon future development of the use of DT in mathematics classrooms. The use of hand held technology will especially increase in the next years (Guin et al. 2005). In the meantime, euphoric initial expectations are supressed by pragmatic attitudes. Despite of the use of digital technologies in the public and business world and despite of the tremendous number of research and practical classroom papers, the use of technologies in mathematics education and the impact on a change of curricula is still limited.

In the current ICME Study 17 “Mathematics Education and Technology - Rethinking the terrain” (Hoyles & Lagrange 2010), disappointment is quite often expressed that - despite of the countless ideas, classroom suggestions, lesson plans and research reports – the use of SCs have succeeded, as many had expected in the beginning of 1990s. Some quotations from the ICME Study:

“Technology still plays a marginal role in mathematics classrooms” (S. 312)

“The impact of this technology (CAS) on most curricula is weak today” (s. 426)

“The situation is not so brilliant and no one would claim that the expectations expressed at the time of the first study (20 years ago) have been fulfilled.” (S. 464)
This study gives a good overview of the numerous activities of the last years concerning the use of new technologies in mathematics education (see also Weigand 2010). But the book is not a vision, it rather poses questions, which are, however, quite similar or very similar to those 20 years before. One may interpret that as - partly - resignation, but one can see it also as an indicator of how hard these questions are to be answered. Finally, one can also understand it as a request and as a challenge to develop new ideas - visions - in order to make progress with the integration of new technologies in mathematics education.

It is surely crucial whether these tools are allowed in examinations and, above all, whether they might be used in final or state-wide examinations. World-wide most diverse models can be found. In the last years in Germany, there is a tendency to allow graphing calculators, however, the use of symbolic calculators is only voluntary.

In the following, three theses are set up, which are more strongly a pragmatic view than a vision to strengthen the influence of new technologies on mathematics education.

**Concerning the Future**

Central questions concerning the use of new technologies and concerning a change of teaching and learning have been posed for a long time and they can be answered – of course – not in a final way (Trouche & Drijvers 2010). With new, always changing media, „old“ questions arise in new ways: What is mathematical basic knowledge and how can it be saved? How can traditional working styles and mathematical thinking (proofs, arguing) be preserved and further developed? How can digital tools support the protection of the basic knowledge?

The following is based on the hypothesis that an only isolated point of view of mathematics education and mathematics instruction will not lead to a sustainable basis for future developments. These isolated points of views could be views only on certain subfields of mathematics, an evaluation of the current situation without seeing the relation to its development, a view on pupils without the view on teachers and contents, a view on technologies without a view on pupils, contents and teachers.

Thinking in relations and connections will be the central condition for changes (improvements) in mathematics education. Connectivity will be a key word of the future.

Three theses are set up concerning the use of new technologies and a further positive development of mathematics education. These are not the only aspects which will be important in the near future, but maybe these are the most crucial ones.

1. **Thesis: The use of new technologies requires a global concept of teaching and learning**

An integrated global concept of the use of new technologies has to follow different aspects: It concerns

- the interaction of different digital components such as laptop, netbooks, the Internet, pocket computers under technical aspects.
- the use of class room materials. It is crucial to build the relationship between traditional and digital materials, between paper-pencil and digital tools, between schoolbooks and (interactive) e-books and worksheets.
- internal-school aspects. The cooperation of all groups represents the school: pupils, teachers, the head of the school and parents.
- over-school aspects such as the cooperation between teachers of different schools, between school and school administration as well as between school and university.

The following collage shows these connections and relationships of new technologies (computers and hand held technology) – starting in the middle and going clockwise from down left – with computers, laptops and notebooks, smart-phones, whiteboards, a navigation system in the classroom, the Internet (and the necessity of technical support), traditional books, teachers, head of the school, parents, other schools and professional development.
Future developments and the question of a wider integration into classroom technology have to be discussed in the context of an extended mathematical learning environment (classroom, home, digital learning environments). But over all, we will not forget that a change in teaching and learning does not automatically mean a change to a better learning and understanding.

Example: A current example of the effectiveness of the use of new media and the necessity of a global concept is the use of interactive whiteboards. It is surely a surplus that interactive acting is possible with this technology in a direct dynamic way, like it is not the case with traditional presentations. However, only presenting contents with an interactive whiteboard will not lead to a provable increase of pupils’ knowledge. The use of this medium must be integrated into a global concept of new digital media usage. If all pupils have laptops or netbooks available in the classroom, and if there is a class-internal digital wireless navigation system, the interaction between pupils and teachers might change. Interactive whiteboards can support this change: The work and the acting of an individual pupil can activate – supported by the whiteboard presentation – a whole class discussion at any time. Moreover, if there is a digital communication system in the classroom, and pupils also have access to a learning platform over the internet at home, in class developed whiteboard sketches can be given to the students for working on them at home. Homework can already be made by pupils under the aspect that it will be presented interactively and dynamically.

Technology and whiteboards can be catalysts for a change in the classroom. But they have to be integrated into a global concept of learning and teaching in school.

2. Thesis: While using new technologies the important “traditional” goals as well as basic skills, facilities and knowledge have to be preserved.

It is sufficiently well-known that effective learning can only develop on a solid basis of knowledge and abilities. Knowledge is cumulatively acquired. A good structure and network of already existing knowledge is the basis for the acquisition of new knowledge. It gives the learner security and acquaintance, it develops meaning by connecting contents and showing a common thread in the learning process. You may talk about a “genetic learning process”

There is especially a need for
• preserving a flexible availability of basic arithmetic, geometric, algebraic and stochastic knowledge, and
• preserving and developing mathematical working styles like proving, arguing, constructing.

There are two important questions for the future: What is the basic knowledge, what are the basic abilities, the basic skills pupils and students should know or should be able to do
• without the use of technology?
• as the basis and condition for the (later) use of DT?

There is a variety of answers to these questions. But they cannot be answered in an absolute way, like Herget et al. tried in 2001. The questions always have to be answered in relation to the goals and aims of learning and teaching.

Example: There are many examples concerning the success of this kind of networked learning. We quote Philipp Melanchthon (1497-1560), who demanded „ad fontes“ (lat.: back to the sources) in his inaugural speech with the topic „Concerning the improvement of the studies of the youth“ in the year 1518 in Wittenberg. At Melanchthon’s time, „ad fontes“ meant studying the ancient Greek philosophers. Today, „ad fontes“ can also be regarded as guidance for working with digital tools. The “sources” are knowledge and the basis of a more than thousand years old mathematical knowledge. New technologies give new possibilities for an access to these sources.

3. Thesis: The effects and the results of the use of digital technologies in mathematics classrooms have to be continually evaluated, new concepts have to be developed and integrated into professional development.

Mathematics education, regarded as an “engineer science” (Freudenthal 1978) or a „design science“ (Wittmann 1995) has to develop and evaluate appropriate teaching and learning concepts. The basis for these concepts are firstly existing empirical results, secondly theoretical analyses of existing concepts and thirdly creative ideas and visions of a new kind of learning. Concerning the use of new technologies, there exist many theoretical concepts and classroom suggestions, but there still is a lack of especially long-standing empirical investigations. To collect personal meanings of teachers concerning existing lesson plans and contents is for sure important. But only a reflected (scientific) evaluation gives a basis for a specific and aimed development of new teaching and learning strategies in the classroom.

Developing and evaluating concepts is one aspect. Another important aspect is to spread the existing ideas into the classrooms. Especially schoolbooks are crucial for this process. But moreover, without permanent teacher education and teacher training, new ideas will not come into the classrooms.

Example: The EdUmatics-project (European Development for the Use of Mathematics Technology in Classrooms, see Bardini & Bauer et al. 2011) aims to increase the integration of ICT in European mathematics classrooms. An online training course is constructed to provide learning and teaching material for in-service and pre-service secondary teachers.

Conclusion

It is one or even the central goal of mathematics education to develop – based on theoretical considerations and empirical evaluations – concepts for future developments in mathematics classrooms. It is important to have visions and develop visionary concepts, but you also have to take care that these visions will not be illusions. Connectivity will be a key word of future developments to avoid disillusionments.

The meaning of hand held technology will increase in the next years. “Ad fontes” gives the security of having a basic ground, new technologies can be catalyst of coming a little bit closer to the old – but still important – goals of mathematics teaching.
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Linking IT-based semi-automatic marking of student mathematics responses and meaningful feedback to pedagogical objectives

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The purposes of an online system to auto-mark students’ responses to mathematics test items are to expedite the marking process, to enhance consistency in marking, and to alleviate teacher assessment workload. We propose that a semi-automatic marking and customisable feedback system better serves pedagogical objectives than a fully automatic one. The two pedagogical objectives to be addressed are that teachers should know about the range of students’ solutions and that they should provide meaningful feedback to students through the utilisation of “customisable feedback”. Both objectives are aligned with using assessment data for learning. Our proposed IT-based system consists of a marking component and a feedback component, and it is designed to provide close linkage between IT-based marking and these pedagogical objectives.

1. Introduction

The rapid advancement in IT has changed the approach to mathematics education and mathematics assessment over the past few decades. For a successful integration of IT into mathematics assessment, the first question that comes to mind is whether a computer can successfully replace humans in judging the correctness of mathematical expressions captured online (McGuire & Youngson, 2002). We hope to contribute toward answering this question through our current project, called, the Singapore Mathematics Assessment and Pedagogy Project (SMAPP). It is a large-scale project undertaken at the National Institute of Education, Singapore, with the aims to develop and investigate an innovative and systematic approach to assessment suited to the needs of mathematics teaching and learning in Singapore schools. These needs include developing high-order thinking skills, problem solving abilities, reasoning skills, communication skills, and other desired learning outcomes for students in the 21st century. Some of these skills may be developed by getting students to work through real-life related mathematics assessment tasks, which are delivered through an IT platform. In 2010, the mathematics tasks were trialled by 925 Grade 7 students from 27 classes in five Singapore schools, and the IT-based system discussed below will be piloted with new groups of several hundred students and their teachers in the coming months. The main components of the IT platform are summarised in Figure 1.

Each mathematics task consists of several questions, and they are classified as closed or open questions. Closed questions are multiple-choice items or require only specific numbers or single words without attention to the workings (or solutions). It is quite easy to automatically mark these closed questions.
Open questions require several steps constructed by the students and may include explanations input as texts. In the following sections, we describe the IT-based semi-automatic marking system. This system consists of a marking component and a feedback component. The main purposes of this system are to expedite the marking process, to enhance consistency in marking, and to alleviate teacher assessment workload; the last point is of pragmatic consideration in order not to over-burden the demands on teachers who voluntarily agree to participate in this project. The feedback component, in particular, is designed to serve two pedagogical objectives, namely to help teachers know about the range of their students’ solutions and to enable them to provide meaningful feedback to students through the utilisation of customisable feedback. Both objectives are aligned with the trend to use assessment data for learning.

2. The Marking Component

Several attempts have been made by researchers in Singapore and elsewhere to automatically assess mathematics answers using computer-mediated system (e.g., Ashton & Beevers, 2002; Bennett, Morley & Quardt, 2000; McGuire & Youngson, 2002; Soh & Subramanian, 2008). For open questions, they face the difficulty of entering and accepting mathematical symbols and graphs and marking these entries with numerous possible combinations. McGuire and Youngson (2002), for example, proposed modifying a paper-pencil item to a computer version by reducing the number of possibilities. This is not entirely satisfactory because this requires the design of two versions of the same task. In Singapore, the Math Explorer (Soh & Subramanian, 2008) is an intelligent learning system that can grade student constructed, long answers to mathematics problems, and the feedback has been generally positive (Seow & Hu, 2003). For pedagogical considerations and other constraints, we do not aim to produce a fully automatic marking system; instead we develop a semi-automatic marking system as explained below.

Short open questions usually have a few expected answers. For each of this type of questions, we will have about four likely answers. If the student’s answer matches one of these possible answers, the online system will automatically mark it, but it allows the teacher to review this marking. The teacher can choose to review the marking by students or by questions. In the latter case, the teacher will have a better grasp of the more common incomplete or wrong answers given by the students for that question and this can serve the important pedagogical purpose of alerting the teacher to areas for possible remediation. Figure 2 shows a screen shot of several student answers to the same question. In this example, the teacher may note that these students have different ways of handling units, some of which are wrong, and these differences need to be discussed with the class.

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>STUDENT</th>
<th>CORRECT ANSWER</th>
<th>STUDENT’S ANSWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1c1l</td>
<td>ICTMT10 Demo Student 1</td>
<td>80g/m² × 0.0625m² = 5g</td>
<td></td>
</tr>
<tr>
<td>Q1c1l</td>
<td>ICTMT10 Demo Student 2</td>
<td>80g × 0.0625m² = 5g/m²</td>
<td></td>
</tr>
<tr>
<td>Q1c1l</td>
<td>ICTMT10 Demo Student 3</td>
<td>80 × 0.0625 = 5g</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Screen shot of several student answers from “Mark By Question” option

For open questions that do not have expected answers such as explanations and personal opinions, the teachers have to mark them based on some given guidelines. This invariably introduces inconsistency in the grades, but the more important consideration is for the teachers to read the student texts carefully to understand their thinking. For these questions (only one or two per task), several feedback options are still provided.
3. The Customisable Feedback Component

The literature about formative assessment has constantly stressed the importance of giving meaningful feedback to students as part of the learning process (e.g., Black & William, 1998; Hattie, 2009; William, 2007). According to William (2007), feedback in education refers to “any information that a student is given about their performance” (p. 1061). The gap between what the student currently knows or can do and the expected goals should be made explicit so that the student can understand the goals and is given the appropriate actions to close the gap (Sadler, 1989). Teachers often give marks or grades or number of correct ticks as the main form of feedback to indicate this gap. Although this piece of information may be motivating to some students, it is not particularly helpful to promote further learning because there are no clear directions of what to do next. Indeed, William (2007) cited short-duration studies that show that students given only comments without the scores achieved gains in performance and improved attitude from one lesson to the next, whereas those given only the scores or scores and comments did not show similar gains. We have designed our system to test this claim by allowing the teacher to select only comments without scores as the feedback to be given to their students.

Mason and Bruning (2001) described eight types of feedback that can be delivered through computer-based instruction. These types range from “no feedback” (a performance score) to “bug-related feedback” that helps students to identify their erroneous procedures without giving them the correct answers. They suggested that students should be given some control over the kind of feedback they wish to receive. While we are not able to follow their suggestion to allow this choice, we have provided the option of asking students to inform us what they would do after they have read the teacher feedback; this is explained in section 4 below.

On the basis of some of these prior studies, we provide several types of feedback that the teachers can select. When the answer is fully correct, the system is able to auto-grade it, and the feedback begins with a short praise, such as “excellent” or “well done”, followed by one of the following types of comments:

(a) A succinct comment about the correct procedure, e.g., “You know how to find the area of a rectangle”, to help students who may have arrived at the correct answer by the wrong procedure or have just guessed it.

(b) Interesting information, e.g., “Four people on the moon need about 1500 kg of oxygen per year to survive!” to arouse their curiosity.

(c) An extension of the problem, e.g., “With this distance that you have calculated, how many times do you think you can travel from one end of Singapore to the other?” to encourage more practice.

These comments encourage the students to “look back” at their correct answers and to try to build on it, which is the last step of the problem solving process advocated by Polya (1957) and adopted in the Singapore mathematics curriculum.

For partially correct or totally wrong answer, the teacher can select from several comments that provide hints for correction. These comments depend on the questions asked. Two examples are given below:

(a) Try to use ratio or proportion or model drawing to solve this problem.

(b) You had found one correct value. Read the poster carefully and try to identify the other items.

The teacher can modify these comments, and their modifications will be automatically stored online for future use by the teacher and colleagues in the same school. This is the section where the teacher has to read the student work carefully and then selects or modifies the comments so that the feedback is likely to be meaningful to that student. We believe that this form of customisable feedback serves the pedagogical objective better than a fully automatic one.

For fairly straightforward questions, students may be able to obtain the correct answer without showing their workings. To help them build the desirable habit to show their workings, the standard feedback is automatically given: “F99: Next time, show your working.”
Another standard feedback is given when the student has skipped the question, i.e., “F0: You seem to have skipped this question. Next time, try to answer all the questions.”

The screen shot in Figure 3 shows the arrangement of the marking and feedback components. The teacher selects the relevant marks and feedback through the pull-down menus. Figure 4 shows a new customisable feedback added by the teacher.

4. Student Responses

After the teacher has completed the marking and feedback components, the students will be asked to log into the system to view the teacher feedback without the scores. They will then select one of these three options about what they might do next: (1) Now, I understand. (2) I still do not understand, so I will discuss with my teacher. (3) I still do not understand, so I will discuss with my friends. See Figure 5. These student responses will be captured for further analysis by the research team. Working through this feedback output will hopefully help the students to correct their mathematical mistakes and to develop the habits of self-reflection.
5. Use of Assessment Data

As suggested by Schofield and Ashton (2005), all the online data are captured and can be downloaded into EXCEL format for more sophisticated analysis to answer specific research questions. The teacher can examine the output by students or by questions. Examining the reports about individual students, the teacher can know more about the performance of these students and provide more targeted remediation. On the other hand, studying the responses by questions allows the teacher to identify patterns of errors that suggest underlying misconceptions related to the specific skills. The teachers are encouraged to work with the research team to plan follow-up activities.

The research team will also mark all the scripts using the same system in order to establish inter-rater reliability of the student scores and to pin-point any shortcomings of the system to be rectified for the next task.

6. Concluding Remarks

In the above sections, we have described the pedagogical considerations underpinning our IT platform, and we believe that these pedagogical factors are more important than IT design features. Nevertheless, our IT system has included the relatively new features of semi-automatic marking, customisable feedback, and student responses to teacher feedback. These features are integrated with the more well-known online processes of delivering mathematics tasks, capturing student responses, and allowing download of data. This results in a coherent system of novel and standard features used in online assessment. This system is currently being tested for its efficacy to satisfy the requirements for using assessment data and processes to improve mathematics learning.

7. References


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A long-term educational treatment using dynamic geometry software

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This paper reports on a pilot study to investigate problem solving processes of 12 ninth graders (ages 14 to 16) and the subsequent long-term study, which is in progress and is being carried out in two classes over a period of at least two years. It began in the seventh grade with an acquaintance phase, in which we assessed the mathematical abilities of the students. With the beginning of the treatment in the eighth grade both classes will be taught by the author. This treatment consists of the systematic use of what we refer to as electronic worksheets. On top of that one of the classes will get an extra training in problem solving strategies. Two further classes that are not being taught by the author will serve as control groups.

Theoretical Background

Dragging in dynamic geometry

According to Hölzl (1994, p. 65) the dragging mode is the “main attraction” of a dynamic geometry software (abbreviated DGS, Hölzl used Cabri géomètre) besides the possibilities of programming macros, tracing points of the construction and visualizing their loci. In the dragging mode pupils can manipulate the geometrical objects they create on the screen and experiment with their constructions. Such activities are also requested by modern curricula of the federal German states like the one of Lower Saxony, where the studies are being carried out.

In reality the integration of electronic media into the daily routine of mathematic education in schools is far more difficult and long-winded as written papers of the educational authorities might suggest. Beside structural problems like the availability of computer rooms and the number of workstations in these rooms the experience shows that it takes a lot of time and requires intensive and frequent work with the electronic media until pupils and teachers get able to use the whole range of possibilities of these modern tools. Artigue (2002, p. 250) calls this process “instrumental genesis”. The results of a survey of more than 600 publications and reports from the late 1990s “clearly show that the complexity of instrumental genesis has been widely under-estimated in research [...] until quite recently” (ibid., p. 253). Laborde (2001) accompanied four teachers over a period of three years and found a “dichotomy between conjecture and explanation or proof”:

The most obvious contribution of Cabri is the possibility of dynamic visualisation of geometrical relations preserved by the drag mode. Teachers (even the novice in using technology) immediately exploited this possibility by asking students to conjecture properties from what they could see. However, when the students were asked to justify, the teachers did not mention the possibility of using Cabri to find a reason or to elaborate a proof. It is as if there was no interaction between visualisation and proving. (ibid., p. 306)

To reach that goal it seems to be required that the pupils get a deeper understanding of what happens beneath the visible surface of the monitor. For dynamic geometry and especially the role of dragging Hölzl (1994, p. 68ff.) proposes a relational model which distinguishes between drawings and figures. While a drawing is only one momentarily appearance of a geometrical object on the screen, the affiliated figure also comprehends all the mathematical relations that exist between the several components of the geometrical object. One immediate consequence is the existence of different types of points in dynamic geometry. Those without any relation to other geometrical objects are called free points, basis points or draggable points, because they can be dragged to any position on the screen. Points can also be bound to other geometrical objects like lines, circles etc., those points are called object points or semi-draggable points, because they can only be dragged along the objects they are bound to. The third type of points are intersections. Those points cannot be dragged at all. Referring to drawings and figures Hölzl (2001) defines the process of dragging as follows:

A drawing is the materialised representation of an ideal geometric figure only characterised by its internal relationships. Dragging is a tool to find different representations of one and...
the same figure in continuous transition. Because dragging acts on a drawing with the effect being determined by the figure, a mediating function emerges. (p. 83)

He derives at least two different possibilities of using this mediating function. Dragging can be used as a testing device for conjectures or it can be used to investigate the changing appearance of the drawing in order to find invariants of the figure or to produce conjectures.

Arzarello et al. (1998) and Olivero (1999) adopt a theoretical model from Gallo (1994) to describe problem solving processes. The model distinguishes between ascending and descending control movements or modalities. During ascending movements the problem solver investigates the given problem in order to formulate conjectures and thereby moves from a perceptive level to a theoretical one. Working with a dynamic geometry programme the problem solver might observe what happens to the drawing when dragging on it and eventually discovers invariant properties on the perceptive level that lead to conjectures on the theoretical level. The problem solver enters the descending control movement by trying to validate the conjecture through targeted dragging of certain points in a given construction or even through the creation of a decisive construction and, subsequently, a well-directed manipulation. By doing so the problem solver returns to the perceptive level. According to Arzarello (1998, p. 28) “any process of exploration-conjecturing-proving is featured by a complex switching from one modality to the other and back, which requires a high flexibility in tuning to the right one”. Special attention should be turned to the transition between the movements which are ruled by abduction. In these important turning points Arzarello discovered an important difference between a paper and pencil environment and a DGS environment: “While in the former the abductions are produced because of the genuity of the subjects, in Cabri the dragging process can mediate them” (Arzarello et al. 2002, p.67). So both Hölzl and Arzarello emphasise the mediating function of the drag mode.

**Coding of problem solving processes**

Schoenfeld (1985, chapter 9) presents a framework for the analysis of videotaped problem solving processes. Schoenfeld parses the processes into so-called episodes. An episode is “a period of time during which an individual or a problem-solving group is engaged in one large task [...] or a closely related body of tasks in the service of the same goal [...]” (ibid., p. 292).

Schoenfeld differentiates between six categories of episodes: Reading, Analysing, Exploring, Planning, Implementing and Verifying. In his studies 60% of the processes he detected were of a type he called “wild goose chase”, where the problem solvers “read the problem, picked a particular direction to work on, and pursued that direction until they ran out of time” (Schoenfeld 1992, p. 190f.). Rott (2011) operationalises this type of process by the solely appearance of the episode-categories Reading, Analysis and Exploring in any constellation. Rott (ibid., p. 4) also adapts Schoenfelds episode-categories to the phases of the solution process of a problem described by Pólya (1945, p. 5ff.).

**The electronic worksheets**

Introducing dynamic geometry into the classroom initially perturbs the sensible “equilibrium” (Laborde 2001) of the complex teaching system. The teacher has to ensure the reconstitution of a new equilibrium. Amongst the choices that have to be made Laborde states “the interaction between teacher and students: by whom is the technology used and for what purposes? For example, is it to be used to support the discourse of the teacher and to illustrate some points of the curriculum; or is it to support the learning of the students?” (ibid., p. 285)

When designing the electronic worksheets Elschenbroich and Seebach decided themselves for the latter. In their introduction (Elschenbroich & Seebach 2002, p. 5 - 9) they state a range of problems introducing dynamic geometry into mathematical education that have to be met. In addition to the new mathematical topics pupils must learn how to handle the software and gain a software-specific view on geometrical objects and their relations. They have to learn how to construct complex geometrical objects with the software and will often experience that the construction they made does not satisfy the demands of the task. That might lead to frustration, especially if the pupils realize the deficits after a long while of constructing. That is why the electronic worksheets, of which an
example is given below, are ready constructed, so that the students can go straight into exploring the given tasks.

Drag on C.

What do you realize referring to the plotted angles? (Hint: Observe the triangle AMC)

Proceed in the same way with the triangle MBC and the other partial angles at M and C.

What is the consequence from your observations for the magnitude of the whole angle at C?

Figure 1: Example for an electronic worksheet (translated by the author)

Research questions

Arzarello (1998, p. 38) considers four main questions worthwhile studying. Amongst them is the application of the model of ascending and descending control modalities and the switching between them “within other media used to approach geometry” (ibid.). This leads me to my first research question:

Can the described model be applied to processes in which electronic worksheets are used?

Hölzl formulates two research perspectives:

First, it is necessary to find long term arrangements for the integration of DGS in the learning of geometry that enable students to grasp a geometric situation where a drawing that is dragged is not only one drawing which moves and changes but a sequence of different drawings with certain common properties. Second, we need to find ways to help students focus on invariants rather than focus on details which suppress the overall impression of a drawing in its concentration on local relationships between parts of a figure. (2001, p. 84)

Therefore my second question is:

Do the settings in my long-term study provide such an arrangement and will there be positive effects on the students performances?

Many electronic worksheets refer to certain steps of mathematical proofs and thereby guide the students through these proves. Therefore my third question sensu Laborde (2001) is:

Will the students use the software more often as a tool to find reasons for conjectures or to elaborate a proof?

Pilot-study

Design

To gain experience in the ascertainment of data and for the development of suitable instruments for the long term study a pilot study was being carried out. In November 2009 six pairs of students (P01 to P06) were taken from a ninth grade class of a secondary school in Hanover. The students were chosen by their marks in mathematics over the past four years and arranged in pairs of the same gender. They attended four lessons in which they were instructed in the use of the dynamic geometry
software GeoGebra (Hohenwarter, 2002). Afterwards they were given six tasks (T01 to T06), T03 and T04 as well as T05 and T06 were of a similar type. The available media (DGS plus paper & pencil vs. only paper and pencil, abbreviated D, P respectively) were systematically changed within a symmetric crossover-design, which can be seen at a glance in the overview (Table 1):

<table>
<thead>
<tr>
<th></th>
<th>T01</th>
<th>T02</th>
<th>T03</th>
<th>T04</th>
<th>T05</th>
<th>T06</th>
</tr>
</thead>
<tbody>
<tr>
<td>P01</td>
<td>P</td>
<td>D</td>
<td>P</td>
<td>D</td>
<td>D</td>
<td>P</td>
</tr>
<tr>
<td>P02</td>
<td>P</td>
<td>D</td>
<td>D</td>
<td>P</td>
<td>P</td>
<td>D</td>
</tr>
<tr>
<td>P03</td>
<td>P</td>
<td>D</td>
<td>P</td>
<td>D</td>
<td>D</td>
<td>P</td>
</tr>
<tr>
<td>P04</td>
<td>P</td>
<td>D</td>
<td>D</td>
<td>P</td>
<td>P</td>
<td>D</td>
</tr>
<tr>
<td>P05</td>
<td>P</td>
<td>D</td>
<td>P</td>
<td>D</td>
<td>D</td>
<td>P</td>
</tr>
<tr>
<td>P06</td>
<td>P</td>
<td>D</td>
<td>D</td>
<td>P</td>
<td>P</td>
<td>D</td>
</tr>
</tbody>
</table>

Table 1: Design of the pilot study

**Methodology**

All problem solving processes were videotaped. For each task the whole range of possible discoveries and proofs of the given problems was written down and divided up into so called “cognitive units”. The finding of a complete proof is a greater achievement for a student than drawing of an auxiliary element into a given figure. Therefore a complete proof with a sequence of logical steps has to be honoured with a corresponding amount of cognitive units. The number of achieved units were counted for each pupil as a measurement of the quality of the products of the solution process. Processes with an above average assessment were classified “successful”.

The parsing of the processes was carried out by a research assistant and the author, approximately 10% of the processes were coded by both independently with a high reliability.

The author also started to parse the processes into episodes that are determined by the use of the different dragging modalities. Therefore a manual with definitions for the different types of dragging based on the Arzarello categories was written. For this is an ongoing process results have to be stated in a later publication.

**Results**

50 of the 72 observed processes have been parsed so far. 28 of them correspond to the “wild goose chase” type. With a percentage of 56% that is the same order of magnitude Schoenfeld found.

I also examined if an interrelation between the process-type and the quality of the products can be found. 20 processes with the medium DGS have been assessed so far.

<table>
<thead>
<tr>
<th></th>
<th>(below) average</th>
<th>Above average</th>
</tr>
</thead>
<tbody>
<tr>
<td>“wild goose chase”</td>
<td>10 (8)</td>
<td>6 (8)</td>
</tr>
<tr>
<td>miscellaneous</td>
<td>0 (2)</td>
<td>4 (2)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2: Contingency table – process type and product success

A $\chi^2$-test showed a significant correlation between the process type “wild goose chase” and a below average quality of the product of the process ($p=.025 < .05$).

It is also remarkable that the episode type Verification was only found in approximately 10% of the examined processes.
Consequences for the Long-term-study

The main aim of the Pilot-study was to test potential instruments and technology for the Long-term study. Schemes to describe and assess problem solving processes have been tried and optimised. The Schoenfeld scheme turned out to be an appropriate tool to distinguish different types of processes. The type Schoenfeld called “wild goose chase” was strongly recognized in our videos. The results of the Pilot-study indicate a connection of this type with poor achievements of the students. Therefore a decrease of the percentage of this type over the period of the Long-term-study should be an indicator for an improvement of the students achievements. Another result was that the students hardly ever verified their results or solution paths, in Pólyas (1945) terms “they didn’t look back”. For the Long-term-study a special strategy-training for the students will be developed. One indicator of the success of that training will be an increase of the appearance of that episode type.

Long-term-study

Design

Two classes of a secondary school in Hanover will be accompanied over a period of at least two years. During an acquaintance phase in the seventh grade the mathematical abilities of the students, their motivation and other items have been assessed. From the eighth grade on both classes will be taught by the author.

Both classes will intensively use DGS and work with the electronic worksheets, one of the classes will get an extra training in problem solving strategies, symbolized by the treatment-boxes for class 1 in figure 2. The students work mainly in the computer room with permanent access to the DGS. The electronic worksheets are a permanent feature of the lessons. Due to the curriculum of the eighth form the course will start with a geometry unit about circles and inscribed angles followed by an algebraic unit about systems of linear equations. The students will keep record of their intuitions and discoveries as well as their moods and feelings about their work in a learning diary. The preferred method throughout the lessons will be “Think-Pair-Share” (Barzel et al., p. 118ff.) which means that the students start working on their own, discuss the results afterwards with their neighbour and finally present it in front of the whole class. This method triggers communication about the mathematical topics.

Elschenbroich & Seebach (2002) emphasize that “the computer does not make mathematics education speechless, but on the contrary clears a space for a linguistic component” (ibid. p. 1, translated by the author).

Figure 2: Overview of the Long-term-study
Methodology

The units will be beta-tested by the author with students from Leibniz-University. A few of those lessons will be attended by Mr. Elschenbroich, one of the authors of the electric worksheets, who also advises the author during the development phase of the units.

During the implementation in school data will be collected from both classes after the treatment phases, the instruments therefore have been tested throughout the pilot study as described above.

Figure 3: The first unit

Specimen from the unit “circles and inscribed angles”

The unit begins with some exercises on defining, this is a good occasion for speaking about mathematics. In the subsequent lesson the pupils shall discover and proof the Isosceles Triangle Theorem with the support of an electronic worksheet.

This theorem is needed for the proof of Thales’ Theorem in the following lesson. The discovery and proof of this theorem by the students is also assisted by electronic worksheets. The worksheet which is supposed to inspire the finding of the proof of Thales’ Theorem is presented in Figure 1.

The unit continues with the converse of Thales’ theorem. It follows the discovery and proof of the Inscribed Angle Theorem and ends with its converse, each accompanied by electronic worksheets.

Nine electronic worksheets are involved in the unit, six of them were taken from Elschenbroich & Seebach (2002), the remaining ones are designs of the author.

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